L3 –Completing the Square Unit 4
MPM2D
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Part 1: Perfect Square Trinomials

Completing the square is a process for changing a standard form quadratic equation into vertex form

$$y = ax^2 + bx + c \rightarrow y = a(x-h)^2 + k$$

Notice that vertex form contains a $(x-h)^2$. A binomial squared can be obtained when factoring a perfect square trinomial:

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

The process of completing the square involves creating this perfect square trinomial within the standard form equation so that it can be factored to create the vertex form equation.

Let's start by analyzing the following perfect square trinomials. Specifically notice how the middle term is 2 times the product of the square roots of the first and last terms.

$x^2 + 10x + 25$

$x^2 - 12x + 36$

Example 1: Determine the value of k that would make each quadratic a perfect square trinomial. Then factor the trinomial.

a) $x^2 + 14x + k$

b) $x^2 - 24x + k$

Tip: You can calculate the constant term that makes the quadratic a PST by squaring half of the coefficient of the x term.

Note: *this only works when the coefficient of x^2 is 1.*

Part 2: Completing the Square

Completing the Square Steps

$$ax^2 + bx + c \rightarrow a(x-h)^2 + k$$

1) Put brackets around the first 2 terms
2) Factor out the constant in front of the x^2 term
3) Look at the last term in the brackets, divide it by 2 and then square it
4) Add AND subtract that term behind the last term in the brackets
5) Move the negative term outside the brackets by multiplying it by the $'a'$ value
6) Simplify the terms outside the brackets
7) Factor the perfect square trinomial

$$a^2 + 2ab + b^2 = (a+b)^2$$

Example 2: Rewrite each quadratic in vertex form by completing the square. Then state the vertex, whether it is a max or min point, and the axis of symmetry.

a) $y = x^2 + 2x + 7$

b) $y = 5x^2 - 30x + 41$

c) $y = -5x^2 + 20x + 2$

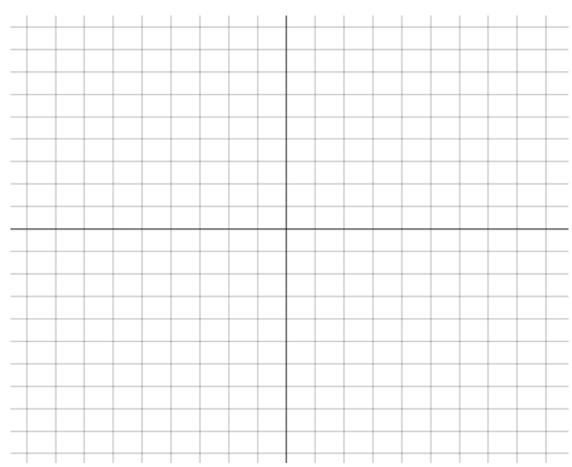
d) $y = 3x^2 + 8x - 5$

Example 3: For each of the following functions, **i)** convert to vertex form by completing the square, **ii)** complete the table of properties, **iii)** graph the function by making a table of values

a) $y = x^2 + 6x + 8$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

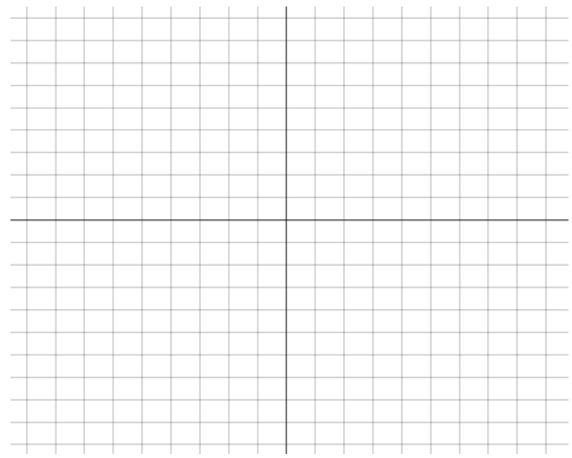
x	y



b) $y = 3x^2 - 24x + 11$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

x	y



c) $y = 6x - 3x^2$

Vertex	
Axis of Symmetry	
Direction of Opening	
Values x may take (domain)	
Values y may take (range)	

x	y