L4 –Quadratics in Factored Form Unit 4
MPM2D
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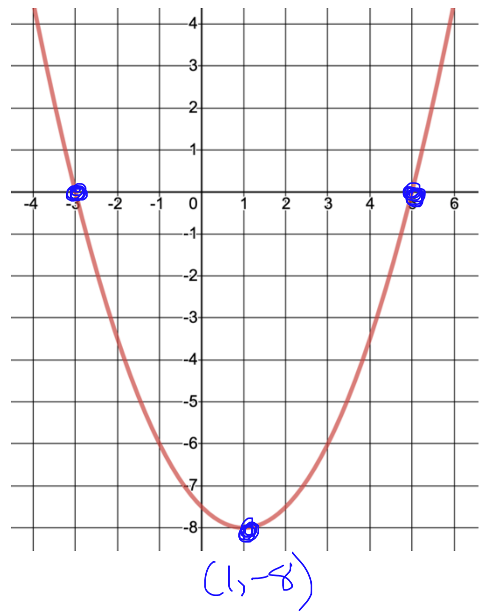
Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x-h)^2 + k$

Factored Form: $y = a(x-r)(x-s)$

Part 1: Analysis of a Quadratic in Factored Form

Example 1: Given the graph of $y = 2(x+3)(x-5)$



a) What are the x-intercepts and how do they relate to the equation?

x-int: $x = -3, 5$

They are the zeros of the equation.

b) What is the vertex? How does the x-coordinate of the vertex relate to the x-intercepts?

Vertex: $(1, -8)$

The x-coordinate of the vertex is the average of the x-intercepts.

c) What is the equation of the axis of symmetry?

$x = 1$

d) What is direction of opening?

up

Properties of $y = a(x-r)(x-s)$

- x-intercepts at r and s (the values of x that make each factor equal 0)
- $a > 0$, opens up
- $a < 0$, opens down
- Axis of symmetry at $y = \frac{r+s}{2}$
- The x-coordinate of the vertex is the average of the x-intercepts $\left(\frac{r+s}{2}\right)$

Example 1: Given the following quadratic equations, determine the **i)** x-intercepts using the zero product rule, **ii)** the axis of symmetry, **iii)** the vertex **iv)** graph the quadratic

> **Zero product rule:** The product of factors is zero if one or more of the factors are zero.
>
> $ab = 0$ if $a = 0$ or $b = 0$ (or both)

a) $y = 2(x+1)(x-3)$

i) $0 = 2(x+1)(x-3)$

$x+1=0 \quad x-3=0$

$x=-1 \quad x=3$

$(-1,0) \quad (3,0)$

ii) aos: $x = \frac{-1+3}{2}$

$x = 1$

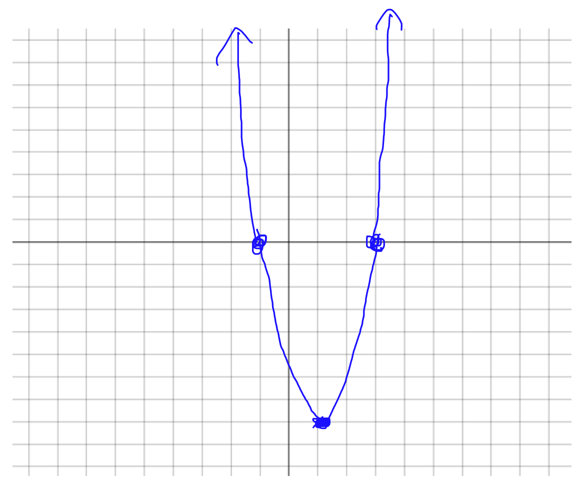
iii) x-vertex $= 1$

y-vertex $= 2(1+1)(1-3)$

$= 2(2)(-2)$

$= -8$

$(1,-8)$



b) $y = \frac{1}{2}(x+6)(x+2)$

i) $0 = \frac{1}{2}(x+6)(x+2)$

$x+6=0 \quad x+2=0$

$x=-6 \quad x=-2$

$(-6,0) \quad (-2,0)$

ii) aos: $x = \frac{-6+(-2)}{2}$

$x = -4$

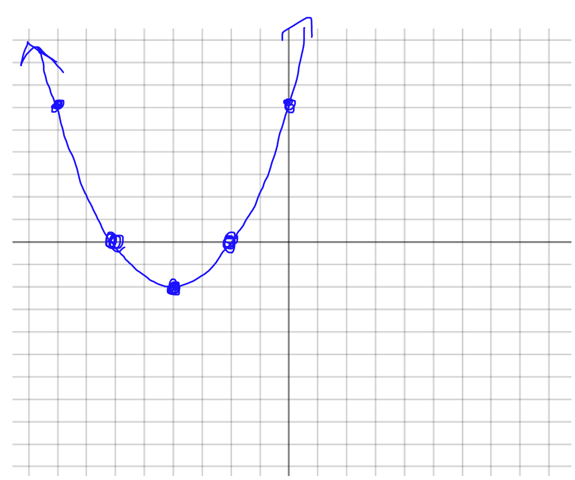
iii) x-vertex $= -4$

y-vertex $= \frac{1}{2}(-4+6)(-4+2)$

$= \frac{1}{2}(2)(-2)$

$= -2$

$(-4,-2)$



c) $y = x^2 + 2x - 8$

$\frac{4}{4} \; \frac{x}{+} \; \frac{-2}{-2} \; = -8$
$= 2$

Note: Factor the standard form quadratic in to factored form so that you can more easily find the x-intercepts.

i) $0 = (x+4)(x-2)$

$x+4=0 \quad x-2=0$

$x=-4 \quad x=2$

$(-4,0) \quad (2,0)$

ii) aos: $x = \frac{-4+2}{2}$

$x=-1$

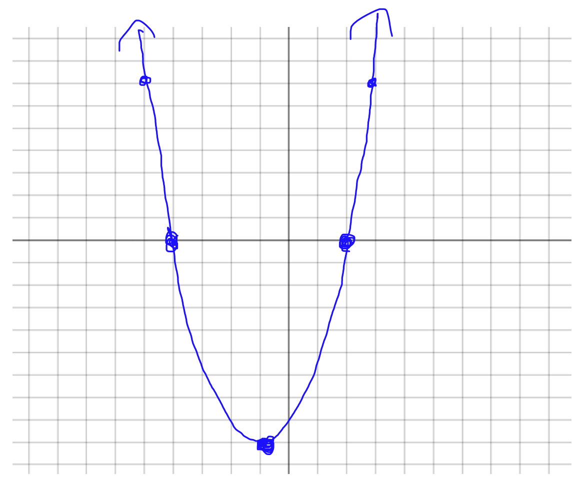
iii) x-~~vertex~~ $= -1$

y-vertex $= (-1)^2 + 2(-1) - 8$

$= 1-2-8$

$= -9$

$(-1,-9)$



d) $y = x^2 - 9$

i) $0 = (x)^2 - (3)^2$

$0 = (x-3)(x+3)$

$x-3=0 \quad x+3=0$

$x=3 \quad x=-3$

$(3,0) \quad (-3,0)$

ii) aos: $x = \frac{3+(-3)}{2}$

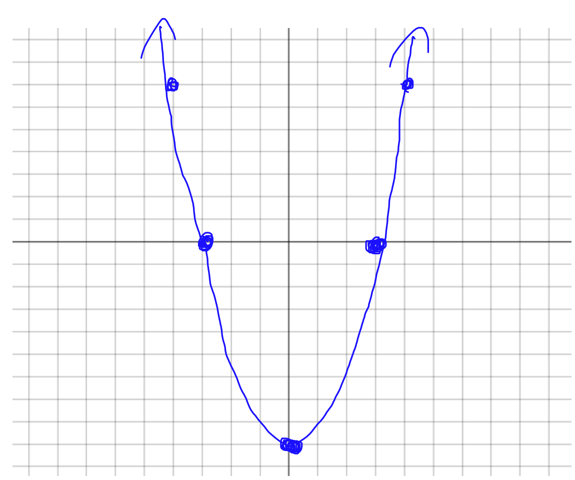
$x=0$

iii) x-~~vertex~~ $= 0$

y-vertex $= (0)^2 - 9$

$= -9$

$(0,-9)$

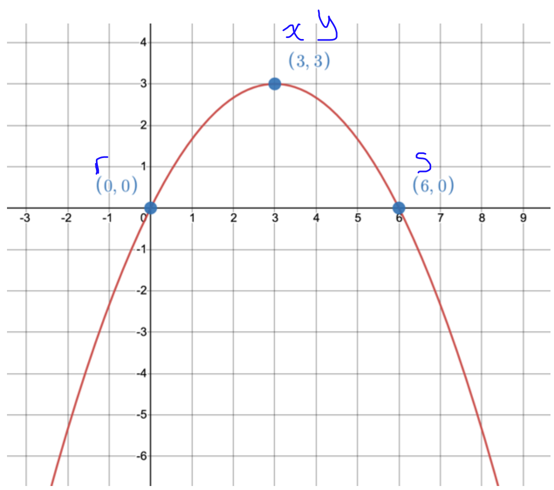


Algorithm for Determining Factored Form Equation from a Graph

- Find the x-intercepts (r and s)
- Find another point on the graph (x, y)
- Plug the values of r, s, x, and y in to $y = a(x-r)(x-s)$ and solve for a
- Write the final equation by plugging in a, r, and s. NOT x and y.

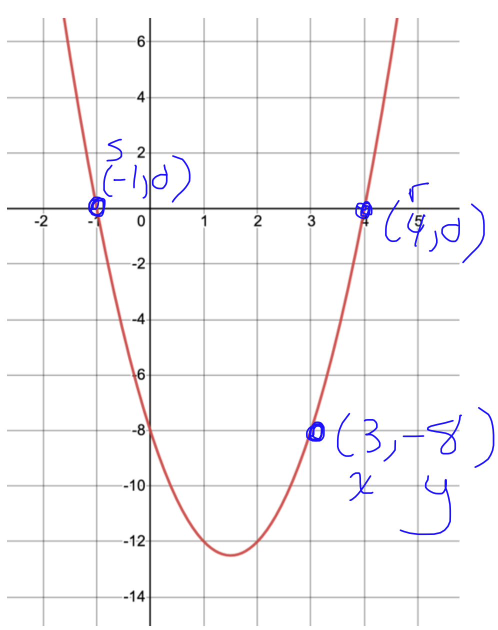
Example 2: Determine the factored form equation of each of the following quadratic relations.

a)



$$y = a(x-r)(x-s)$$
$$3 = a(3-0)(3-6)$$
$$3 = a(3)(-3)$$
$$3 = -9a$$
$$\frac{3}{-9} = a$$
$$a = -\frac{1}{3}$$
$$\boxed{y = -\frac{1}{3}x(x-6)}$$

b)



$$y = a(x-r)(x-s)$$
$$-8 = a[3-(-1)](3-4)$$
$$-8 = a(4)(-1)$$
$$-8 = -4a$$
$$\frac{-8}{-4} = a$$
$$a = 2$$
$$\boxed{y = 2(x+1)(x-4)}$$

Example 3: Determine the factored form equation of the parabola with x-intercepts at $\overset{r}{-3}$ and $\overset{s}{-5}$ and passes through the point $(\underset{x}{-4},\underset{y}{1})$.

$$y = a(x-r)(x-s)$$

$$1 = a[-4-(-3)][-4-(-5)]$$

$$1 = a(-1)(1)$$

$$1 = -1a$$

$$a = -1$$

$$\boxed{y = -(x+3)(x+5)}$$