L6 – Sine Law Unit 3

MPM2D

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When solving for sides and angles in triangles, there are 4 main tools that can be used. Pythagorean theorem and SOHCAHTOA can only be used with **Right Triangles**. Sine Law and Cosine Law can be used with **oblique and right-angle triangles**. In this lesson we will focus on the Sine Law.

Rule	When to Use It	
Pythagorean Theorem $a^2 + b^2 = c^2$	Right Triangle Know: 2 sides Want: 3rd side	
SOHCAHTOA $S\frac{O}{H}C\frac{A}{H}T\frac{O}{a}$	Right Triangle Know: 2 sides Want: Angle (use inverse ratio)	Right Triangle Know: 1 side, 1 angle Want: Side
Sine Law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Oblique Triangle (no right angle) Know: 2 sides and opposite angle Want: Angle	Oblique Triangle (no right angle) Know: 1 side and all angles Want: Side
Cosine Law $a^2 = b^2 + c^2 - 2bc(\cos A)$ $\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$	Oblique Triangle Know: 2 sides and contained angle Want: 3rd side (use top formula)	Oblique Triangle Know: All 3 sides Want: Angle (use bottom formula)

Section 1: Proof

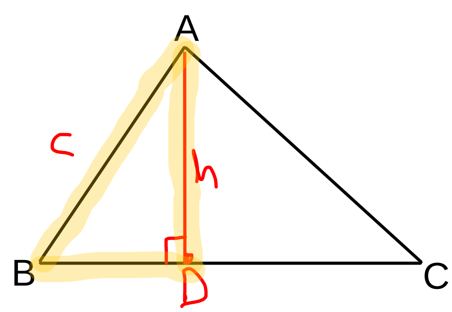
In an acute triangle, when two angles and a side are given, the other sides can be found using the sine law, which can be developed as follows.

In ΔABC, draw AD perpendicular to BC. AD is the altitude or height, h, of ΔABC.

Looking at ΔABD...

$\sin(B) = \frac{h}{c}$

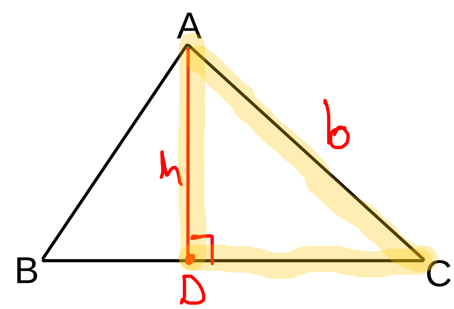
$c\sin(B) = h$



Looking at ΔADC

$\sin(C) = \frac{h}{b}$

$b\sin(C) = h$



$\therefore b\sin(C) = c\sin(B)$

$$\boxed{\frac{b}{\sin(B)} = \frac{c}{\sin(C)}}$$

Sine Law: the relationship between the sides and their opposite angles in any acute ΔABC is…

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

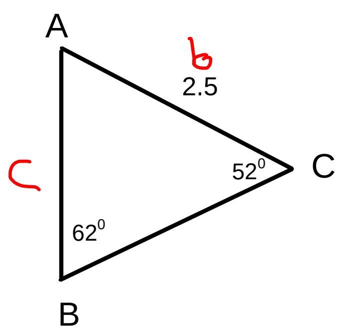
Note: Even though there are 3 parts to this equation, you only use two parts at a time. The choice of what to use depends on the information given. Make sure in the equation you create there is only 1 unknown.

Section 2: Find Side Lengths

__Note:__ Sine Law can be used to solve for a side length when you know 1 side and 2 angles.

Example 1: Find the measure of each indicated side

a) Find the length of side $'c'$



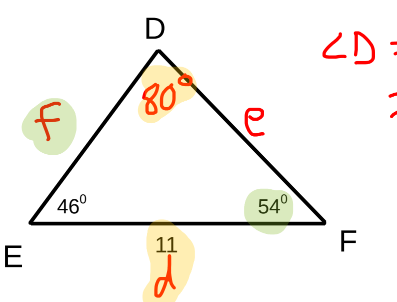
$$\frac{c}{\sin(C)} = \frac{b}{\sin(B)}$$

$$\frac{c}{\sin(52°)} = \frac{2.5}{\sin(62°)}$$

$$c = \frac{2.5\sin(52°)}{\sin(62°)}$$

$$\boxed{c \simeq 2.23}$$

b) Find the length of side $'f'$



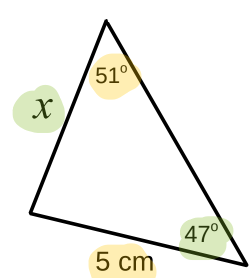
$\angle D = 180 - 54 - 46$
$= 80$

$$\frac{11}{\sin(80)} = \frac{f}{\sin(54)}$$

$$f = \frac{11\sin(54)}{\sin(80)}$$

$$\boxed{f \simeq 9.04}$$

c) Solve for the length of side x



$$\frac{5}{\sin(51)} = \frac{x}{\sin(47)}$$

$$x = \frac{5\sin(47)}{\sin(51)}$$

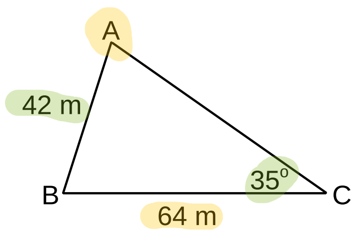
$$\boxed{x \simeq 4.71}$$

Section 3: Find Angles

> ***Note:*** *Sine Law can be used to solve for an angle if you know 2 sides and 1 of their opposite angles.*

Example 2: Find the measure of each indicated angle

a) Find the measure of angle A



$$\frac{64}{\sin(A)} = \frac{42}{\sin(35)}$$

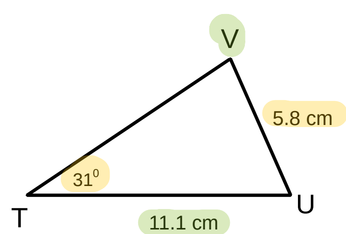
$$64\sin(35) = 42\sin(A)$$

$$\frac{64\sin(35)}{42} = \sin(A)$$

$$A = \sin^{-1}\left[\frac{64\sin(35)}{42}\right]$$

$$\boxed{A \simeq 60.93^\circ}$$

b) Find the measure of angle V



$$\frac{5.8}{\sin(31)} = \frac{11.1}{\sin(V)}$$

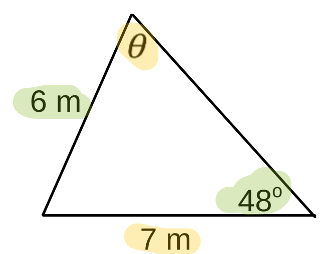
$$5.8\sin(V) = 11.1\sin(31)$$

$$\sin(V) = \frac{11.1\sin(31)}{5.8}$$

$$V = \sin^{-1}\left[\frac{11.1\sin(31)}{5.8}\right]$$

$$\boxed{V \simeq 80.29^\circ}$$

c) Find the measure of angle θ



$$\frac{6}{\sin(48)} = \frac{7}{\sin(\theta)}$$

$$6\sin(\theta) = 7\sin(48^\circ)$$

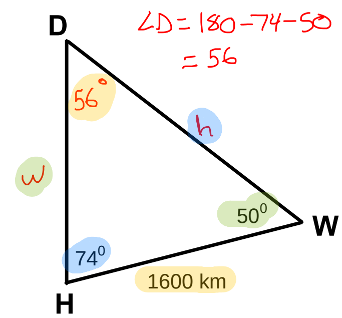
$$\sin(\theta) = \frac{7\sin(48^\circ)}{6}$$

$$\theta = \sin^{-1}\left[\frac{7\sin(48)}{6}\right]$$

$$\boxed{\theta \simeq 60.11^\circ}$$

Example 3: Find the perimeter of the Bermuda triangle

$$\angle D = 180 - 74 - 50$$
$$= 56$$



$$\frac{1600}{\sin(56)} = \frac{w}{\sin(50)}$$

$$w = \frac{1600\sin(50)}{\sin(56)}$$

$$w \simeq 1478.4 \text{ km}$$

$$\frac{1600}{\sin(56)} = \frac{h}{\sin(74)}$$

$$h = \frac{1600\sin(74)}{\sin(56)}$$

$$h \simeq 1855.2 \text{ km}$$

$$\boxed{\text{The perimeter} = 1600 + 1478.4 + 1855.2 = 4933.6 \text{ km}}$$