L7 – Cosine Law Unit 3
MPM2D
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When solving for sides and angles in triangles, there are 4 main tools that can be used. Pythagorean theorem and SOHCAHTOA can only be used with Right Triangle. Sine Law and Cosine Law can be used with Right and Oblique triangles. In this lesson we will focus on the Cosine Law.

Rule	When to Use It	
Pythagorean Theorem $a^2 + b^2 = c^2$	Right Triangle Know: 2 sides Want: 3rd side	
SOHCAHTOA $S\frac{O}{H}C\frac{A}{H}T\frac{O}{a}$	Right Triangle Know: 2 sides Want: Angle (use inverse ratio)	Right Triangle Know: 1 side, 1 angle Want: Side
Sine Law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Oblique Triangle (no right angle) Know: 2 sides and opposite angle Want: Angle	Oblique Triangle (no right angle) Know: 1 side and all angles Want: Side
Cosine Law $a^2 = b^2 + c^2 - 2bc(\cos A)$ $\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$	Oblique Triangle Know: 2 sides and contained angle Want: 3rd side (use top formula)	Oblique Triangle Know: All 3 sides Want: Angle (use bottom formula)

Section 1: Proof

Cosine law can be developed as follows

In ΔABC, draw CD perpendicular to AB. CD is the altitude, h, of ΔABC.

Looking at ΔACD...

$x^2 + h^2 = b^2$

$\cos(A) = \frac{x}{b}$

$b\cos(A) = x$

Looking at ΔCDB...

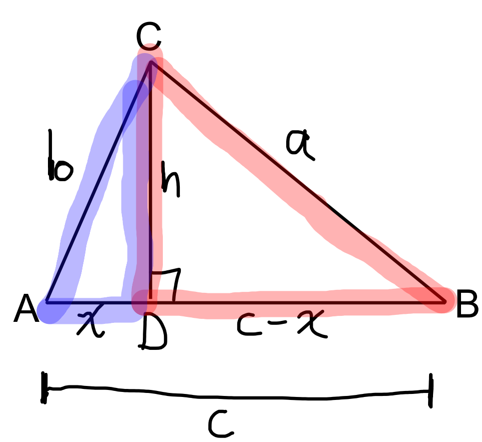
$(c-x)^2 + h^2 = a^2$

$c^2 - 2cx + x^2 + h^2 = a^2$

$c^2 - 2cx + b^2 = a^2$

$c^2 - 2cb\cos(A) + b^2 = a^2$

$a^2 = b^2 + c^2 - 2bc\cos(A)$



$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$

Cosine Law:

the relationship between two sides and their contained angle in any acute ΔABC is…

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

the relationship between the sides and one of their opposite angles in any acute ΔABC is…

$$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$$

Section 2: Finding Sides

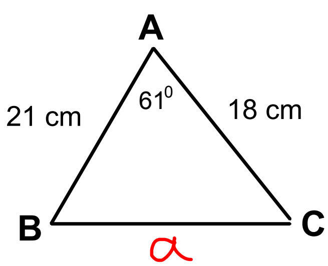
Cosine Law can be used to solve for a side length when you know 2 sides and the angle contained by those 2 sides.

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

Note: *sides b and c are interchangeable. It just matters that they are the two known sides and angle A is contained by the two sides.*

Example 1: Find the measure of the indicated side

a) Find the length of side $'a'$

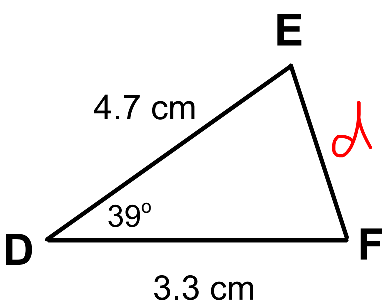


$$a^2 = 18^2 + 21^2 - 2(18)(21)\cos(61^\circ)$$

$$a^2 = 398.4839271$$

$$a \simeq 19.96$$

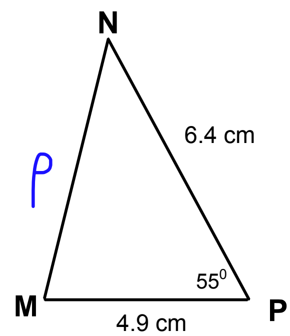
b) Find the length of side d



$d^2 = 4.7^2 + 3.3^2 - 2(4.7)(3.3)\cos(39°)$

$d \simeq 2.98\,cm$

c) Find the length of side p



$p^2 = 4.9^2 + 6.4^2 - 2(4.9)(6.4)\cos(55)$

$p \simeq 5.38\,cm$

Section 3: Finding Angles

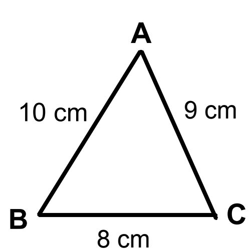
Note: *The rearranged version of Cosine Law can be used to solve for an angle if you know all 3 sides of a triangle.*

$$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$$

> ***Note:*** *Notice in this formula that the side opposite from the angle you are finding comes first in the numerator. The order of the other 2 known sides does not matter.*

Example 2: Solve for the indicated angle

a) Find the measure of angle A

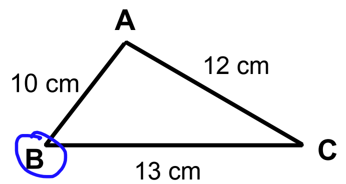


$$\cos(A) = \frac{8^2 - 10^2 - 9^2}{-2(10)(9)}$$

$$\angle A = \cos^{-1}\left[\frac{-117}{-180}\right]$$

$$\angle A \simeq 49.46°$$

b) Find the measure of angle B

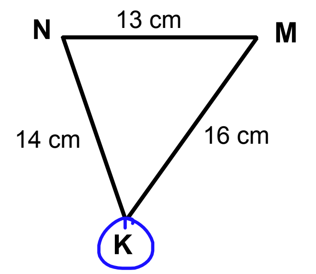


$$\cos(B) = \frac{12^2 - 10^2 - 13^2}{-2(10)(13)}$$

$$\angle B = \cos^{-1}\left(\frac{-125}{-260}\right)$$

$$\angle B \simeq 61.26^\circ$$

c) Find the measure of angle K

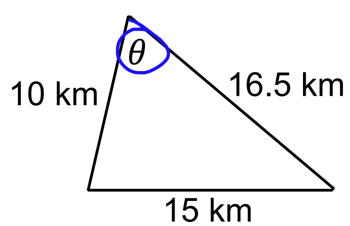


$$\cos(K) = \frac{13^2 - 14^2 - 16^2}{-2(14)(16)}$$

$$\angle K = \cos^{-1}\left(\frac{-283}{-448}\right)$$

$$\angle K \simeq 50.82^\circ$$

d) Find the measure of angle θ

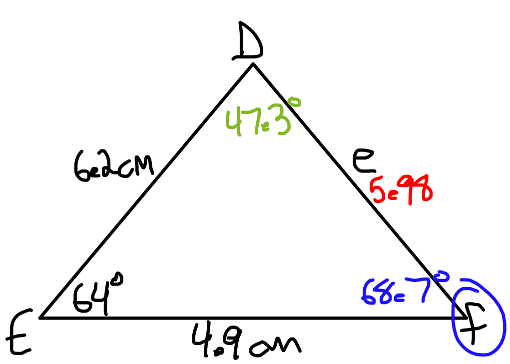


$$\cos(\theta) = \frac{15^2 - 10^2 - 16.5^2}{-2(10)(16.5)}$$

$$\theta = \cos^{-1}\left(\frac{-147.25}{-330}\right)$$

$$\theta \simeq 63.5^\circ$$

Example 3: In acute ΔDEF, $d = 4.9\ cm$, $f = 6.2\ cm$, and $\angle E = 64°$. Solve ΔDEF.



$$e^2 = 6.2^2 + 4.9^2 - 2(6.2)(4.9)\cos(64°)$$

$$\boxed{e \simeq 5.98\text{cm}}$$

$$\cos(F) = \frac{6.2^2 - 4.9^2 - 5.98^2}{-2(4.9)(5.98)}$$

$$\boxed{\angle F \simeq 68.7°}$$

$$\angle D = 180 - 64 - 68.7$$

$$\boxed{\angle D \simeq 47.3°}$$