W5 – Special Products Unit 3
MPM2D
Jensen

1) Expand

a) $(x-4)(x+4)$

$= x^2 - (4)^2$

$= x^2 - 16$

b) $(3y-2)(3y+2)$

$= (3y)^2 - (2)^2$

$= 9y^2 - 4$

c) $(5x-1)(5x+1)$

$= (5x)^2 - (1)^2$

$= 25x^2 - 1$

d) $(x+4)^2$

$= (x+4)(x+4)$

$= x^2 + 4x + 4x + 16$

$= x^2 + 8x + 16$

e) $(3x+2)^2$

$= (3x+2)(3x+2)$

$= 9x^2 + 6x + 6x + 4$

$= 9x^2 + 12x + 4$

f) $(3x+7y)^2$

$= (3x+7y)(3x+7y)$

$= 9x^2 + 21xy + 21xy + 49y^2$

$= 9x^2 + 42xy + 49y^2$

2) Factor

a) $x^2 - 25$

$= (x)^2 - (5)^2$

$= (x-5)(x+5)$

b) $y^2 - 49$

$= (y)^2 - (7)^2$

$= (y-7)(y+7)$

c) $9k^2 - 1$

$= (3k)^2 - (1)^2$

$= (3k-1)(3k+1)$

d) $16k^2 - 49$

$= (4k)^2 - (7)^2$

$= (4k-7)(4k+7)$

e) $25w^2 - 36$

$= (5w)^2 - (6)^2$

$= (5w-6)(5w+6)$

f) $4 - 9w^2$

$= (2)^2 - (3w)^2$

$= (2-3w)(2+3w)$

3) Factor

a) $x^2 - y^2$

$= (x-y)(x+y)$

b) $36x^2 - y^2$

$= (6x)^2 - (y)^2$

$= (6x-y)(6x+y)$

c) $25r^2 - 36s^2$

$= (5r)^2 - (6s)^2$

$= (5r-6s)(5r+6s)$

d) $144r^2 - 49s^2$

$= (12r)^2 - (7s)^2$

$= (12r-7s)(12r+7s)$

e) $121x^2 - 9y^2$

$= (11x)^2 - (3y)^2$

$= (11x-3y)(11x+3y)$

f) $100r^2 - 81s^2$

$= (10r)^2 - (9s)^2$

$= (10r-9s)(10r+9s)$

4) Factor

a) $x^2 + 14x + 49$

$\underline{7} \times \underline{7} = 49$

$\underline{7} + \underline{7} = 14$

$= (x+7)(x+7)$

$= (x+7)^2$

b) $x^2 - 6x + 9$

$\underline{-3} \times \underline{-3} = 9$

$\underline{-3} + \underline{-3} = -6$

$= (x-3)(x-3)$

$= (x-3)^2$

c) $x^2 - 8x + 16$

$\underline{-4} \times \underline{-4} = 16$

$\underline{-4} + \underline{-4} = -8$

$= (x-4)(x-4)$

$= (x-4)^2$

d) $100 - 20x + x^2$

$\underline{-10} \times \underline{-10} = 100$

$\underline{-10} + \underline{-10} = -20$

$= x^2 - 20x + 100$

$= (x-10)(x-10)$

$= (x-10)^2$

e) $4x^2 - 12xy + 9y^2$

$\underline{-6} \times \underline{-6} = 36$

$\underline{-6} + \underline{-6} = -12$

$= 4x^2 - 6xy - 6xy + 9y^2$

$= 2x(2x-3y) - 3y(2x-3y)$

$= (2x-3y)(2x-3y)$

$= (2x-3y)^2$

f) $49x^2 + 56xy + 16y^2$

$\underline{28} \times \underline{28} = 784$

$\underline{28} + \underline{28} = 56$

$= 49x^2 + 28xy + 28xy + 16y^2$

$= 7x(7x+4y) + 4y(7x+4y)$

$= (7x+4y)(7x+4y)$

$= (7x+4y)^2$

5) Factor if possible

a) $2a^2 + 12a + 18$

$\underline{3} \times \underline{3} = 9$

$\underline{3} + \underline{3} = 6$

$= 2(a^2+6a+9)$

$= 2(a+3)(a+3)$

$= 2(a+3)^2$

b) $25x^2 - 16y$

NOT Factorable

c) $75x^2 + 210xy + 147y^2$

$= 3(25x^2 + 70xy + 49y^2)$

$\underline{35} \times \underline{35} = 1225$

$\underline{35} + \underline{35} = 70$

$= 3(25x^2 + 35xy + 35xy + 49y^2)$

$= 3[5x(5x+7y) + 7y(5x+7y)]$

$= 3(5x+7y)(5x+7y)$

$= 3(5x+7y)^2$

d) $9x^3y - 16xy^3$

$= xy(9x^2 - 16y^2)$

$= xy[(3x)^2 - (4y)^2]$

$= xy(3x-4y)(3x+4y)$

e) $36m^2 - 96mn + 64n^2$

$-12 \times -12 = 144$

$-12 + -12 = -24$

$= 4(9m^2 - 24mn + 16n^2)$

$= 4(9m^2 - 12mn - 12mn + 16n^2)$

$= 4[3m(3m-4n) - 4n(3m-4n)]$

$= 4(3m-4n)(3m-4n)$

$= 4(3m-4n)^2$

f) $20x^2 + 20xy + 5y^2$

$2 \times 2 = 4$

$2 + 2 = 4$

$= 5(4x^2 + 4xy + y^2)$

$= 5(4x^2 + 2xy + 2xy + y^2)$

$= 5[2x(2x+y) + y(2x+y)]$

$= 5(2x+y)(2x+y)$

$= 5(2x+y)^2$

6) Determine the value(s) of k such that each trinomial is a perfect square.

a) $x^2 + kx + 16$

$= x^2 + kx + (4)^2$

$kx = 2(x)(4)$

$kx = 8x$

$k = \pm 8$

b) $9x^2 + kx + 49$

$= (3x)^2 + kx + (7)^2$

$kx = 2(3x)(7)$

$kx = 42x$

$k = \pm 42$

c) $x^2 + 4x + k$

$4x = 2(\sqrt{x^2})(\sqrt{k})$

$4x = 2x\sqrt{k}$

$2 = \sqrt{k}$

$k = 4$

d) $4x^2 - 12x + k$

$= (2x)^2 - 12x + (\sqrt{k})^2$

$-12x = 2(2x)(\sqrt{k})$

$-12x = 4x\sqrt{k}$

$-3 = \sqrt{k}$

$k = 9$

e) $kx^2 + 40x + 16$

$= (x\sqrt{k})^2 + 40x + (4)^2$

$2(x\sqrt{k})(4) = 40x$

$8\sqrt{k} = 40$

$\sqrt{k} = 5$

$k = 25$

f) $kx^2 - 24xy + 9y^2$

$= (x\sqrt{k})^2 - 24xy + (3y)^2$

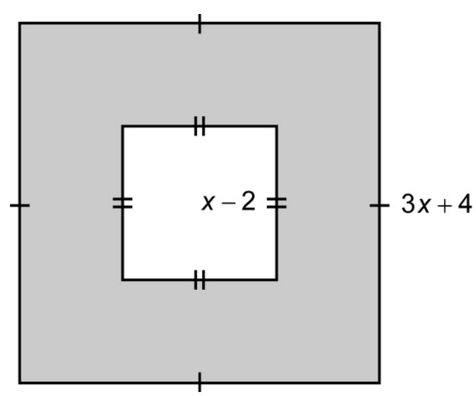
$2(x\sqrt{k})(3y) = -24xy$

$6\sqrt{k} = -24$

$\sqrt{k} = -4$

$k = 16$

7) Find an algebraic expression for the area of the shaded region in factored form



$A = (3x+4)^2 - (x-2)^2$

$= 9x^2 + 24x + 16 - (x^2 - 4x + 4)$

$= 9x^2 + 24x + 16 - x^2 + 4x - 4$

$= 8x^2 + 28x + 12$

$= 4(2x^2 + 7x + 3)$

$= 4(2x^2 + 6x + x + 3)$

$= 4[2x(x+3) + 1(x+3)]$

$= 4(x+3)(2x+1)$

8) The area of a volleyball court can be represented by the trinomial $2x^2 - 4x + 2$.

$-1 \times -1 = 1$
$-1 + -1 = -2$

a) Factor the trinomial completely

$A = 2(x^2 - 2x + 1)$
$= 2(x^2 - x - x + 1)$
$= 2[x(x-1) - 1(x-1)]$
$= 2(x-1)^2$

b) If the length of the court is twice the width, use the factors to write the expressions that represent the length and width.

$A = 2(x-1)(x-1)$

length $= 2(x-1)$

width $= x-1$

c) If $x = 10$ m, what are the length and width of the court.

length $= 2(10-1)$
$= 2(9)$
$= 18$m

width $= 10-1$
$= 9$m

Answers

1)a) $x^2 - 16$ **b)** $9y^2 - 4$ **c)** $25x^2 - 1$ **d)** $x^2 + 8x + 16$ **e)** $9x^2 + 12x + 4$ **f)** $9x^2 + 42xy + 49y^2$
2)a) $(x-5)(x+5)$ **b)** $(y-7)(y+7)$ **c)** $(3k-1)(3k+1)$ **d)** $(4k-7)(4k+7)$ **e)** $(5w-6)(5w+6)$ **f)** $(2-3w)(2+3w)$
3)a) $(x-y)(x+y)$ **b)** $(6x-y)(6x+y)$ **c)** $(5r-6s)(5r+6s)$ **d)** $(12r-7s)(12r+7s)$ **e)** $(11x-3y)(11x+3y)$
f) $(10r-9s)(10r+9s)$
4)a) $(x+7)^2$ **b)** $(x-3)^2$ **c)** $(x-4)^2$ **d)** $(10-x)^2$ **e)** $(2x-3y)^2$ **f)** $(7x+4y)^2$
5)a) $2(a+3)^2$ **b)** not possible **c)** $3(5x+7y)^2$ **d)** $xy(3x-4y)(3x+4y)$ **e)** $4(3m-4n)^2$ **f)** $5(2x+y)^2$
6)a) ± 8 **b)** ± 42 **c)** 4 **d)** 9 **e)** 25 **f)** 16
7) $A = 4(x+3)(2x+1)$
8)a) $2(x-1)^2$ **b)** $2(x-1)$, $x-1$ **c)** 18m by 9m