L1 – Similar Triangles Unit 3
MPM2D
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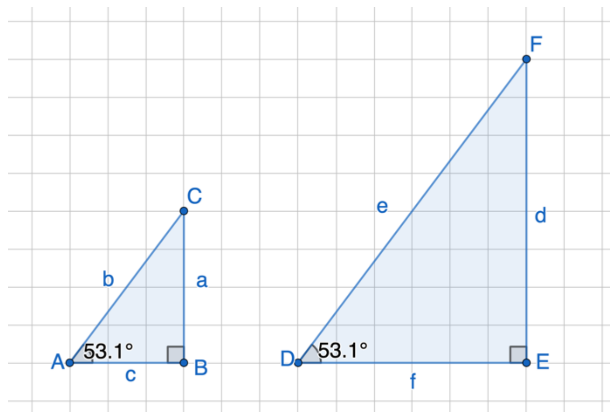
Congruent figures have the exact same shape AND size.

Similar figures have identical shapes, but different sizes.

Part 1: Properties of Similar Triangles

If ΔABC is similar to ΔDEF:

- The corresponding angles are equal
  - $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$
- Ratios of corresponding sides are equal
  - $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



From the diagram we can verify these properties:

Corresponding angles are equal → $53.1° = 53.1°$, $90° = 90°$, $36.9° = 36.9°$

Ratios of corresponding sides are equal → $\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$

Part 2: Proving Triangles are Similar

There are 3 ways to prove triangles are similar:

1) Angle Angle similarity (2 angles are equal)

Note: *If 2 pairs of angles are equal, the 3rd pair must be equal as well*

2) Side Side Side similarity (all ratios of sides are equal)

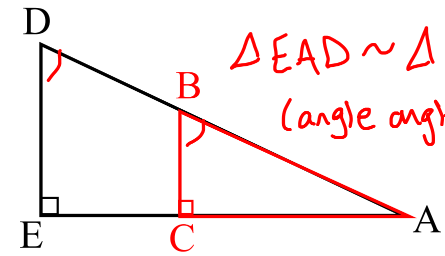
3) Side Angle Side similarity (2 ratios of sides and 1 angle are equal)

If you can prove any of those 3 scenarios, it proves the triangles are SIMILAR meaning they are the exact same shape but not necessarily the same size.

Note: When you write a similarity statement, the order of the vertices must correctly identify pairs of equal angles and pairs of corresponding sides.

Example 1: Prove the following triangles are similar

a)



$\Delta EAD \sim \Delta CAB$

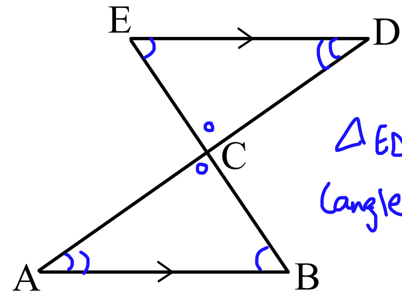
(angle angle similarity)

$\angle E = \angle C$ (90°)

$\angle A = \angle A$ (shared angle)

$\angle D = \angle B$ (corresponding angles)

b)



$\Delta EDC \sim \Delta BAC$

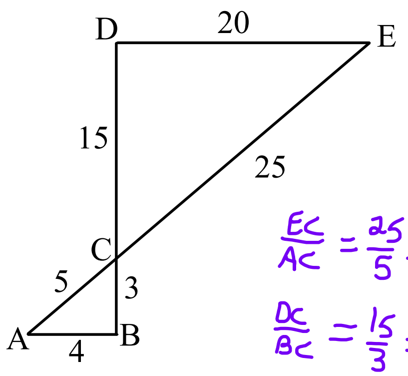
(angle angle similarity)

$\angle E = \angle B$ (alternating angles)

$\angle D = \angle A$ (alternating angles)

$\angle ECD = \angle ACB$ (opposite angles)

c)



$\frac{EC}{AC} = \frac{25}{5} = 5$

$\frac{DC}{BC} = \frac{15}{3} = 5$

$\frac{ED}{AB} = \frac{20}{4} = 5$

$\Delta ECD \sim \Delta ACB$

(side side side similarity)

Part 3: Use Similar Triangles to Solve Problems

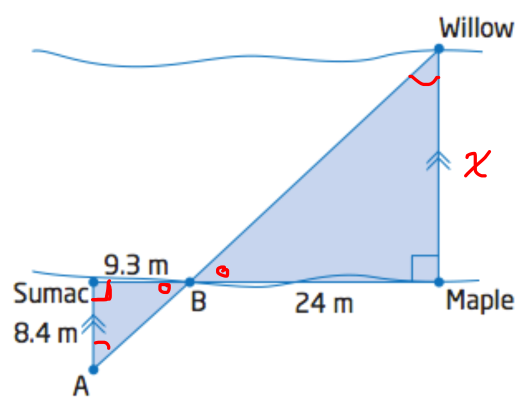
Example 2: To determine the width of a river, Naomi finds a willow tree and a maple tree that are directly across from each other on opposite shores. Using a third tree on the shoreline, Naomi plants two stakes, A and B, and measures the distances shown. Find the width of the river using the information Naomi found.

$\angle SBA = \angle WBM$ (opposite angles)

$\angle S = \angle M$ (90°)

$\angle A = \angle W$ (alternating angles)

$\therefore$ $\Delta BSA \sim \Delta BMW$ (angle angle similarity)



$\frac{SA}{MW} = \frac{BS}{BM}$

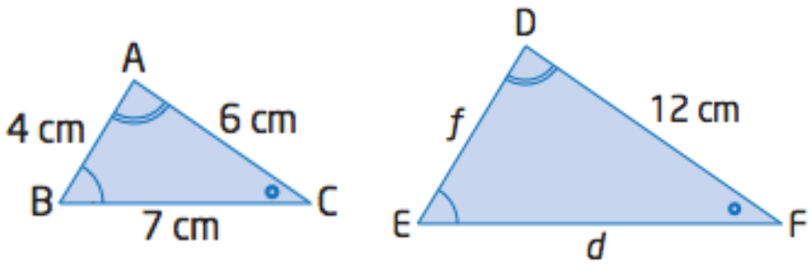
$\frac{8.4}{x} = \frac{9.3}{24}$

$8.4(24) = 9.3x$

$201.6 = 9.3x$

$x \simeq 21.7\,m$

Example 3: The following triangles are similar; find the unknown side lengths.



$\triangle ABC \sim \triangle DEF \rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

$\frac{4}{f} = \frac{6}{12} = \frac{7}{d}$

$\frac{4}{f} = \frac{6}{12}$

$4(12) = 6f$

$48 = 6f$

$\boxed{f = 8}$

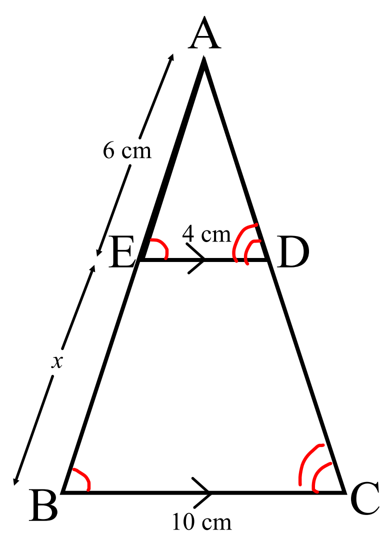
$\frac{6}{12} = \frac{7}{d}$

$6d = 7(12)$

$6d = 84$

$\boxed{d = 14}$

Example 4: Solve for the length of x.



$\angle E = \angle B$ (corresponding angles)

$\angle A = \angle A$ (shared angle)

$\angle D = \angle C$ (corresponding angles)

$\therefore \triangle EAD \sim \triangle BAC$ (angle angle similarity)

$\frac{EA}{BA} = \frac{ED}{BC}$

$\frac{6}{x+6} = \frac{4}{10}$

$6(10) = 4(x+6)$

$60 = 4(x+6)$

$\frac{60}{4} = x+6$

$15 = x+6$

$\boxed{x = 9}$