<mark>L2 –Quadratics in Vertex Form</mark> MPM2D

Jensen

Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x - h)^2 + k$

Factored Form: y = a(x - r)(x - s)

Part 1: Effects of a, h, and k on transforming the graph of $y = x^2$

The effects of the *k* parameter on the graph of $y = x^2 + k$

| Function | Graph | Vertex | Axis of Symmetry | Transformations |
|---------------|-------|--------|---------------------|-----------------|
| $y = x^2 + 3$ | | | | |
| $y = x^2 - 2$ | | | | |

The effects of the *h* parameter on the graph of $y = (x - h)^2$

| Function | Graph | Vertex | Axis of Symmetry | Transformations |
|-----------------|-------|--------|---------------------|-----------------|
| $y = (x - 2)^2$ | | | | |
| $y = (x+3)^2$ | | | | |

| Function | Graph | Vertex | Axis of Symmetry | Transformations |
|-----------------------|-------|--------|---------------------|-----------------|
| $y = 2x^2$ | | | | |
| $y = \frac{1}{2}x^2$ | | | | |
| $y = -\frac{1}{2}x^2$ | | | | |

The effects of the *a* parameter on the graph of $y = ax^2$



Example 1: For each of the following functions, i) describe the transformations compared to $y = x^2$, ii) complete the table of properties, iii) graph the function by making a table of values

a) $y = -3(x+2)^2$

Transformations:

| Vertex | |
|------------------|--|
| Axis of Symmetry | |
| Direction of | |
| Opening | |
| Values x may | |
| take (domain) | |
| Values y may | |
| take (range) | |

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b)
$$y = 2x^2 - 5$$

Transformations:

| Vertex | |
|------------------|--|
| Axis of Symmetry | |
| Direction of | |
| Opening | |
| Values x may | |
| take (domain) | |
| Values y may | |
| take (range) | |

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| -9 | -8 -7 | 7 -6 | -5 | -4 | 1 -1 | 3 -: | 2 - | 1 0 | | 1 : | 2 : | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| -9 | -8 -7 | 7 -6 | -5 | -4 | 1 - 4 | 3 -: | 2 - | 1 0 1 2 3 | | 1 : | 2 : | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| -9 | -8 -7 | 7 -6 | -5 | -4 | | 3 -: | 2 - | 1 0 | | 1 : | 2 : | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| -9 | -8 -7 | 7 -6 | -5 | -4 | | 3 -; | 2 - | 1 0 1 2 3 4 5 | | 1 ; | 2 : | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| -9 | -8 - | 7 -6 | -5 | -4 | | 3 -; | 2 - | 1 0 | | 1 : | 2 : | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| -9 | -8 - | 7 -6 | -5 | | | 3 -: | 2 - | 1 0 1- 2- 3- 4- 5- 6- 6- | | 1 : | 2 : | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| -9 | -8 -7 | 7 -6 | -5 | | | 3 -: | 2 - | 1 0 1- 2- 3- 4- 5- 6- 7- | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
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c) $y = 2(x-3)^2 + 1$

Transformations:

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| | | | | | | | | | 6 | I | | | | | | | | | L |
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Example 2: Determine the vertex form equation of the parabola with its vertex at (1,5) and passes through the point (0,2)

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Example 3: Determine the vertex form equation of the following parabolas



Example 4: The graph of $y = x^2$ is reflected vertically in the *x*-axis, compressed vertically by a factor of $\frac{1}{4}$, shifted 1 unit to the left, and 2 units down. Write the vertex form equation of this parabola.

Example 5: At a fireworks display, a firework is launched from a height of 2 meters above the ground and reaches a max height of 40 meters at a horizontal distance of 10 meters. The firework continues to travel an additional 1 meter horizontally after it reaches its max height before it explodes. What is the height when it explodes?