

Part 1: Review of Operations with Polynomials

Like terms are terms that have the same **variable** factors. You can simplify expressions containing like terms by adding their coefficients and keeping the variable factors the same.

Example 1: Simplify the following expressions

a) $3x + 4x + 2y$

$$= 7x + 2y$$

b) $(x^2 + 4x - 2) + (2x^2 - 6x + 9)$

$$= x^2 + 4x - 2 + 2x^2 - 6x + 9$$

When adding polynomials you can remove the brackets

$$= x^2 + 2x^2 + 4x - 6x - 2 + 9$$

Group like terms together

$$= 3x^2 - 2x + 7$$

Collect the like terms by adding the coefficients and keeping the variable part the same

c) $2(6m^2 - mn + 4) - (7m^2 + 4mn - 2)$

$$= 12m^2 - 2mn + 8 - 7m^2 - 4mn + 2$$

Start by eliminating brackets using distributive property

$$= 12m^2 - 7m^2 - 2mn - 4mn + 8 + 2$$

Group like terms together

$$= 5m^2 - 6mn + 10$$

Collect like terms

When multiplying/dividing monomials, you must multiply/divide the numerical coefficients and then multiply/divide the variables using exponent rules:

PRODUCT RULE: $(x^a)(x^b) = x^{a+b}$

QUOTIENT RULE: $\frac{x^a}{x^b} = x^{a-b}$

Example 2: Simplify the following expressions

a) $3x(2x^2 - 5) + 4x$

$$= 6x^3 - 15x + 4x$$

$$= 6x^3 - 11x$$

b) $5y(2x + y) - 4(2xy - 3y)$

$$= 10xy + 5y^2 - 8xy + 12y$$

$$= 2xy + 5y^2 + 12y$$

Part 2: Multiplying Binomials

When multiplying binomials, you must multiply each term in the first binomial by each term in the second binomial. The acronym FOIL can be used to remember all four products you need to calculate. FOIL stands for First, Outside, Inside, Last. After multiplying (expanding), make sure to simplify by collecting like terms.

How it works:

$$\begin{aligned} & \begin{array}{c} \text{Outside} \\ \text{First} \\ \text{Inside} \\ \text{Last} \end{array} \\ & (x + 2)(x + 3) \\ & = \underbrace{x^2}_{\text{First}} + \underbrace{3x}_{\text{Outside}} + \underbrace{2x}_{\text{Inside}} + \underbrace{6}_{\text{Last}} \\ & = x^2 + 5x + 6 \end{aligned}$$

Example 3: Expand and simplify each of the following

a) $(x + 4)(x - 5)$

$$\begin{aligned} & = x^2 - 5x + 4x - 20 \\ & = x^2 - x - 20 \end{aligned}$$

b) $(3x + 1)(2x + 7)$

$$\begin{aligned} & = 6x^2 + 21x + 2x + 7 \\ & = 6x^2 + 23x + 7 \end{aligned}$$

c) $3(2x - 1)(2x + 5)$

$$\begin{aligned} & = (6x - 3)(2x + 5) \\ & = 12x^2 + 18x - 6x - 9 \\ & = 12x^2 + 12x - 9 \end{aligned}$$

d) $2(3y + 2)(y - 1) - (y - 2)(2y + 1)$

$$\begin{aligned} & = (6y + 4)(y - 1) - (y - 2)(2y + 1) \\ & = 6y^2 - 6y + 4y - 4 - (2y^2 + y - 4y - 2) \\ & = 6y^2 - 6y + 4y - 4 - 2y^2 - y + 4y + 2 \\ & = 4y^2 + y - 2 \end{aligned}$$

e) $4(x - 2)(x - 6) + 3(x + 3)(3x - 2)$

$$\begin{aligned} & = 4(x^2 - 6x - 2x + 12) + 3(3x^2 - 2x + 9x - 6) \\ & = 4(x^2 - 8x + 12) + 3(3x^2 + 7x - 6) \\ & = 4x^2 - 32x + 48 + 9x^2 + 21x - 18 \\ & = 13x^2 - 11x + 30 \end{aligned}$$

f) $2(3x + 4)^2 - (4x + 5)(4x - 5)$

$$\begin{aligned} & = 2(3x + 4)(3x + 4) - (4x + 5)(4x - 5) \\ & = 2(9x^2 + 12x + 12x + 16) - (16x^2 - 20x + 20x - 25) \\ & = 2(9x^2 + 24x + 16) - (16x^2 - 25) \\ & = 18x^2 + 48x + 32 - 16x^2 + 25 \\ & = 2x^2 + 48x + 57 \end{aligned}$$

Note: this question involves 2 **SPECIAL PRODUCTS** that we will learn more about later. 1) perfect square trinomial and 2) difference of squares

Part 3: Multiplying Polynomials

The general rule when multiplying polynomials is that each term in one of the polynomials must be multiplied by each term in the other polynomial.

Example 4: Expand and simplify each of the following

a) $(x + 2y)(3x - 4y + 5)$

$$\begin{aligned} &= 3x^2 - 4xy + 5x + 6xy - 8y^2 + 10y \\ &= 3x^2 - 8y^2 + 2xy + 5x + 10y \end{aligned}$$

b) $(x^2 - 2x + 1)(x^2 + 5x + 6)$

$$\begin{aligned} &= x^2(x^2 + 5x + 6) - 2x(x^2 + 5x + 6) + 1(x^2 + 5x + 6) \\ &= x^4 + 5x^3 + 6x^2 - 2x^3 - 10x^2 - 12x + x^2 + 5x + 6 \\ &= x^4 + 3x^3 - 3x^2 - 7x + 6 \end{aligned}$$

c) $(4x - 3)^3$

$$\begin{aligned} &= (4x - 3)(4x - 3)(4x - 3) \\ &= (4x - 3)(16x^2 - 12x - 12x + 9) \\ &= (4x - 3)(16x^2 - 24x + 9) \\ &= 64x^3 - 96x^2 + 36x - 48x^2 + 72x - 27 \\ &= 64x^3 - 144x^2 + 108x - 27 \end{aligned}$$

IMPORTANT!

$$(a - b)^3 \neq a^3 - b^3$$