L3 – Factor $x^2 + bx + c$ Unit 3

MPM2D

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To learn how to factor trinomials of the form $x^2 + bx + c$, let's study the expansion of $(x + m)(x + n)$

$$(x + m)(x + n) = x^2 + nx + mx + mn$$

$$= x^2 + mx + nx + mn$$

$$= x^2 + (m + n)x + mn$$

Compare the result above to the general expression $x^2 + bx + c$

$$x^2 + (m + n)x + mn$$

$$x^2 + bx + c$$

So to factor $x^2 + bx + c$, you must find the numbers that add to b and multiply to c.

General Rule:

$$x^2 + bx + c = (x + m)(x + n)$$

Where $b = m + n$ and $c = mn$

Example 1: Factor each of the following

a) $x^2 + 7x + 12$

$3 + 4 = 7$

$3 \times 4 = 12$

$= (x+3)(x+4)$

b) $x^2 + 8x + 15$

$3 + 5 = 8$

$3 \times 5 = 15$

$= (x+3)(x+5)$

c) $x^2 - 29x + 28$

$-28 + -1 = -29$

$-28 \times -1 = 28$

$= (x-28)(x-1)$

d) $x^2 + 3x - 18$

$6 + -3 = 3$

$6 \times -3 = -18$

$= (x+6)(x-3)$

e) $2x^2 - 8x - 42$

$= 2(x^2 - 4x - 21)$

$= 2(x-7)(x+3)$

$-7 + 3 = -4$

$-7 \times 3 = -21$

f) $-2x^2 + 8x + 42$

$= -2(x^2 - 4x - 21)$

$= -2(x-7)(x+3)$

g) $x^2 + 11xy + 24y^2$

$= (x+8y)(x+3y)$

$8 + 3 = 11$

$8 \times 3 = 24$

h) $x^2 + 10x + 25$

$= (x+5)(x+5)$

$= (x+5)^2$

$5 + 5 = 10$

$5 \times 5 = 25$

Note: This is a special product called a Perfect Square Trinomial

$(a+b)^2 = a^2 + 2ab + b^2$

i) $x^4 + 4x^2 + 3$

$= (x^2)^2 + 4(x^2) + 3$ Let $k = x^2$

$= k^2 + 4k + 3$

$= (k+3)(k+1)$

$= (x^2+3)(x^2+1)$

$3 + 1 = 4$

$3 \times 1 = 3$