L2 – Common Factoring Unit 3
MPM2D
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Factoring a polynomial is the OPPOSITE of expanding

Expanding → Multiplying

Factoring → Dividing

To factor a polynomial, remove the greatest common factor as the first factor, then **divide** each term by the greatest common factor to obtain the second factor.

A greatest common factor is the greatest number and/or variable that is a factor (divides evenly into) of all terms in a set.

Part 1: Monomial Common Factor

Example 1: Factor each of the following expressions

a) $-5x + 20$

$= -5\left(\frac{-5x + 20}{-5}\right)$

$= -5\left(\frac{-5x}{-5} + \frac{20}{-5}\right)$

$= -5(x - 4)$

b) $8x^2 - 7x$

$= x\left(\frac{8x^2 - 7x}{x}\right)$

$= x\left(\frac{8x^2}{x} - \frac{7x}{x}\right)$

$= x(8x - 7)$

c) $25x^6 + 15x^4$

Note: When common factoring a variable, choose the one with the smallest exponent.

$= 5x^4\left(\frac{25x^6 + 15x^4}{5x^4}\right)$

$= 5x^4(5x^2 + 3)$

d) $21x^4y^3 - 28x^2y^5 + 7xy^3$

$= 7xy^3\left(\frac{21x^4y^3 - 28x^2y^5 + 7xy^3}{7xy^3}\right)$

$= 7xy^3(3x^3 - 4xy^2 + 1)$

e) $4x^2y^3 + 10x^4y^2 - 12x^3y^2$

$= 2x^2y^2\left(\frac{4x^2y^3 + 10x^4y^2 - 12x^3y^2}{2x^2y^2}\right)$

$= 2x^2y^2(2y + 5x^2 - 6x)$

f) $8x^3 - 6x^2y^2 + 4x^2y$

$= 2x^2\left(\frac{8x^3 - 6x^2y^2 + 4x^2y}{2x^2}\right)$

$= 2x^2(4x - 3y^2 + 2y)$

Part 2: Binomial Common Factor and Factoring by Grouping

A greatest common factor is not necessarily a monomial.

Example 2: Factor each of the following expressions

a) $3x(y+1)+7(y+1)$

$= (y+1)\left[\frac{3x(y+1)+7(y+1)}{y+1}\right]$

$= (y+1)(3x+7)$

b) $2x(x-3)-5(x-3)$

$= (x-3)\left[\frac{2x(x-3)-5(x-3)}{x-3}\right]$

$= (x-3)(2x-5)$

c) $5x(x^2+2x+7)-4(x^2+2x+7)$

$= (x^2+2x+7)\left[\frac{5x(x^2+2x+7)-4(x^2+2x+7)}{x^2+2x+7}\right]$

$= (x^2+2x+7)(5x-4)$

Some polynomials do not have a common factor but can be factored by **grouping**. When factoring by grouping:

1) group pairs of terms with a common factor (always separate groups with an addition sign)
2) remove a common factor from each group
3) factor the common binomial (or other type of polynomial) from the expression

Example 3: Factor each of the following expressions

a) $ac+bc+ad+bd$

$= (ac+bc)+(ad+bd)$

$= c(a+b)+d(a+b)$

$= (a+b)(c+d)$

b) $9x^2+15x+3x+5$

$= (9x^2+15x)+(3x+5)$

$= 3x(3x+5)+1(3x+5)$

$= (3x+5)(3x+1)$