

**W4 – Factor  $ax^2 + bx + c$  where  $a \neq 1$** 

Unit 3

MPM2D

Jensen

1) Factor, if possible.

$$\begin{aligned} \text{a) } & 2x^2 + 7x + 5 \\ & = 2x^2 + 2x + 5x + 5 \\ & = 2x(x+1) + 5(x+1) \\ & = (x+1)(2x+5) \end{aligned}$$

$$\begin{array}{l} \underline{2} \times \underline{5} = 10 \\ \underline{2} + \underline{5} = 7 \end{array}$$

$$\begin{aligned} \text{b) } & 6y^2 + 19y + 8 \\ & = 6y^2 + 16y + 3y + 8 \\ & = 2y(3y+8) + 1(3y+8) \\ & = (3y+8)(2y+1) \end{aligned}$$

$$\begin{array}{l} \underline{16} \times \underline{3} = 48 \\ \underline{16} + \underline{3} = 19 \end{array}$$

$$\begin{aligned} \text{c) } & 4k^2 + 15k + 9 \\ & = 4k^2 + 12k + 3k + 9 \\ & = 4k(k+3) + 3(k+3) \\ & = (k+3)(4k+3) \end{aligned}$$

$$\begin{array}{l} \underline{12} \times \underline{3} = 36 \\ \underline{12} + \underline{3} = 15 \end{array}$$

$$\begin{aligned} \text{d) } & 3m^2 + 10m + 8 \\ & = 3m^2 + 6m + 4m + 8 \\ & = 3m(m+2) + 4(m+2) \\ & = (m+2)(3m+4) \end{aligned}$$

$$\begin{array}{l} \underline{6} \times \underline{4} = 24 \\ \underline{6} + \underline{4} = 10 \end{array}$$

$$\begin{array}{l} \text{e) } 10w^2 + 15w + 3 \\ \text{not factorable} \end{array}$$

$$\begin{array}{l} \underline{\quad} \times \underline{\quad} = 30 \\ \underline{\quad} + \underline{\quad} = 15 \end{array}$$

$$\begin{aligned} \text{f) } & 12q^2 + 17q + 6 \\ & = 12q^2 + 9q + 8q + 6 \\ & = 3q(4q+3) + 2(4q+3) \\ & = (4q+3)(3q+2) \end{aligned}$$

$$\begin{array}{l} \underline{9} \times \underline{8} = 72 \\ \underline{9} + \underline{8} = 17 \end{array}$$

$$\begin{aligned} \text{g) } & 4x^2 - 11x + 6 \\ & = 4x^2 - 8x - 3x + 6 \\ & = 4x(x-2) - 3(x-2) \\ & = (x-2)(4x-3) \end{aligned}$$

$$\begin{array}{l} \underline{-8} \times \underline{-3} = 24 \\ \underline{-8} + \underline{-3} = -11 \end{array}$$

$$\begin{aligned} \text{h) } & 5n^2 - 11n + 6 \\ & = 5n^2 - 6n - 5n + 6 \\ & = n(5n-6) - 1(5n-6) \\ & = (5n-6)(n-1) \end{aligned}$$

$$\begin{array}{l} \underline{-6} \times \underline{-5} = 30 \\ \underline{-6} + \underline{-5} = -11 \end{array}$$

$$\begin{aligned} \text{i) } & 9b^2 - 24b + 7 \\ & = 9b^2 - 21b - 3b + 7 \\ & = 3b(3b-7) - 1(3b-7) \\ & = (3b-7)(3b-1) \end{aligned}$$

$$\begin{array}{l} \underline{-21} \times \underline{-3} = 63 \\ \underline{-21} + \underline{-3} = -24 \end{array}$$

$$\begin{aligned} \text{j) } 3y^2 + 4y - 7 & \quad \boxed{\begin{array}{l} 7 \times -3 = -21 \\ 7 + -3 = 4 \end{array}} \\ & = 3y^2 + 7y - 3y - 7 \\ & = y(3y+7) - 1(3y+7) \\ & = (3y+7)(y-1) \end{aligned}$$

$$\begin{aligned} \text{k) } 8k^2 - 6k - 5 & \quad \boxed{\begin{array}{l} -10 \times 4 = -40 \\ -10 + 4 = -6 \end{array}} \\ & = 8k^2 - 10k + 4k - 5 \\ & = 2k(4k-5) + 1(4k-5) \\ & = (4k-5)(2k+1) \end{aligned}$$

$$\begin{aligned} \text{l) } 5h^2 - 14h - 3 & \quad \boxed{\begin{array}{l} -15 \times 1 = -15 \\ -15 + 1 = -14 \end{array}} \\ & = 5h^2 - 15h + h - 3 \\ & = 5h(h-3) + 1(h-3) \\ & = (h-3)(5h+1) \end{aligned}$$

$$\begin{aligned} \text{m) } 3x^2 + 7xy + 2y^2 & \quad \boxed{\begin{array}{l} 6 \times 1 = 6 \\ 6 + 1 = 7 \end{array}} \\ & = 3x^2 + 6xy + 1xy + 2y^2 \\ & = 3x(x+2y) + y(x+2y) \\ & = (x+2y)(3x+y) \end{aligned}$$

$$\begin{aligned} \text{n) } 2p^2 - 11pq + 5q^2 & \quad \boxed{\begin{array}{l} -10 \times -1 = 10 \\ -10 + -1 = -11 \end{array}} \\ & = 2p^2 - 10pq - 1pq + 5q^2 \\ & = 2p(p-5q) - q(p-5q) \\ & = (p-5q)(2p-q) \end{aligned}$$

$$\begin{aligned} \text{o) } 9x^2 - 9xy - 4y^2 & \quad \boxed{\begin{array}{l} -12 \times 3 = -36 \\ -12 + 3 = -9 \end{array}} \\ & = 9x^2 - 12xy + 3xy - 4y^2 \\ & = 3x(3x-4y) + y(3x-4y) \\ & = (3x-4y)(3x+y) \end{aligned}$$

$$\begin{aligned} \text{p) } 8k^2 - 16k + 6 & \quad \boxed{\begin{array}{l} -6 \times -2 = 12 \\ -6 + -2 = -8 \end{array}} \\ & = 2(4k^2 - 8k + 3) \\ & = 2(4k^2 - 6k - 2k + 3) \\ & = 2[2k(2k-3) - 1(2k-3)] \\ & = 2(2k-3)(2k-1) \end{aligned}$$

$$\begin{aligned} \text{q) } 6m^2 - 14m - 12 & \quad \boxed{\begin{array}{l} -9 \times 2 = -18 \\ -9 + 2 = -7 \end{array}} \\ & = 2(3m^2 - 7m - 6) \\ & = 2(3m^2 - 9m + 2m - 6) \\ & = 2[3m(m-3) + 2(m-3)] \\ & = 2(m-3)(3m+2) \end{aligned}$$

$$\begin{aligned} \text{r) } 10r^2 - 22r + 4 & \quad \boxed{\begin{array}{l} -10 \times -1 = 10 \\ -10 + -1 = -11 \end{array}} \\ & = 2(5r^2 - 11r + 2) \\ & = 2(5r^2 - 10r - 1r + 2) \\ & = 2[5r(r-2) - 1(r-2)] \\ & = 2(r-2)(5r-1) \end{aligned}$$

$$\text{s) } 2x^3 + 9x^2 + 4x$$

$$= x(2x^2 + 9x + 4)$$

$$= x(2x^2 + 8x + 1x + 4)$$

$$= x[2x(x+4) + 1(x+4)]$$

$$= x(x+4)(2x+1)$$

$$\begin{array}{l} \underline{8} \times \underline{1} = 8 \\ \underline{8} + \underline{1} = 9 \end{array}$$

$$\text{t) } 5x^2y - 7xy + 2y$$

$$= y(5x^2 - 7x + 2)$$

$$= y(5x^2 - 2x - 5x + 2)$$

$$= y[x(5x-2) - 1(5x-2)]$$

$$= y(5x-2)(x-1)$$

$$\begin{array}{l} \underline{-2} \times \underline{-5} = 10 \\ \underline{-2} + \underline{-5} = -7 \end{array}$$

2) A rectangle has area defined by  $6x^2 + 13x - 8$ .

a) Factor to find algebraic expressions for the length and width

$$A = 6x^2 + 13x - 8$$

$$= 2x(3x+8) - 1(3x+8)$$

$$= (3x+8)(2x-1)$$

$$\text{Length} = 3x+8$$

$$\text{Width} = 2x-1$$

$$\begin{array}{l} \underline{16} \times \underline{-3} = -48 \\ \underline{16} + \underline{-3} = 13 \end{array}$$

Width?

Area is  
 $6x^2 + 13x - 8$

Length?

b) If  $x = 10$  cm, what is the perimeter and area of the rectangle?

$$P = 2l + 2w$$

$$P = 2(3x+8) + 2(2x-1)$$

$$P = 2[3(10)+8] + 2[2(10)-1]$$

$$P = 2(38) + 2(19)$$

$$P = 114 \text{ cm}$$

$$A = lw$$

$$A = (3x+8)(2x-1)$$

$$A = [3(10)+8][2(10)-1]$$

$$A = (38)(19)$$

$$A = 722 \text{ cm}^2$$

### Answers

1) a)  $(2x+5)(x+1)$  b)  $(3y+8)(2y+1)$  c)  $(4k+3)(k+3)$  d)  $(3m+4)(m+2)$  e) not possible f)  $(3q+2)(4q+3)$

g)  $(x-2)(4x-3)$  h)  $5n-6)(n-1)$  i)  $(3b-1)(3b-7)$  j)  $(3y+7)(y-1)$  k)  $(2k+1)(4k-5)$  l)  $(5h+1)(h-3)$

m)  $(3x+y)(x+2y)$  n)  $(2p-q)(p-5q)$  o)  $(3x+y)(3x-4y)$  p)  $2(2k-1)(2k-3)$  q)  $2(3m+2)(m-3)$

r)  $2(5r-1)(r-2)$  s)  $x(2x+1)(x+4)$  t)  $y(5x-2)(x-1)$

2) length is  $3x+8$ ; width is  $2x-1$  b)  $P = 114 \text{ cm}$ ;  $A = 722 \text{ cm}^2$