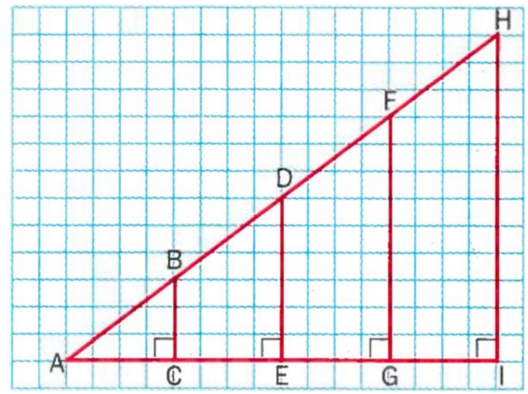
L2 – SOHCAHTOA Unit 3
MPM2D
Jensen

Trigonometry is the branch of mathematics that deals with the relationships between the sides and the angles of triangles, and the calculations based on them.

Part 1: Investigation



Given the four nested triangles

a) Are the triangles similar? Explain.

Yes, this can be shown through angle angle similarity. All four triangles share angle A, and have a 90 degree angle.

b) Which angle do all 4 triangles share?

angle A

c) From that shared angle, find the ratio of the side opposite to that angle to the side adjacent to that angle in each triangle.

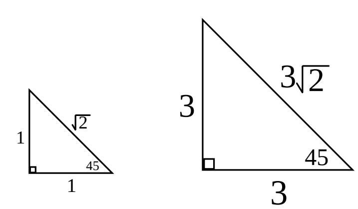
Triangle	ΔABC	ΔADE	ΔAFG	ΔAHI
Ratio	$\frac{BC}{AC} = \frac{3}{4}$	$\frac{DE}{AE} = \frac{6}{8} = \frac{3}{4}$	$\frac{FG}{AG} = \frac{9}{12} = \frac{3}{4}$	$\frac{HI}{AI} = \frac{12}{16} = \frac{3}{4}$

Findings:

What you just found is called the TANGENT ratio from angle A. The word tangent means the ratio of $\frac{opposite}{adjacent}$ from a reference angle. Notice that the ratio was equivalent for each of these triangles. This is because the triangles are SIMILAR. If you have two right triangles with an equivalent reference angle, they must be similar and therefore have equivalent ratios of pairs of sides.

Your calculator has ratios of sides stored in it for any reference angle you choose.

By typing $\tan(45°)$, you are asking your calculator for the ratio of $\frac{opposite\ side\ length}{adjacent\ side\ length}$ from the 45° angle of a right triangle. Your calculator does not know what size triangle you have, but it knows it is similar to all other right triangles with the same 45° reference angle, and therefore has the same ratio of side lengths. This is why your calculator is able to tell you that $\tan(45°) = 1$.



Example 1: Calculate the tan ratio, to the nearest thousandth, for each angle.

a) $40°$

$\tan(40°)$

$\simeq 0.84$

b) $55°$

$\tan(55°)$

$\simeq 1.43$

c) $73°$

$\tan(73°)$

$\simeq 3.27$

d) $89°$

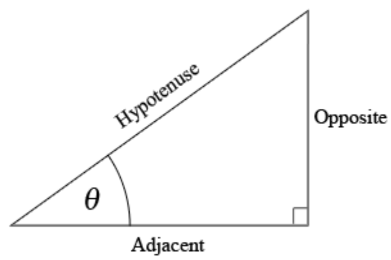
$\tan(89°)$

$\simeq 57.29$

A tangent ratio is just one of three possible ratios of sides you can make from a triangle. The other two ratios have names as well. Look to the next section to learn them!

Section 2: What is $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$?

If we know a right-angle triangle has an angle of θ, all other right-angle triangles with an angle of θ are **similar** and therefore have equivalent ratios of corresponding sides. The three primary ratios are shown in the diagram to the right.



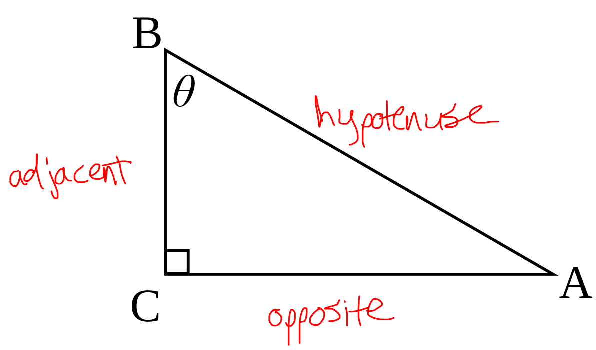
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

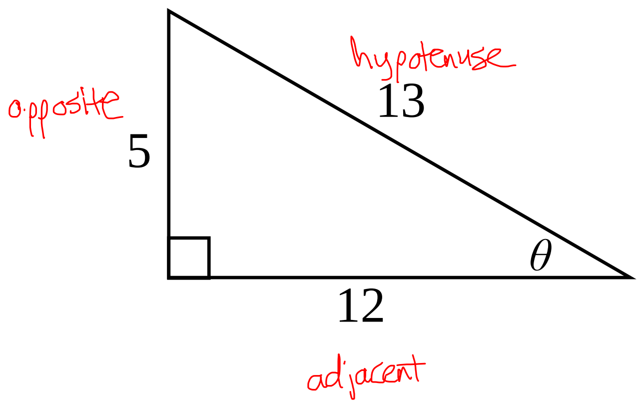
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Geogebra demonstration

Example 2: Label the adjacent, opposite, and hypotenuse of the triangle from the reference angle θ.



Example 3: State the sine, cosine, and tangent ratios for θ.



$\sin\theta = \frac{5}{12}$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

Example 4: Calculate each of the following

a) $\sin 30°$

$= \frac{1}{2}$

b) $\cos 60°$

$= \frac{1}{2}$

c) $\tan 10°$

$\simeq 0.18$