L2 –Quadratics in Vertex Form Unit 4
MPM2D
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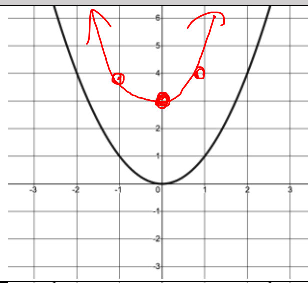
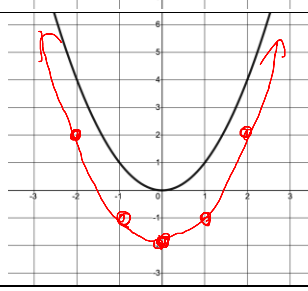
Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x - h)^2 + k$

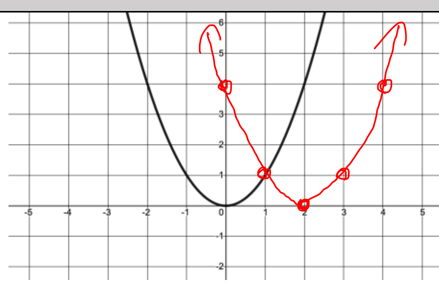
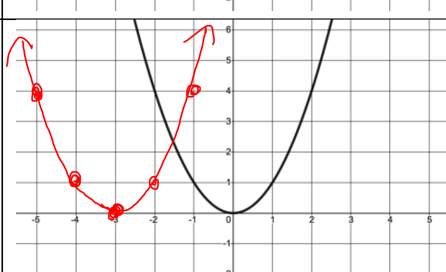
Factored Form: $y = a(x - r)(x - s)$

Part 1: Effects of a, h, and k on transforming the graph of $y = x^2$

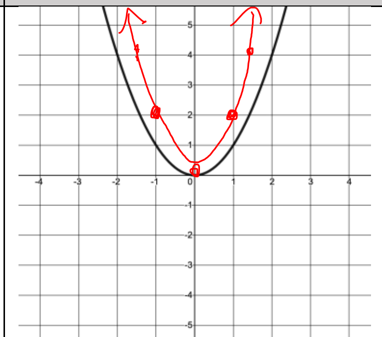
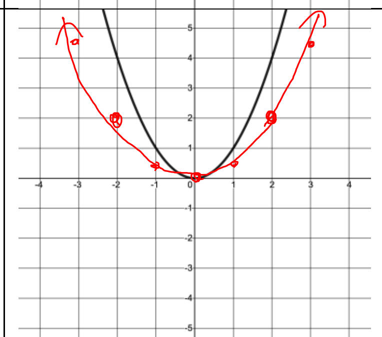
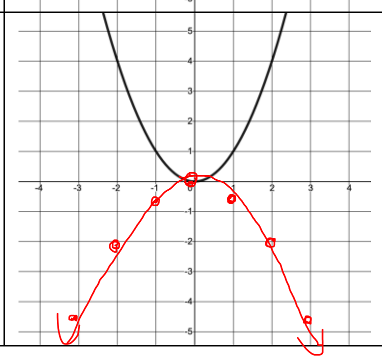
The effects of the k parameter on the graph of $y = x^2 + k$

Function	Graph	Vertex	Axis of Symmetry	Transformations
$y = x^2 + 3$		$(0, 3)$	$x = 0$	shift up 3 units
$y = x^2 - 2$		$(0, -2)$	$x = 0$	shift down 2 units

The effects of the h parameter on the graph of $y = (x - h)^2$

Function	Graph	Vertex	Axis of Symmetry	Transformations
$y = (x - 2)^2$		$(2, 0)$	$x = 2$	shift Right 2 units
$y = (x + 3)^2$		$(-3, 0)$	$x = -3$	shift Left 3 units

The effects of the a parameter on the graph of $y = ax^2$

Function	Graph	Vertex	Axis of Symmetry	Transformations
$y = 2x^2$		$(0,0)$	$x=0$	Vertical Stretch by a factor of 2.
$y = \frac{1}{2}x^2$		$(0,0)$	$x=0$	Vertical Compression by a factor of $\frac{1}{2}$
$y = -\frac{1}{2}x^2$		$(0,0)$	$x=0$	vertical compression by a factor of $\frac{1}{2}$ AND a vertical reflection.

Properties of $y = a(x - h)^2 + k$

$a > 0$ → opens up
$a < 0$ → opens down; vertical reflection in the x-axis
$a > 1$ or $a < -1$ → vertical stretch by a factor of $|a|$
$-1 < a < 1$ → vertical compression by a factor of $|a|$

$h > 0$ → shift RIGHT h units
$h < 0$ → shift LEFT $|h|$ units

$k > 0$ → shift up k units
$k < 0$ → shift down $|k|$ units

Vertex is at (h, k)

Axis of symmetry is at $x = h$

The domain (values x may take) of all quadratic functions is $X \in \mathbb{R}$

The range (values y may take) depends on the location of the vertex

Example 1: For each of the following functions, **i)** describe the transformations compared to $y = x^2$, **ii)** complete the table of properties, **iii)** graph the function by making a table of values

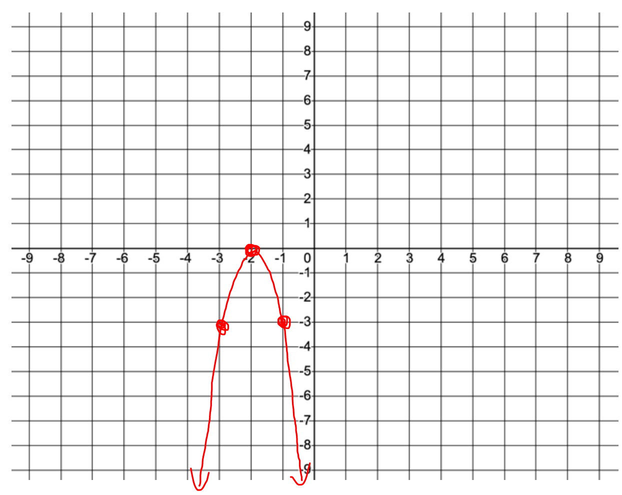
a) $y = -3(x + 2)^2$

Transformations:

- vertical stretch by a factor of 3
- vertical reflection
- shift left 2 units

Vertex	$(-2, 0)$
Axis of Symmetry	$x = -2$
Direction of Opening	down
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{y \in \mathbb{R} \mid y \leq 0\}$

x	y
-4	-12
-3	-3
-2	0
-1	-3
0	-12



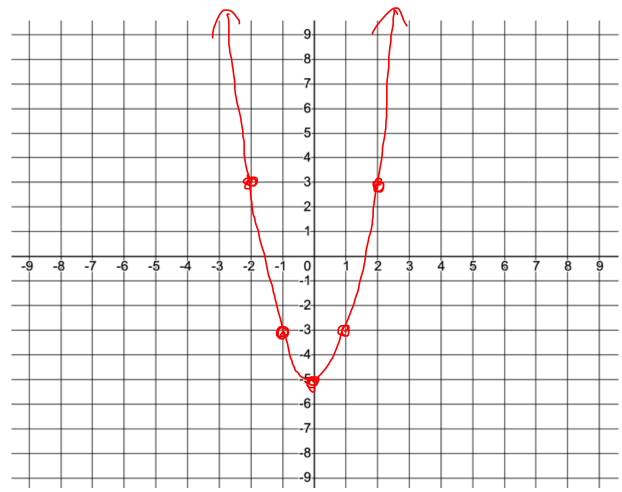
b) $y = 2x^2 - 5$ $= 2(x-0)^2 - 5$

Transformations:

- vertical stretch by a factor of 2
- shift down 5 units

Vertex	(0,-5)
Axis of Symmetry	x=0
Direction of Opening	up
Values x may take (domain)	{X∈ℝ}
Values y may take (range)	{Y∈ℝ / y≥-5}

x	y
-2	3
-1	-3
0	-5
1	-3
2	3



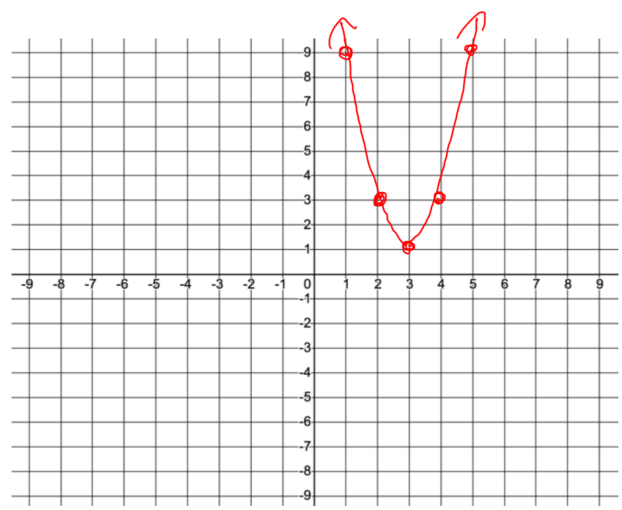
c) $y = 2(x-3)^2 + 1$

Transformations:

- vertical stretch by a factor of 2
- shift right 3 units
- shift up 1 unit

Vertex	(3,1)
Axis of Symmetry	x=3
Direction of Opening	up
Values x may take (domain)	{X∈ℝ}
Values y may take (range)	{Y∈ℝ / y≥1}

x	y
1	9
2	3
3	1
4	3
5	9



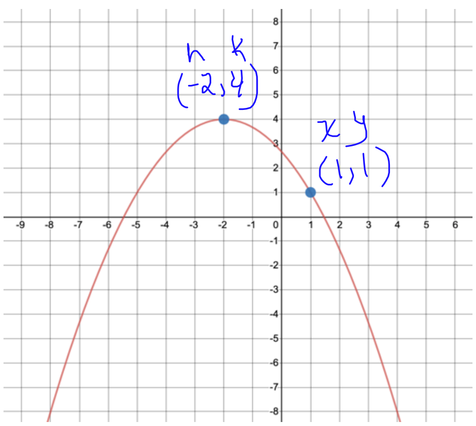
Example 2: Determine the vertex form equation of the parabola with its vertex at $(\overset{h}{1},\overset{k}{5})$ and passes through the point $(\underset{x}{0},\underset{y}{2})$

$$y = a(x-h)^2 + k$$
$$2 = a(0-1)^2 + 5$$
$$2 = a(1) + 5$$
$$2 - 5 = a$$
$$a = -3$$

$$\boxed{y = -3(x-1)^2 + 5}$$

Example 3: Determine the vertex form equation of the following parabolas

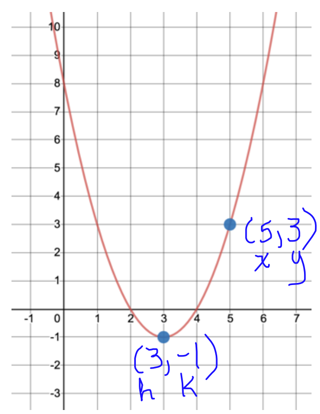
a)



$$y = a(x-h)^2 + k$$
$$1 = a[1-(-2)]^2 + 4$$
$$1 = a(3)^2 + 4$$
$$1 = 9a + 4$$
$$1 - 4 = 9a$$
$$-3 = 9a$$
$$\frac{-3}{9} = a$$
$$a = \frac{-1}{3}$$

$$\boxed{y = -\frac{1}{3}(x+2)^2 + 4}$$

b)



$$y = a(x-h)^2 + k$$
$$3 = a(5-3)^2 + (-1)$$
$$3 = a(4) - 1$$
$$3 + 1 = 4a$$
$$4 = 4a$$
$$a = \frac{4}{4}$$
$$a = 1$$

$$\boxed{y = (x-3)^2 - 1}$$

Example 4: The graph of $y = x^2$ is reflected vertically in the x-axis, compressed vertically by a factor of $\frac{1}{4}$, shifted 1 unit to the left, and 2 units down. Write the vertex form equation of this parabola.

$a = -\frac{1}{4}$

$h = -1$

$k = -2$

$y = a(x-h)^2 + k$

$$\boxed{y = \frac{-1}{4}(x+1)^2 - 2}$$

Example 5: At a fireworks display, a firework is launched from a height of 2 meters above the ground and reaches a max height of 40 meters at a horizontal distance of 10 meters. The firework continues to travel an additional 1 meter horizontally after it reaches its max height before it explodes. What is the height when it explodes?

vertex: $(\overset{h}{10}, \overset{k}{40})$

y-int: $(\underset{x}{0}, \underset{y}{2})$

$y = a(x-h)^2 + k$

$2 = a(0-10)^2 + 40$

$2 = a(100) + 40$

$-38 = 100a$

$a = \frac{-38}{100}$

$a = \frac{-19}{50}$

$y = \frac{-19}{50}(x-10)^2 + 40$

calculate height when $x = 11$

$y = \frac{-19}{50}(11-10)^2 + 40$

$y = \frac{-19}{50} + \frac{2000}{50}$

$y = \frac{1981}{50}$

$\boxed{\text{The height is 39.62 m when it explodes.}}$