W3 –Completing the Square Unit 4
MPM2D
Jensen

1) For each quadratic that is in standard form, determine the value of 'c' that makes each expression a perfect square trinomial (remember, the 'c' value is half of the 'b' value squared)

a) $x^2 + 6x + c$

$c = \left(\frac{6}{2}\right)^2$
$= 9$

b) $x^2 - 12x + c$

$c = \left(\frac{12}{2}\right)^2$
$= 36$

c) $x^2 + 2x + c$

$c = \left(\frac{2}{2}\right)^2$
$= 1$

2) Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square

a) $y = x^2 + 6x - 1$

$y = (x^2 + 6x) - 1$
$y = (x^2 + 6x + 9 - 9) - 1$
$y = (x^2 + 6x + 9) - 9 - 1$
$y = (x^2 + 6x + 9) - 10$
$y = (x + 3)(x + 3) - 10$
$y = (x + 3)^2 - 10$

b) $y = x^2 + 10x + 20$

$y = (x^2 + 10x) + 20$
$y = (x^2 + 10x + 25 - 25) + 20$
$y = (x^2 + 10x + 25) - 25 + 20$
$y = (x + 5)(x + 5) - 5$
$y = (x + 5)^2 - 5$

c) $y = x^2 - 6x - 4$

$y = (x^2 - 6x) - 4$
$y = (x^2 - 6x + 9 - 9) - 4$
$y = (x^2 - 6x + 9) - 9 - 4$
$y = (x - 3)(x - 3) - 13$
$y = (x - 3)^2 - 13$

d) $y = x^2 - 12x + 8$

$y = (x^2 - 12x) + 8$
$y = (x^2 - 12x + 36 - 36) + 8$
$y = (x^2 - 12x + 36) - 36 + 8$
$y = (x - 6)(x - 6) - 28$
$y = (x - 6)^2 - 28$

e) $y = -x^2 + 80x - 100$

$y = (-x^2 + 80x) - 100$

$y = -(x^2 - 80x) - 100$

$y = -(x^2 - 80x + 1600 - 1600) - 100$

$y = -(x^2 - 80x + 1600) + 1600 - 100$

$y = -(x - 40)(x - 40) + 1500$

$y = -(x - 40)^2 + 1500$

f) $y = 3x^2 + 90x + 50$

$= (3x^2 + 90x) + 50$

$= 3(x^2 + 30x) + 50$

$= 3(x^2 + 30x + 225 - 225) + 50$

$= 3(x^2 + 30x + 225) - 675 + 50$

$= 3(x + 15)(x + 15) - 625$

$= 3(x + 15)^2 - 625$

g) $y = -7x^2 + 14x - 3$

$y = (-7x^2 + 14x) - 3$

$y = -7(x^2 - 2x) - 3$

$y = -7(x^2 - 2x + 1 - 1) - 3$

$y = -7(x^2 - 2x + 1) + 7 - 3$

$y = -7(x - 1)(x - 1) + 4$

$y = -7(x - 1)^2 + 4$

h) $y = 4x^2 + 64x + 156$

$y = (4x^2 + 64x) + 156$

$y = 4(x^2 + 16x) + 156$

$y = 4(x^2 + 16x + 64 - 64) + 156$

$y = 4(x^2 + 16x + 64) - 256 + 156$

$y = 4(x + 8)(x + 8) - 100$

$y = 4(x + 8)^2 - 100$

3) Find the maximum or minimum point of each parabola by completing the square.

a) $y = -x^2 - 10x - 9$

$y = (-x^2 - 10x) - 9$

$y = -(x^2 + 10x) - 9$

$y = -(x^2 + 10x + 25 - 25) - 9$

$y = -(x^2 + 10x + 25) + 25 - 9$

$y = -(x^2 + 10x + 25) + 16$

$y = -(x + 5)(x + 5) + 16$

$y = -(x + 5)^2 + 16$

Max at $(-5, 16)$

b) $y = 2x^2 + 120x + 75$

$y = (2x^2 + 120x) + 75$

$y = 2(x^2 + 60x) + 75$

$y = 2(x^2 + 60x + 900 - 900) + 75$

$y = 2(x^2 + 60x + 900) - 1800 + 75$

$y = 2(x + 30)(x + 30) - 1725$

$y = 2(x + 30)^2 - 1725$

Min at $(-30, -1725)$

4) The path of a ball is modeled by the equation $y = -x^2 + 4x + 1$, where x is the horizontal distance, in meters, travelled and y is the height, in meters, of the ball above the ground. What is the maximum height of the ball, and at what horizontal distance does it occur?

$y = (-x^2 + 4x) + 1$

$y = -(x^2 - 4x) + 1$

$y = -(x^2 - 4x + 4 - 4) + 1$

$y = -(x^2 - 4x + 4) + 4 + 1$

$y = -(x-2)(x-2) + 5$

$y = -(x-2)^2 + 5$

Max height of 5 meters at a horizontal distance of 2 meters.

5) The path of a rocket is given by the equation, $h = -3t^2 + 30t + 73$, where 'h' is the height in meters and 't' is the time in seconds.

a) What is the max height of the rocket

$h = (-3t^2 + 30t) + 73$

$h = -3(t^2 - 10t) + 73$

$h = -3(t^2 - 10t + 25 - 25) + 73$

$h = -3(t^2 - 10t + 25) + 75 + 73$

$h = -3(t-5)(t-5) + 148$

$h = -3(t-5)^2 + 148$

Max height of 148 m.

b) At what time does the rocket reach its maximum height

5 seconds.

6) For each of the following functions, **i)** convert to vertex form by completing the square, **ii)** complete the table of properties, **iii)** graph the function by making a table of values

$y = 2x^2 - 12x + 22$

$y=(2x^2-12x)+22$

$y=2(x^2-6x)+22$

$y=2(x^2-6x+9-9)+22$

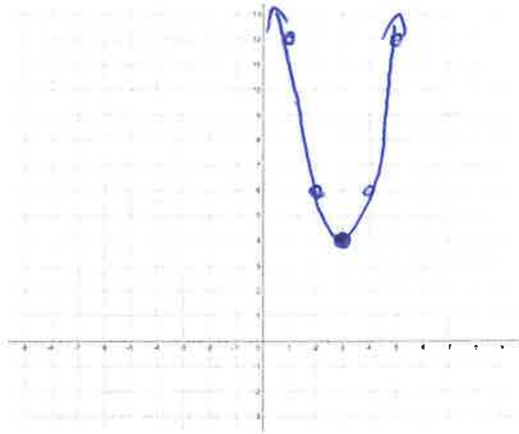
$y=2(x^2-6x+9)-18+22$

$y=2(x-3)(x-3)+4$

$y=2(x-3)^2+4$

Vertex	(3, 4)
Axis of Symmetry	$x=3$
Direction of Opening	up
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} \mid y \geq 4\}$

x	y
1	12
2	6
3	4
4	6
5	12



b) $y = \frac{1}{2}x^2 - 4x - 7$

$y=(\frac{1}{2}x^2-4x)-7$

$y=\frac{1}{2}(x^2-8x)-7$

$y=\frac{1}{2}(x^2-8x+16-16)-7$

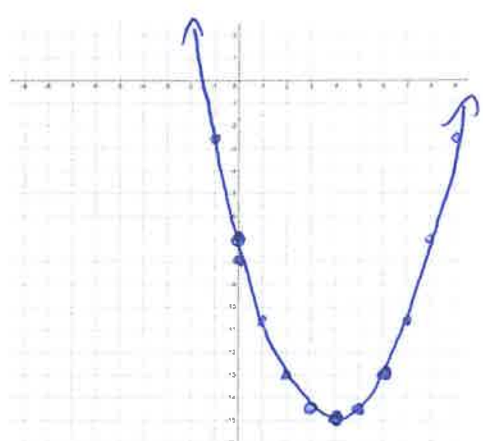
$y=\frac{1}{2}(x^2-8x+16)-8-7$

$y=\frac{1}{2}(x-4)(x-4)-15$

$y=\frac{1}{2}(x-4)^2-15$

Vertex	(4, -15)
Axis of Symmetry	$x=4$
Direction of Opening	up
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} \mid y \geq -15\}$

x	y
2	-13
3	-14.5
4	-15
5	-14.5
6	-13

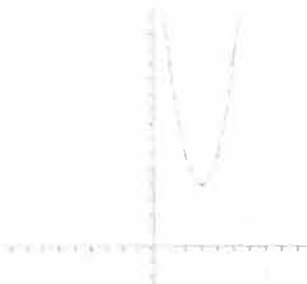


Answers

1)a) 9 **b)** 36 **c)** 1
2)a) $y = (x+3)^2 - 10$ **b)** $y = (x+5)^2 - 5$ **c)** $y = (x-3)^2 - 13$ **d)** $y = (x-6)^2$
e) $y = -(x-40)^2 + 1500$ **f)** $y = 3(x+15)^2 - 625$ **g)** $y = -7(x-1)^2 + 4$ **h)** $y = 4(x+8)^2 - 100$
3)a) max at $(-5,15)$ **b)** min at $(-30,-1725)$.
4) max height of 5m occurs at a horizontal distance of 2m
5)a) 148m **b)** 5 seconds
6)a)

Vertex	$(3,4)$
Axis of Symmetry	$x = 3$
Direction of Opening	Up
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} \mid y \geq 4\}$

x	y
1	12
2	6
3	4
4	6
5	12



b)

Vertex	$(4,-15)$
Axis of Symmetry	$x = 4$
Direction of Opening	Up
Values x may take (domain)	$\{X \in \mathbb{R}\}$
Values y may take (range)	$\{Y \in \mathbb{R} \mid y \geq -15\}$

x	y
2	-13
3	-14.5
4	-15
5	-14.5
6	-13