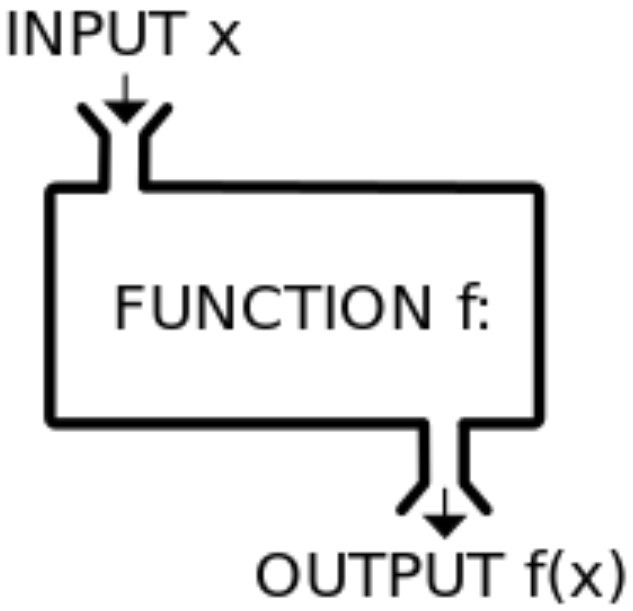
Chapter 1- Functions

Workbook Package

MCR3U



Chapter 1 Workbook Checklist

Worksheet	Check ✓
1.1 – Functions, Domain, and Range	
1.2 – Functions and Function Notation	
1.3 – Max or Min of a Quadratic Function	
1.3 – Extra Practice (optional)	
1.4 – Working with Radicals	
1.5 – Solving Quadratic Equations – Part 1: Factoring	
1.5 – Solving Quadratic Equations – Part 2: QF	
1.7 – Solve Linear Quadratic Systems	
Chapter 1 Review	
Chapter 1 Mini Practice Test	

Mark /10	0-2	3-5	6-8	9-10
Work completion for chapter 1	Little to know homework done throughout chapter.	Some homework completed. Solutions are unorganized or not shown in full.	Most homework completed. Solutions are clear and organized.	All homework completed accurately. Solutions are well organized and shown in full.

Mark /4	1	2	3	4
In Class Work for Chapter 1	Class time not used well for work completion. Inattentive during lessons. Need to improve at limiting distractions.	Some work completed during class. Sometimes distracted during lessons.	Works well during class. Minimal distractions. Good attention during lessons.	Always uses class time efficiently. Pays attention and contributes to lessons.

Comments:

1.1 Functions, Domain, and Range - Worksheet

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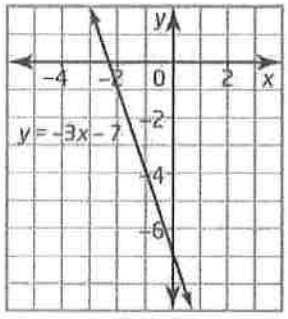
SOLUTIONS

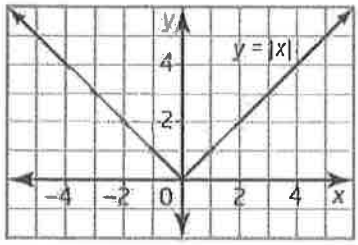
1) Which graphs represent functions? Justify your answer.

a)



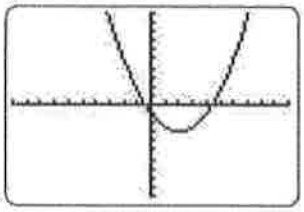
function

b)



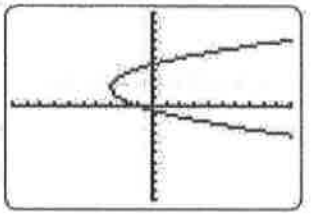
function

c)



function

d)



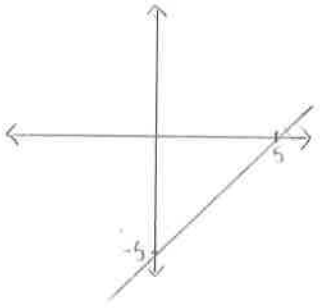
Not a function

- fails vertical line test.

2) Is each relation a function? Explain and make a rough sketch of the graph of each.

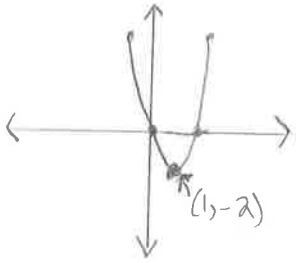
a) $y = x - 5$

function



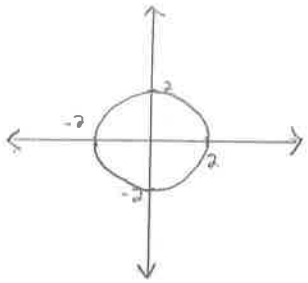
b) $y = 2(x-1)^2 - 2$

function



c) $x^2 + y^2 = 4$

NOT a function



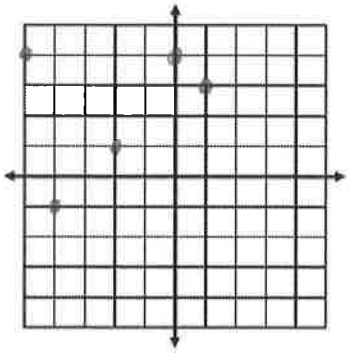
3) State the domain and range. Represent as a table and graph. Then state if it is a function.

a) {(-5, 4), (-4, -1), (-2, 1), (0, 4), (1, 3)}

Domain: $\{x \in \mathbb{R} \mid x = -5, -4, -2, 0, 1\}$

Range: $\{y \in \mathbb{R} \mid y = -1, 1, 3, 4\}$

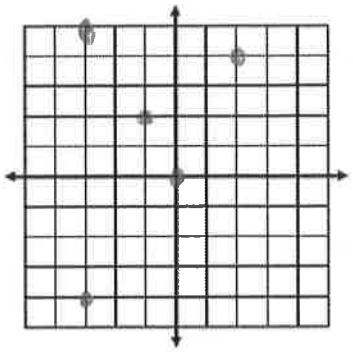
x	y
-5	4
-4	-1
-2	1
0	4
1	3



Is this relation a function? Yes

b) {(-3, -4), (-1, 2), (0, 0), (-3, 5), (2, 4)}

x	y
-3	-4
-1	2
0	0



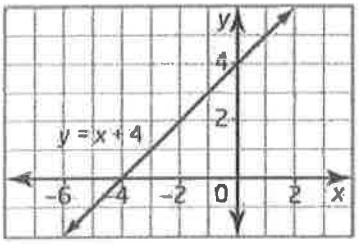
Domain: $\{x \in \mathbb{R} \mid x = -3, -1, 0, 2\}$

Range: $\{y \in \mathbb{R} \mid y = -4, 0, 2, 4, 5\}$

Is this relation a function? NO

4) State the domain and range of each relation. Then state if the relation is a function.

a)

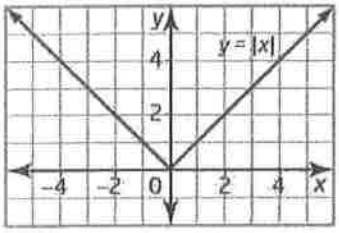


D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$

Function

b)

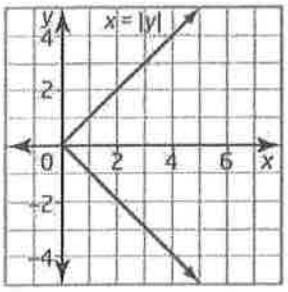


D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

Function

c)

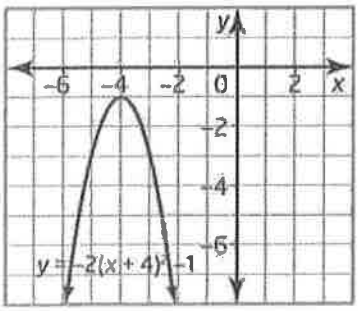


D: $\{x \in \mathbb{R} \mid x \geq 0\}$

R: $\{y \in \mathbb{R}\}$

NOT a function

d)

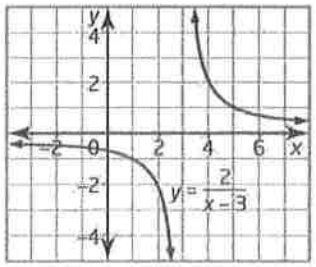


D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \leq -1\}$

Function

e)

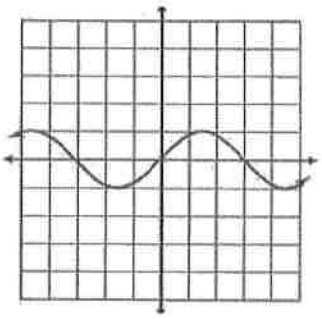


D: $\{x \in \mathbb{R} \mid x \neq 3\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$

Function

f)

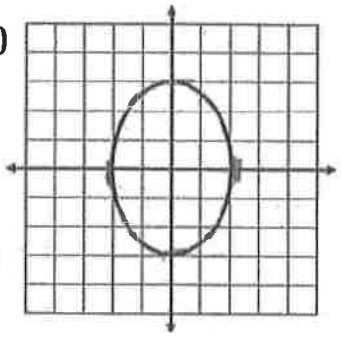


D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$

Function

g)



D: $\{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$

R: $\{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$

Not a function

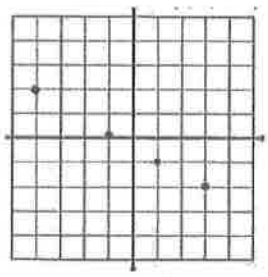
5) Which of the following relations are functions?

a)

x	y
2	−3
−1	0
5	5
3	2
2	1

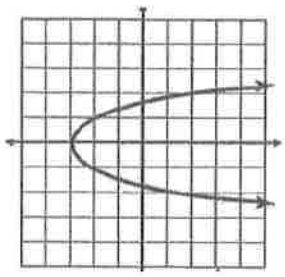
Not a function

b)



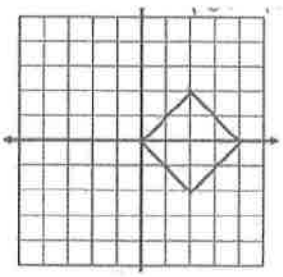
Function

c)



Not a function

d)



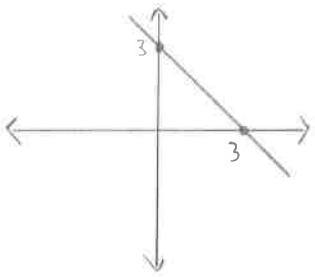
Not a function

6) Determine the domain and range of each of the following relations. Use a graphing calculator or a graphing app to help if necessary. Make a rough sketch of the graph.

a) $y = -x + 3$

D: $\{x \in \mathbb{R}\}$

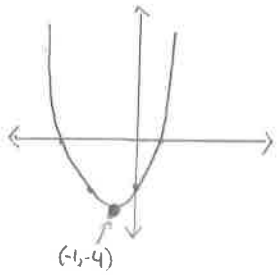
R: $\{y \in \mathbb{R}\}$



b) $y = (x+1)^2 - 4$

D: $\{x \in \mathbb{R}\}$

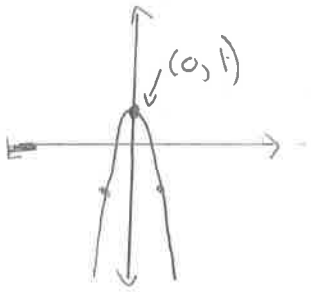
R: $\{y \in \mathbb{R} \mid y \geq -4\}$



c) $y = -3x^2 + 1$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \leq 1\}$

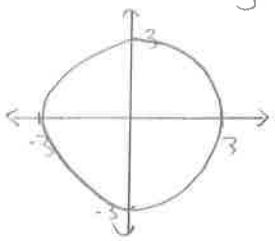


d) $x^2 + y^2 = 9$

$r = 3$

D: $\{x \in \mathbb{R} \mid -3 \leq x \leq 3\}$

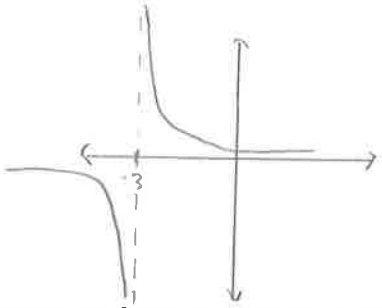
R: $\{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$



e) $y = \frac{1}{x+3}$

D: $\{x \in \mathbb{R} \mid x \neq -3\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$



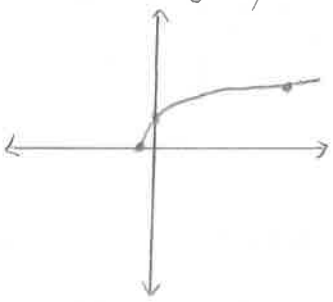
f) $y = \sqrt{2x+1}$

$2x+1 < 0$

$x < -\frac{1}{2}$

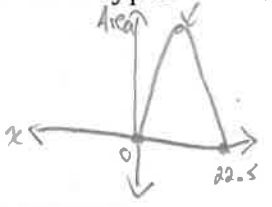
D: $\{x \in \mathbb{R} \mid x \geq -0.5\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

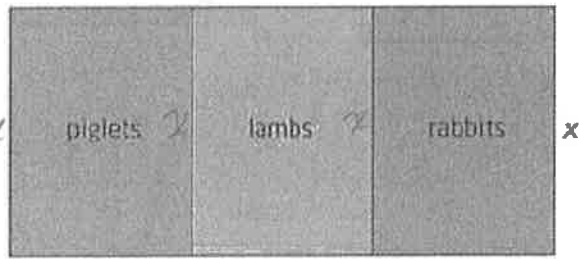


7) Pam has 90 m of fencing to enclose an area in a petting zoo with two dividers to separate three types of young animals. The three pens are to have the same area.

a) Express the area function for the three pens in terms of x.
b) Determine the domain and range for the area function.



$\frac{90-4x}{2}$



$\frac{90-4x}{2}$

a) $A = \left(\frac{90-4x}{2}\right)(x)$

$= (45-2x)(x)$

$= 45x - 2x^2$

$= -2x^2 + 45x$

x-int

$x = 0$ or $45 - 2x = 0$

$45 = 2x$

$22.5 = x$

x-vertex $= \frac{-45}{2(-2)}$

$= 11.25$

y-vertex $= -2(11.25)^2 + 45(11.25)$

$= 253.125$

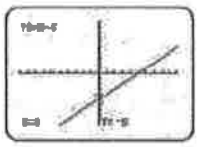
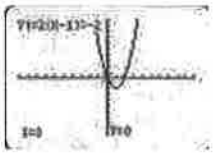
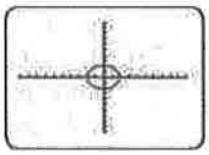
vertex is $(11.25, 253.125)$

b) D: $\{x \in \mathbb{R} \mid 0 < x < 22.5\}$

R: $\{y \in \mathbb{R} \mid 0 < y \leq 253.125\}$

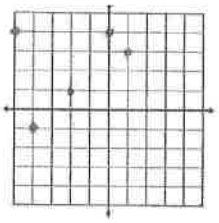
Answers

1) a, b, and c are functions. d is not a function.

2) a) function  **b)** function  **c)** not a function 

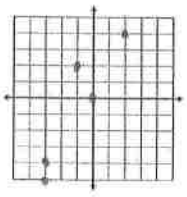
3) a) D: $\{X \in \mathbb{R} | x = -5, -4, -2, 0, 1\}$ R: $\{Y \in \mathbb{R} | y = -1, 1, 3, 4\}$

x	y
-5	4
-4	-1
-2	1
0	4
1	3



b) D: $\{X \in \mathbb{R} | x = -3, -1, 0, 2\}$ R: $\{Y \in \mathbb{R} | y = -4, 0, 2, 4, 5\}$

x	y
-3	-4
-1	2
0	0
-3	5
2	4



4) a) D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R}\}$ this relation is a function
b) D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} | y \geq 0\}$ this relation is a function
c) D: $\{X \in \mathbb{R} | x \geq 0\}$ R: $\{Y \in \mathbb{R}\}$ this relation is NOT a function
d) D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} | y \leq -1\}$ this relation is a function
e) D: $\{X \in \mathbb{R} | x \neq 3\}$ R: $\{Y \in \mathbb{R} | y \neq 0\}$ this relation is a function
f) D: $\{X \in \mathbb{R}\}$ R: $\{Y \in \mathbb{R} | -1 \leq y \leq 1\}$ this relation is a function
g) D: $\{X \in \mathbb{R} | -2 \leq x \leq 2\}$ R: $\{Y \in \mathbb{R} | -3 \leq y \leq 3\}$ this relation is NOT a function

5) b is the only relation that is a function

6) **a)** domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$
b) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -4\}$
c) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 1\}$
d) domain $\{x \in \mathbb{R}, -3 \leq x \leq 3\}$; range $\{y \in \mathbb{R}, -3 \leq y \leq 3\}$
e) domain $\{x \in \mathbb{R}, x \neq -3\}$, range $\{y \in \mathbb{R}, y \neq 0\}$
f) domain $\{x \in \mathbb{R}, x \geq -0.5\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

7) a) $A = -2x^2 + 45x$ **b)** D: $\{X \in \mathbb{R} | 0 < x < 22.5\}$ R: $\{Y \in \mathbb{R} | 0 < y \leq 253.1\}$

1.2 Functions and Function Notation – Worksheet

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SOLUTIONS

1) For each function, determine $f(4)$, $f(-5)$, and $f\left(-\frac{2}{3}\right)$.

a) $f(x) = \frac{2}{5}x + 11$

$f(4) = \frac{2}{5}(4) + 11$

$= \frac{8}{5} + \frac{55}{5}$

$= \frac{63}{5}$

$f(-5) = \frac{2}{5}(-5) + 11$

$= -2 + 11$

$= 9$

$f\left(-\frac{2}{3}\right) = \left(\frac{2}{5}\right)\left(-\frac{2}{3}\right) + 11$

$= -\frac{4}{15} + \frac{165}{15}$

$= \frac{161}{15}$

b) $f(x) = 3x^2 + 2x + 1$

$f(4) = 3(4)^2 + 2(4) + 1$

$= 48 + 8 + 1$

$= 57$

$f(-5) = 3(-5)^2 + 2(-5) + 1$

$= 75 - 10 + 1$

$= 66$

$f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 + 2\left(-\frac{2}{3}\right) + 1$

$= \frac{12}{9} - \frac{4}{3} + 1$

$= \frac{12}{9} - \frac{12}{9} - \frac{9}{9}$

$= 1$

c) $f(x) = 2(x + 4)^2$

$f(4) = 2(4+4)^2$

$= 2(64)$

$= 128$

$f(-5) = 2(-5+4)^2$

$= 2(1)$

$= 2$

$f\left(-\frac{2}{3}\right) = 2\left(-\frac{2}{3} + \frac{12}{3}\right)^2$

$= 2\left(\frac{10}{3}\right)^2$

$= \frac{200}{9}$

d) $f(x) = -6$

$f(4) = -6$ $f(-5) = -6$ $f\left(-\frac{2}{3}\right) = -6$

e) $f(x) = \frac{1}{x}$

$f(4) = \frac{1}{4}$ $f(-5) = -\frac{1}{5}$ $f\left(-\frac{2}{3}\right) = \frac{1}{\left(-\frac{2}{3}\right)}$

$= \frac{1}{1} \div \frac{-2}{3}$

$= \frac{1}{1} \times \frac{3}{-2}$

$= -\frac{3}{2}$

f) $f(x) = \sqrt{x+5}$

$f(4) = \sqrt{4+5}$

$= 3$

$f(-5) = \sqrt{-5+5}$

$= 0$

$f\left(-\frac{2}{3}\right) = \sqrt{-\frac{2}{3} + \frac{15}{3}}$

$= \sqrt{\frac{13}{3}}$

2) If $f(x) = x^2 + 2$, state the following.

a) $f(1) = (1)^2 + 2$
$= 3$

b) $f(0) = (0)^2 + 2$
$= 2$

c) $f(2) = (2)^2 + 2$
$= 6$

d) $f(-2) = (-2)^2 + 2$
$= 6$

e) $f(3) = (3)^2 + 2$
$= 11$

f) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 2$
$= \frac{1}{4} + \frac{8}{4}$
$= \frac{9}{4}$

3) State $f(4)$ for each of the following functions.

a) $f(x) = 4 + 5x$
$f(4) = 4 + 5(4)$
$= 24$

b) $f(x) = x^2 - 6$
$f(4) = (4)^2 - 6$
$= 10$

c) $f(t) = 9 - t$
$f(4) = 9 - 4$
$= 5$

d) $f(x) = 10$
$f(4) = 10$

e) $f(z) = z^3$
$f(4) = (4)^3$
$= 64$

f) $f(x) = 8(5 - x)$
$f(4) = 8(5 - 4)$
$= 8(1)$
$= 8$

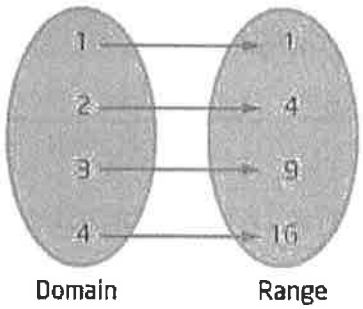
g) $f(x) = \frac{1}{x}$
$f(4) = \frac{1}{4}$

h) $f(x) = \sqrt{13 - x}$
$f(4) = \sqrt{13 - 4}$
$= \sqrt{9}$
$= 3$

i) $f(t) = \frac{1}{t^2}$
$f(4) = \frac{1}{4^2}$
$= \frac{1}{16}$

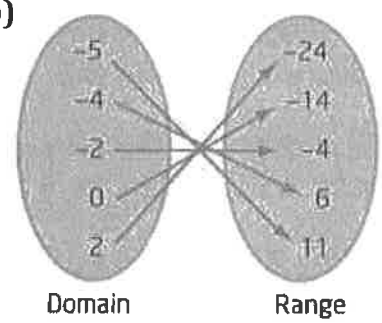
4) Write the ordered pairs associated with each mapping diagram. Then state if the relation is a function.

)



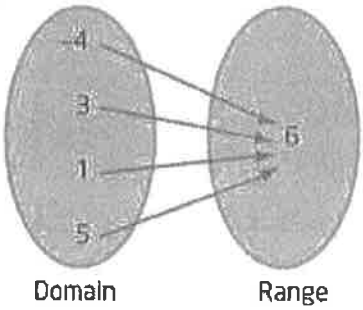
$\{(1,1),(2,4),(3,9),(4,16)\}$ Function

b)



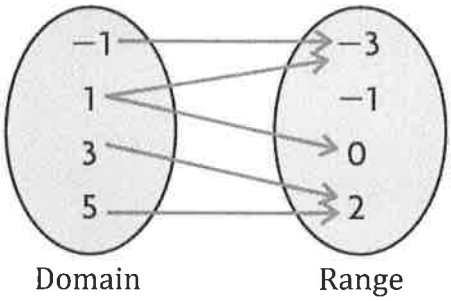
$\{(-5,11),(-4,6),(-2,-4),(0,-14),(2,-24)\}$ Function

)



$\{(-4,6),(3,6),(1,6),(5,6)\}$ function

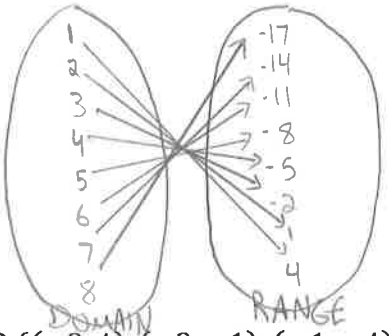
d)



$\{(-1,-3),(1,-3),(1,0),(3,2),(5,2)\}$ NOT a function

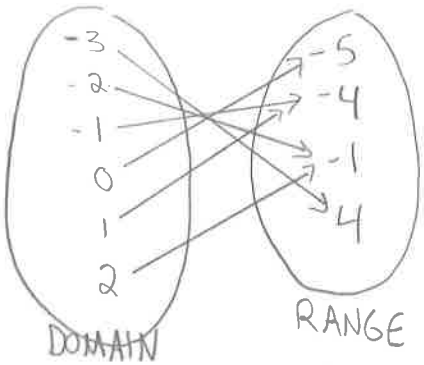
5) Show each set of data in a mapping diagram. Then state if the relation is a function.

a) $\{(1,4),(2,1),(3,-2),(4,-5),(5,-8),(6,-11),(7,-14),(8,-17)\}$



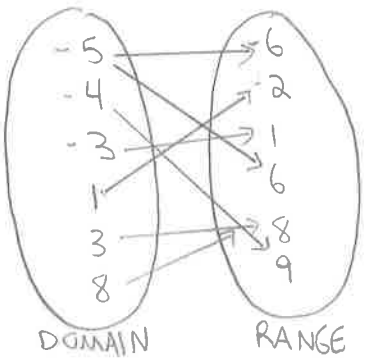
FUNCTION

b) $\{(-3,4),(-2,-1),(-1,-4),(0,-5),(1,-4),(2,-1)\}$



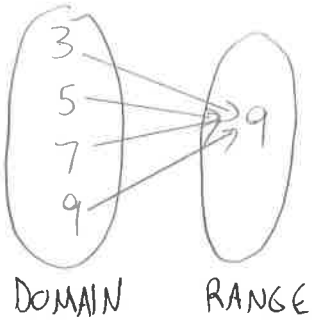
FUNCTION

c) $\{(-5,6),(-4,9),(-3,1),(-5,-6),(1,-2),(3,8),(8,8)\}$



NOT A FUNCTION

d) $\{(9,9),(7,9),(5,9),(3,9)\}$



6) State the domains of the following functions

a) $f(x) = \sqrt{8-x}$

$\{x \in \mathbb{R} \mid x \leq 8\}$

b) $f(x) = \frac{x^2+3}{(x-1)(x+3)}$

$\{x \in \mathbb{R} \mid x \neq 1, -3\}$

Answers

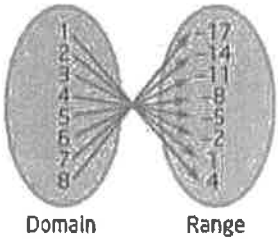
1) a) $\frac{63}{5}$, 9, $\frac{161}{15}$ **b)** 57, 66, 1 **c)** 128, 2, $\frac{200}{9}$ **d)** -6, -6, -6 **e)** $\frac{1}{4}, -\frac{1}{5}, -\frac{3}{2}$ **f)** 3, 0, $\sqrt{\frac{13}{3}}$

2) a) 3 **b)** 2 **c)** 6 **d)** 6 **e)** 11 **f)** $\frac{9}{4}$

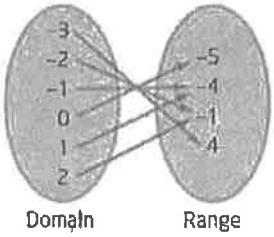
3) a) 24 **b)** 10 **c)** 5 **d)** 10 **e)** 64 **f)** 8 **g)** $\frac{1}{4}$ **h)** 3 **i)** $\frac{1}{16}$

4) a) $\{(1,1),(2,4),(3,9),(4,16)\}$ this relation is a function
b) $\{(-5,11),(-4,6),(-2,-4),(0,-14),(2,-24)\}$ this relation is a function
c) $\{(-4,6),(3,6),(1,6),(5,6)\}$ this relation is a function
d) $\{(-1,-3),(1,-3),(1,0),(3,2),(5,2)\}$ this relation is NOT a function

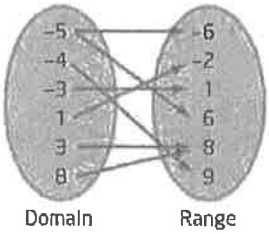
5) a) function



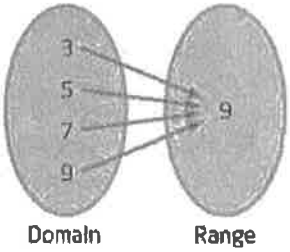
b) function



c) Not a function



d) function



) a) $\{X \in \mathbb{R} | x \leq 8\}$ **b)** $\{X \in \mathbb{R} | x \neq 1, x \neq -3\}$

1.3 Max or Min of a Quadratic Function – Worksheet

MCR3U

Jensen

SOLUTIONS

1) Determine the vertex for each quadratic function by completing the square. State if the vertex is a maximum or a minimum.

a) $f(x) = x^2 + 14x - 14$

$= (x^2+14x) - 14$

$= (x^2+14x+49-49) - 14$

$= (x^2+14x+49) - 49 - 14$

$= (x+7)^2 - 63$

vertex is $(-7, -63)$ and is a minimum.

b) $f(x) = x^2 - 6x + 17$

$= (x^2-6x) + 17$

$= (x^2-6x+9-9) + 17$

$= (x^2-6x+9) - 9 + 17$

$= (x-3)^2 + 8$

vertex is $(3, 8)$ and is a minimum.

c) $f(x) = x^2 + 7x + 11$

$= (x^2+7x+\frac{49}{4}-\frac{49}{4}) + 11$

$= (x^2+7x+\frac{49}{4}) - \frac{49}{4} + \frac{44}{4}$

$= (x+\frac{7}{2})^2 - \frac{5}{4}$

vertex is $(-\frac{7}{2}, -\frac{5}{4})$ and is a minimum

d) $f(x) = 2x^2 + 12x + 16$

$= (2x^2+12x) + 16$

$= 2(x^2+6x+9-9) + 16$

$= 2(x^2+6x+9) - 18 + 16$

$= 2(x+3)^2 - 2$

vertex is $(-3, -2)$ and is a minimum.

e) $f(x) = -3x^2 + 6x + 1$

$= (-3x^2+6x) + 1$

$= -3(x^2-2x+1-1) + 1$

$= -3(x^2-2x+1) + 3 + 1$

$= -3(x-1)^2 + 4$

Vertex is $(1, 4)$ and is a maximum.

f) $f(x) = -\frac{1}{2}x^2 - x + \frac{3}{2}$

$f(x) = (-\frac{1}{2}x^2 - 1x) + \frac{3}{2}$

$= -\frac{1}{2}(x^2+2x+1-1) + \frac{3}{2}$

$= -\frac{1}{2}(x^2+2x+1) + \frac{1}{2} + \frac{3}{2}$

$= \frac{1}{2}(x+1)^2 + 2$

vertex is $(-1, 2)$ and is a max

2) Use partial factoring to determine the vertex of each function. State if the vertex is a max or min.

a) $f(x) = 3x^2 - 6x + 11$

$11 = 3x^2 - 6x + 11$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$3x = 0 \quad x - 2 = 0$

$x = 0 \quad x = 2$

$x\text{-vertex} = \frac{0+2}{2}$

$= 1$

$y\text{-vertex} = 3(1)^2 - 6(1) + 11$

$= 8$

vertex is $(1, 8)$ and is a min.

b) $f(x) = -2x^2 + 8x - 3$

$-3 = -2x^2 + 8x - 3$

$0 = -2x^2 + 8x$

$0 = -2x(x-4)$

$-2x = 0 \quad x - 4 = 0$

$x = 0 \quad x = 4$

$x\text{-vertex} = \frac{0+4}{2}$

$= 2$

$y\text{-vertex} = -2(2)^2 + 8(2) - 3$

$= 5$

vertex is $(2, 5)$ and is a MAX.

c) $h(x) = -x^2 + 2x + 4$

$4 = -x^2 + 2x + 4$

$0 = -x^2 + 2x$

$0 = -1x(x - 2)$

$-1x = 0 \quad x - 2 = 0$

$x = 0 \quad x = 2$

$x\text{-vertex} = \frac{0+2}{2}$

$= 1$

$y\text{-vertex} = -(1)^2 + 2(1) + 4$

$= 5$

vertex is $(1, 5)$ and is a MAX

d) $f(x) = 2x^2 + 12x + 17$

$17 = 2x^2 + 12x + 17$

$0 = 2x^2 + 12x$

$0 = 2x(x + 6)$

$2x = 0 \quad x + 6 = 0$

$x = 0 \quad x = -6$

$x\text{-vertex} = \frac{0+(-6)}{2}$

$= -3$

$y\text{-vertex} = 2(-3)^2 + 12(-3) + 17$

$= -1$

vertex is $(-3, -1)$ and is a MIN.

e) $f(x) = 4x^2 + 64x + 156$

$156 = 4x^2 + 64x + 156$

$0 = 4x^2 + 64x$

$0 = 4x(x + 16)$

$4x = 0 \quad x + 16 = 0$

$x = 0 \quad x = -16$

$x\text{-vertex} = \frac{0+(-16)}{2}$

$= -8$

$y\text{-vertex} = 4(-8)^2 + 64(-8) + 156$

$= -100$

vertex is $(-8, -100)$ and is a MIN.

f) $f(x) = \frac{1}{2}x^2 - 3x + 8$

$8 = \frac{1}{2}x^2 - 3x + 8$

$0 = \frac{1}{2}x^2 - 3x$

$0 = \frac{1}{2}x(x - 6)$

$\frac{1}{2}x = 0 \quad x - 6 = 0$

$x = 0 \quad x = 6$

$x\text{-vertex} = \frac{0+6}{2}$

$= 3$

$y\text{-vertex} = \frac{1}{2}(3)^2 - 3(3) + 8$

$= \frac{9}{2} - \frac{18}{2} + \frac{16}{2}$

$= \frac{7}{2}$

vertex is $(3, 7/2)$ → MIN.

3) An electronics store sells an average of 60 entertainment systems per month at an average of $800 more than the cost price. For every $20 increase in the selling price, the store sells one fewer system. What amount over the cost price will maximize revenue?

Revenue = (cost)(# sold)

$0 = (800 + 20x)(60 - x)$

$800 + 20x = 0 \quad 60 - x = 0$

$20x = -800 \quad x = 60$

$x = -40$

$x\text{-vertex} = \frac{-40+60}{2}$

$= 10$

$y\text{-vertex} = [800 + 20(10)][60 - 10]$

$= (1000)(50)$

$= 50\,000$

vertex is $(10, 50\,000)$

↑ # of increases ↑ revenue

cost = $800 + 20(10)$

= $1000

4) Last year, a banquet hall charged $30 per person, and 60 people attended the hockey banquet dinner. This year, the hall's manager has said that for every 10 extra people that attend the banquet, they will decrease the price by $1.50 per person. What size group would maximize the profit for the hall this year and what would the maximum profit be?

Profit = (cost)(# of people)

$0 = (30 - 1.50x)(60 + 10x)$

$30 - 1.5x = 0 \quad 60 + 10x = 0$

$30 = 1.5x \quad x = -6$

$x = 20$

$x\text{-vertex} = \frac{20+(-6)}{2}$

$= 7$

$y\text{-vertex} = [30 - 1.5(7)][60 + 10(7)]$

$= (19.5)(130)$

cost × people

$= 2535$

vertex is $(7, 2535)$

↑ # of decreases ↑ profit

A group of 130 people would give a max profit of $2535

5) The path of a rocket is given by the function, $h(t) = -3t^2 + 30t + 73$, where 'h' is the height in meters and 't' is the time in seconds.

) What is the maximum height of the rocket?

$h(t) = -3(t^2 - 10t) + 73$

$= -3(t^2 - 10t + 25 - 25) + 73$

$= -3(t^2 - 10t + 25) + 75 + 73$

$= -3(t-5)^2 + 148$

vertex is $(5, 148)$

time, height

Max height is 148 m.

b) At what time does the rocket reach its maximum height?

5 seconds.

Answers

1) a) (-7,-63) min **b)** (3,8) min **c)** $\left(\frac{-7}{2}, \frac{-5}{4}\right)$ min **d)** (-3, -2) min **e)** (1, 4) max **f)** (-1, 2) max

2) a) (1, 8) min **b)** (2, 5) max **c)** (1, 5) max **d)** (-3, -1) min **e)** (-8, -100) min **f)** $(3, \frac{7}{2})$ min

3) $1000

) A group of 130 would give a max profit of $2535

5) a) 148 m **b)** 5 seconds

1.4 Working with Radicals – Worksheet

MCR3U
Jensen

SOLUTIONS

1) Simplify

a) $3(4\sqrt{5})$

$= 12\sqrt{5}$

b) $\sqrt{5}(-2\sqrt{7})$

$= -2\sqrt{35}$

c) $2\sqrt{3}(3\sqrt{2})$

$= 6\sqrt{6}$

2) Express each as a mixed radical in simplest form

a) $\sqrt{12}$

$= \sqrt{4} \times \sqrt{3}$

$= 2\sqrt{3}$

b) $\sqrt{147}$

$= \sqrt{49 \times 3}$

$= \sqrt{49} \times \sqrt{3}$

$= 7\sqrt{3}$

c) $\sqrt{252}$

$= \sqrt{36 \times 7}$

$= \sqrt{36} \times \sqrt{7}$

$= 6\sqrt{7}$

3) Simplify

a) $2\sqrt{3} - 5\sqrt{3} + 4\sqrt{3}$

$= 1\sqrt{3}$

b) $11\sqrt{5} - 4\sqrt{5} - 5\sqrt{5} - 6\sqrt{5}$

$= -4\sqrt{5}$

c) $\sqrt{6} - 4\sqrt{2} + 3\sqrt{6} - \sqrt{2}$

$= 4\sqrt{6} - 5\sqrt{2}$

d) $2\sqrt{10} - \sqrt{10} - 4\sqrt{10} + \sqrt{5}$

$= -3\sqrt{10} + \sqrt{5}$

4) Add or subtract as indicated

a) $8\sqrt{2} - 4\sqrt{8} + \sqrt{32}$

$= 8\sqrt{2} - 4(\sqrt{4})(\sqrt{2}) + (\sqrt{16})(\sqrt{2})$

$= 8\sqrt{2} - 8\sqrt{2} + 4\sqrt{2}$

$= 4\sqrt{2}$

b) $\sqrt{20} - 4\sqrt{12} - \sqrt{125} + 2\sqrt{3}$

$= (\sqrt{4})(\sqrt{5}) - 4(\sqrt{4})(\sqrt{3}) - (\sqrt{5})(\sqrt{25}) + 2\sqrt{3}$

$= 2\sqrt{5} - 8\sqrt{3} - 5\sqrt{5} + 2\sqrt{3}$

$= -3\sqrt{5} - 6\sqrt{3}$

c) $5\sqrt{3} - \sqrt{72} + \sqrt{243} + \sqrt{8}$

$= 5\sqrt{3} - (\sqrt{36})(\sqrt{2}) + (\sqrt{81})(\sqrt{3}) + (\sqrt{4})(\sqrt{2})$

$= 5\sqrt{3} - 6\sqrt{2} + 9\sqrt{3} + 2\sqrt{2}$

$= 14\sqrt{3} - 4\sqrt{2}$

d) $\sqrt{44} + \sqrt{88} + \sqrt{99} + \sqrt{198}$

$= (\sqrt{4})(\sqrt{11}) + (\sqrt{4})(\sqrt{22}) + (\sqrt{9})(\sqrt{11}) + (\sqrt{9})(\sqrt{22})$

$= 2\sqrt{11} + 2\sqrt{22} + 3\sqrt{11} + 3\sqrt{22}$

$= 5\sqrt{11} + 5\sqrt{22}$

5) Expand and simplify

a) $5\sqrt{6}(2\sqrt{3})$

$= 10\sqrt{18}$

$= 10(\sqrt{9})(\sqrt{2})$

$= 30\sqrt{2}$

b) $8\sqrt{5}(\sqrt{10})$

$= 8\sqrt{50}$

$= 8(\sqrt{25})(\sqrt{2})$

$= 40\sqrt{2}$

c) $11\sqrt{2}(5\sqrt{3})$

$= 55\sqrt{6}$

6) Expand and simplify where possible

a) $3(8 - \sqrt{5})$

$= 24 - 3\sqrt{5}$

b) $\sqrt{3}(\sqrt{6} - \sqrt{3})$

$= \sqrt{18} - \sqrt{9}$

$= (\sqrt{9})(\sqrt{2}) - \sqrt{9}$

$= 3\sqrt{2} - 3$

c) $8\sqrt{2}(2\sqrt{8} + 3\sqrt{12})$

$= 16\sqrt{16} + 24\sqrt{24}$

$= 64 + 24(\sqrt{4})(\sqrt{6})$

$= 64 + 48\sqrt{6}$

7) Expand and simplify where possible

a) $(\sqrt{2} + 5)(\sqrt{2} + 5)$

$= \sqrt{4} + 5\sqrt{2} + 5\sqrt{2} + 25$

$= 2 + 10\sqrt{2} + 25$

$= 27 + 10\sqrt{2}$

b) $(\sqrt{3} + 2\sqrt{2})(5 + 5\sqrt{2})$

$= 5\sqrt{3} + 5\sqrt{6} + 10\sqrt{2} + 10\sqrt{4}$

$= 5\sqrt{3} + 5\sqrt{6} + 10\sqrt{2} + 20$

c) $(1 + \sqrt{5})(1 - \sqrt{5})$ D.O.S.

$= (1)^2 - (\sqrt{5})^2$

$= 1 - 5$

$= -4$

d) $(4 - 3\sqrt{7})(\sqrt{7} + 1)$

$= 4\sqrt{7} + 4 - 3\sqrt{49} - 3\sqrt{7}$

$= 1\sqrt{7} + 4 - 21$

$= \sqrt{7} - 17$

8) Simplify

a) $\frac{1}{4}\sqrt{54} - \frac{1}{4}\sqrt{150}$

$= \frac{1}{4}(\sqrt{9})(\sqrt{6}) - \frac{1}{4}(\sqrt{25})(\sqrt{6})$

$= \frac{3}{4}\sqrt{6} - \frac{5}{4}\sqrt{6}$

$= -\frac{1}{2}\sqrt{6}$

b) $\frac{1}{2}\sqrt{8} + \frac{3}{5}\sqrt{50} - \frac{2}{3}\sqrt{18}$

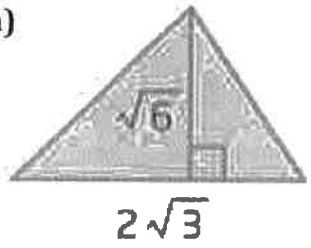
$= \frac{1}{2}(\sqrt{4})(\sqrt{2}) + \frac{3}{5}(\sqrt{25})(\sqrt{2}) - \frac{2}{3}(\sqrt{9})(\sqrt{2})$

$= 1\sqrt{2} + 3\sqrt{2} - 2\sqrt{2}$

$= 2\sqrt{2}$

9) Find a simplified expression for the area of each shape

a)



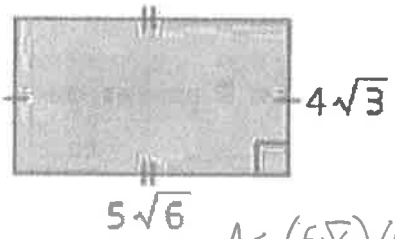
$A = \frac{2\sqrt{3}(\sqrt{6})}{2}$

$= \sqrt{18}$

$= (\sqrt{9})(\sqrt{2})$

$= 3\sqrt{2}$

b)



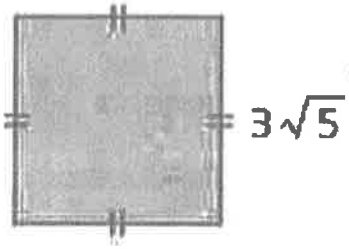
$A = (5\sqrt{6})(4\sqrt{3})$

$= 20\sqrt{18}$

$= 20(\sqrt{9})(\sqrt{2})$

$= 60\sqrt{2}$

c)



$A = (3\sqrt{5})^2$

$= 9(5)$

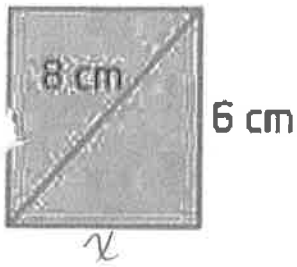
$= 45$

d)

$A = \pi(\sqrt{2})^2$

$= 2\pi$

10) Find the area and perimeter of the rectangle shown. Express your answer in simplified radical form.



$x^2 + 6^2 = 8^2$

$x^2 = 64 - 36$

$x = \sqrt{28}$

$x = 2\sqrt{7}$

$A = (2\sqrt{7})(6)$

$= 12\sqrt{7}\ cm^2$

$P = 2(2\sqrt{7}) + 2(6)$

$= 4\sqrt{7} + 12\ cm$

11) Simplify each of the following

a) $\frac{21-7\sqrt{6}}{7}$

$= \frac{7(3-\sqrt{6})}{7}$

$= 3-\sqrt{6}$

b) $\frac{12-\sqrt{48}}{4}$

$= \frac{12-(\sqrt{16})(\sqrt{3})}{4}$

$= \frac{12-4\sqrt{3}}{4}$

$= \frac{4(3-\sqrt{3})}{4}$

$= 3-\sqrt{3}$

Answers

1) a) $12\sqrt{5}$ **b)** $-2\sqrt{35}$ **c)** $6\sqrt{6}$

2) a) $2\sqrt{3}$ **b)** $7\sqrt{3}$ **c)** $6\sqrt{7}$

3) a) $\sqrt{3}$ **b)** $-4\sqrt{5}$ **c)** $4\sqrt{6}-5\sqrt{2}$ **d)** $-3\sqrt{10}+\sqrt{5}$

4) a) $4\sqrt{2}$ **b)** $-3\sqrt{5}-6\sqrt{3}$ **c)** $14\sqrt{3}-4\sqrt{2}$ **d)** $5\sqrt{11}+5\sqrt{22}$

5) a) $30\sqrt{2}$ **b)** $40\sqrt{2}$ **c)** $55\sqrt{6}$

6) a) $24-3\sqrt{5}$ **b)** $3\sqrt{2}-3$ **c)** $64+48\sqrt{6}$

7) a) $27+10\sqrt{2}$ **b)** $5\sqrt{3}+5\sqrt{6}+10\sqrt{2}+20$ **c)** -4 d) $-17+\sqrt{7}$

8) a) $-\frac{1}{2}\sqrt{6}$ **b)** $2\sqrt{2}$

9) a) $3\sqrt{2}$ **b)** $60\sqrt{2}$ **c)** 45 **d)** 2π

10) area = $12\sqrt{7}$ cm^2; perimeter = $12+4\sqrt{7}$ cm

11) a) $3-\sqrt{6}$ **b)** $3-\sqrt{3}$

1.5 Solving Quadratic Equations – Part 1: Solve by Factoring – Worksheet

MCR3U
Iensen

1) Solve by factoring

a) $x^2 + 8x + 12 = 0$

$(x+6)(x+2) = 0$

$x+6=0 \quad x+2=0$
$x=-6 \quad x=-2$

b) $h^2 + 9h + 18 = 0$

$(h+6)(h+3) = 0$

$h+6=0 \quad h+3=0$
$h=-6 \quad h=-3$

c) $m^2 + 3m = 0$

$m(m+3) = 0$

$m=0 \quad m+3=0$
$m=-3$

d) $w^2 - 18w + 56 = 0$

$(w-14)(w-4) = 0$

$w-14=0 \quad w-4=0$
$w=14 \quad w=4$

e) $x^2 - 2x = 0$

$x(x-2) = 0$

$x=0 \quad x-2=0$
$x=2$

f) $c^2 - 17c + 30 = 0$

$(c-15)(c-2) = 0$

$c-15=0 \quad c-2=0$
$c=15 \quad c=2$

2) Solve

a) $3x^2 + 28x + 9 = 0$ S: 28 P: 27 (27 and 1)

$3x^2 + 27x + 1x + 9 = 0$

$(3x^2 + 27x) + (1x + 9) = 0$

$3x(x+9) + 1(x+9) = 0$

$(x+9)(3x+1) = 0$

$x+9=0 \quad 3x+1=0$
$x=-9 \quad x=-1/3$

b) $4k^2 + 19k + 15 = 0$ S: 19 P: 60 (4 and 15)

$4k^2 + 4k + 15k + 15 = 0$

$(4k^2 + 4k) + (15k + 15) = 0$

$4k(k+1) + 15(k+1) = 0$

$(k+1)(4k+15) = 0$

$k+1=0 \quad 4k+15=0$
$k=-1 \quad k=-15/4$

d) $16b^2 - 1 = 0$ D.O.S.

$(4b-1)(4b+1) = 0$

$4b-1=0$ $\quad$ $4b+1=0$

$b = 1/4$ $\quad$ $b = -1/4$

f) $4x^2 - 12x + 9 = 0$ P.S.T. $a^2 - 2ab + b^2 = (a-b)^2$

$(2x-3)^2 = 0$

$2x-3=0$

$x = 3/2$

3) Solve each quadratic equation by factoring

a) $x^2 + 2x - 3 = 0$ S:2 P:-3 (3 and -1)

$(x+3)(x-1) = 0$

$x+3=0$ $\quad$ $x-1=0$

$x=-3$ $\quad$ $x=1$

b) $x^2 + 3x - 10 = 0$ S:3 P:-10 (5 and -2)

$(x+5)(x-2) = 0$

$x+5=0$ $\quad$ $x-2=0$

$x=-5$ $\quad$ $x=2$

c) $4x^2 - 36 = 0$ D.O.S.

$(2x-6)(2x+6) = 0$

$2x-6=0$ $\quad$ $2x+6=0$

$x=3$ $\quad$ $x=-3$

d) $6x^2 - 14x + 8 = 0$ S:-14 P:48 (-6 and -8)

$6x^2 - 6x - 8x + 8 = 0$

$(6x^2 - 6x) + (-8x + 8) = 0$

$6x(x-1) - 8(x-1) = 0$

$(x-1)(6x-8) = 0$

$x-1=0$ $\quad$ $6x-8=0$

$x=1$ $\quad$ $x=4/3$

e) $15x^2 - 8x + 1 = 0$ S:-8 P:15 (-5 and -3)

$15x^2 - 5x - 3x + 1 = 0$

$(15x^2 - 5x) + (-3x + 1) = 0$

$5x(3x-1) - 1(3x-1) = 0$

$(3x-1)(5x-1) = 0$

$3x-1=0$ $\quad$ $5x-1=0$

$x=1/3$ $\quad$ $x=1/5$

f) $6x^2 + 19x + 10 = 0$ S:19 P:60 (15 and 4)

$6x^2 + 15x + 4x + 10 = 0$

$(6x^2 + 15x) + (4x + 10) = 0$

$3x(2x+5) + 2(2x+5) = 0$

$(2x+5)(3x+2) = 0$

$2x+5=0$ $\quad$ $3x+2=0$

$x=-5/2$ $\quad$ $x=-2/3$

4) Solve by factoring

a) $-x^2 - 10x - 16 = 0$

$-1(x^2+10x+16) = 0$

$x^2+10x+16 = 0$

$(x+2)(x+8) = 0$

$x+2=0 \quad x+8=0$

$x=-2 \quad x=-8$

b) $6d^2 + 15d = -9$

$6d^2+15d+9=0$

$3(2d^2+5d+3)=0$

$2d^2+5d+3=0$

$2d^2+2d+3d+3=0$

$(2d^2+2d)+(3d+3)=0$

$2d(d+1)+3(d+1)=0$

$(d+1)(2d+3)=0$

$d+1=0 \quad 2d+3=0$

$d=-1 \quad d=-3/2$

5) A rectangle has dimensions x+10 and 2x-3. Determine the value of x that gives an area of 54 cm²

$(x+10)(2x-3) = 54$

$2x^2-3x+20x-30=54$

$2x^2+17x-84=0$

$2x^2+24x-7x-84=0$

$(2x^2+24x)+(-7x-84)=0$

$2x(x+12)-7(x+12)=0$

$(x+12)(2x-7)=0$

$x+12=0$

$x=-12$

$2x-7=0$

$x=7/2$ or 3.5 cm

Answers

1) a) -2, -6 **b)** -3, -6 **c)** 0, -3 **d)** 14, 4 **e)** 0, 2 **f)** 15, 2

2) a) $\frac{-1}{3}$, -9 **b)** -1, $\frac{-15}{4}$ **d)** $\frac{1}{4}, \frac{-1}{4}$ **f)** $\frac{3}{2}$

3) a) -3, 1 **b)** -5, 2 **c)** -3, 3 **d)** $\frac{4}{3}$, 1 **e)** $\frac{1}{5}, \frac{1}{3}$ **f)** $-\frac{5}{2}, -\frac{2}{3}$

4) a) -8, -2 **b)** -1, $\frac{-3}{2}$

5) 3.5

1.5 Solving Quadratic Equations – Part 2: Solve Using the Q.F. - Worksheet

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Iensen

SOLUTIONS

1) Use the discriminant to determine the number of roots for each quadratic equation.

a) $x^2 - 10x + 25 = 0$

$b^2-4ac = (-10)^2-4(1)(25)$

$= 0$

∴ 1 root

b) $3x^2 + 4x + \frac{4}{3} = 0$

$b^2-4ac = (4)^2-4(3)(\frac{4}{3})$

$= 0$

∴ 1 root

c) $2x^2 - 8x + 9 = 0$

$b^2-4ac = (-8)^2-4(2)(9)$

$= -8$

∴ No Roots

d) $-2x^2 + 0.75x + 5 = 0$

$b^2-4ac = (0.75)^2-4(-2)(5)$

$= 40.5625$

∴ 2 Roots

2) Solve each quadratic using the quadratic formula. Give exact answers.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

a) $x^2 + 4x - 3 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2-4(1)(-3)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{28}}{2}$

$x = \frac{-4 \pm 2\sqrt{7}}{2}$

$x = \frac{2(-2 \pm \sqrt{7})}{2}$

$x = -2+\sqrt{7}$ and $x = -2-\sqrt{7}$

b) $-x^2 + 12 = 9x$

$-x^2 - 9x + 12 = 0$

$x = \frac{9 \pm \sqrt{(-9)^2-4(-1)(12)}}{2(-1)}$

$x = \frac{9 \pm \sqrt{129}}{-2}$

$x = \frac{9+\sqrt{129}}{-2}$ $\quad x = \frac{9-\sqrt{129}}{-2}$

c) $x^2 = -5x + 2$

$x^2 + 5x - 2 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2-4(1)(-2)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{33}}{2}$

$x = \frac{-5+\sqrt{33}}{2}$ $\quad x = \frac{-5-\sqrt{33}}{2}$

d) $x^2 - 3x + 1 = 6$

$x^2 - 3x - 5 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2-4(1)(-5)}}{2(1)}$

$x = \frac{3 \pm \sqrt{29}}{2}$

$x = \frac{3+\sqrt{29}}{2}$ $\quad x = \frac{3-\sqrt{29}}{2}$

e) $x^2 + 6x + 9 = 0$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{0}}{2}$

$x = -3$

f) $4x^2 - 6x - 1 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(-1)}}{2(4)}$

$x = \frac{6 \pm \sqrt{52}}{8}$

$x = \frac{6 \pm 2\sqrt{13}}{8}$

$x = \frac{2(3 \pm \sqrt{13})}{8}$

$x = \frac{3 \pm \sqrt{13}}{4}$

$x = \frac{3+\sqrt{13}}{4}$ $x = \frac{3-\sqrt{13}}{4}$

g) $5x^2 - 3x - 1 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$

$x = \frac{3 \pm \sqrt{29}}{10}$

$x = \frac{3+\sqrt{29}}{10}$ $x = \frac{3-\sqrt{29}}{10}$

h) $-x^2 + 7x - 18 = 0$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(-1)(-18)}}{2(-1)}$

$x = \frac{-7 \pm \sqrt{-23}}{-2}$

∞ No Roots

i) $4x^2 - 25 = 0$

$4x^2 + 0x - 25 = 0$

$x = \frac{0 \pm \sqrt{(0)^2 - 4(4)(-25)}}{2(4)}$

$x = \frac{\pm 20}{8}$

$x = \frac{5}{2}$ $x = \frac{-5}{2}$

j) $3x^2 - 7x - 4 = x^2 - 4x$

$2x^2 - 3x - 4 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$

$x = \frac{3 \pm \sqrt{41}}{4}$

$x = \frac{3+\sqrt{41}}{4}$ $x = \frac{3-\sqrt{41}}{4}$

k) $8x^2 + 4x - 5 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(8)(-5)}}{2(8)}$

$x = \frac{-4 \pm \sqrt{176}}{16}$

$x = \frac{-4 \pm 4\sqrt{11}}{16}$

$x = \frac{4(-1 \pm \sqrt{11})}{16}$

$x = \frac{-1+\sqrt{11}}{4}$

$x = \frac{-1-\sqrt{11}}{4}$

l) $4x^2 - 18x = 0$

$x = \frac{18 \pm \sqrt{(-18)^2 - 4(4)(0)}}{2(4)}$

$x = \frac{18 \pm 18}{8}$

$x = \frac{18+18}{8}$ $x = \frac{18-18}{8}$

$x = 9/2$ $x = 0$

3) Solve each quadratic equation using any method

a) $3x^2 - 12x = 0$

$3x(x-4)=0$

$3x=0 \quad x-4=0$

$x=0 \quad x=4$

b) $2x^2 + 4x - 6 = 0$ 5:4 p:-12 (6 and -2)

$2x^2+6x-2x-6=0$

$(2x^2+6x)+(-2x-6)=0$

$2x(x+3)-2(x+3)=0$

$(x+3)(2x-2)=0$

$x=-3 \quad x=1$

c) $3x^2 + 5x - 2 = 0$ 5:5 p:-6 (6 and -1)

$3x^2-1x+6x-2=0$

$x(3x-1)+2(3x-1)=0$

$(3x-1)(x+2)=0$

$x=1/3 \quad x=-2$

d) $4x^2 - 11x - 8 = 0$

$x=\frac{11\pm\sqrt{(-11)^2-4(4)(-8)}}{2(4)}$

$x=\frac{11\pm\sqrt{249}}{8}$

$x=\frac{11+\sqrt{249}}{8} \quad x=\frac{11-\sqrt{249}}{8}$

4) Three lengths of pipe measuring 24 cm, 31 cm, and 38 cm will be used to create a right triangle. The same length of pipe will be cut off each of the three pipes to allow a right triangle to be created. What is that length?

$(24-x)^2+(31-x)^2=(38-x)^2$

$576-48x+x^2+961-62x+x^2=1444-76x+x^2$

$x^2-34x+93=0$

$(x-3)(x-31)=0$

$x=3 \quad x=31$

↑ reject because can't cut 31 cm from 24 cm.

3 cm

Answers

1) a) one **b)** one **c)** none **d)** two

2) a) $-2+\sqrt{7}, -2-\sqrt{7}$ **b)** $\frac{9+\sqrt{129}}{-2}, \frac{9-\sqrt{129}}{-2}$ **c)** $\frac{-5+\sqrt{33}}{2}, \frac{-5-\sqrt{33}}{2}$ **d)** $\frac{3+\sqrt{29}}{2}, \frac{3-\sqrt{29}}{2}$ **e)** -3 **f)** $\frac{3+\sqrt{13}}{4}, \frac{3-\sqrt{13}}{4}$

g) $\frac{3+\sqrt{29}}{10}, \frac{3-\sqrt{29}}{10}$ **h)** no roots **i)** $\frac{5}{2}, -\frac{5}{2}$ **j)** $\frac{3+\sqrt{41}}{4}, \frac{3-\sqrt{41}}{4}$ **k)** $\frac{-1+\sqrt{11}}{4}, \frac{-1-\sqrt{11}}{4}$ **l)** $\frac{9}{2}, 0$

3) a) $x = 0$ and $x = 4$ **b)** $x = 1$ and $x = -3$ **c)** $x = \frac{1}{3}$ and $x = -2$ **d)** $x = \frac{11 \pm \sqrt{249}}{8}$

4) 3 cm

1.7 Solve Linear-Quadratic Systems – Worksheet

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SOLUTIONS

1) Determine if each quadratic function will intersect once, twice, or not at all with the given linear function.

a) $y = 2x^2 - 2x + 1$ and $y = 3x - 5$

$3x - 5 = 2x^2 - 2x + 1$

$0 = 2x^2 - 5x + 6$

$b^2 - 4ac = (-5)^2 - 4(2)(6)$

$= -23$

∴ They DON'T intersect

b) $y = -x^2 + 3x - 5$ and $y = -x - 1$

$-x - 1 = -x^2 + 3x - 5$

$0 = -x^2 + 4x - 4$

$b^2 - 4ac = (4)^2 - 4(-1)(-4)$

$= 0$

∴ 1 POI

c) $y = \frac{1}{2}x^2 + 4x - 2$ and $y = x + 3$

$x + 3 = \frac{1}{2}x^2 + 4x - 2$

$0 = \frac{1}{2}x^2 + 3x - 5$

$b^2 - 4ac = (3)^2 - 4\left(\frac{1}{2}\right)(-5)$

$= 19$

∴ 2 POI's.

d) $y = -\frac{2}{3}x^2 + x + 3$ and $y = x$

$x = -\frac{2}{3}x^2 + x + 3$

$0 = -\frac{2}{3}x^2 + 3$

$b^2 - 4ac = (0)^2 - 4\left(-\frac{2}{3}\right)(3)$

$= 8$

∴ 2 POI's.

2) Determine the coordinates of the point(s) of intersection of each linear-quadratic system.

a) $y = x^2 - 7x + 15$ and $y = 2x - 5$

$2x - 5 = x^2 - 7x + 15$

$0 = x^2 - 9x + 20$

$0 = (x - 4)(x - 5)$

$x - 4 = 0$ $\quad x - 5 = 0$

$x = 4$ $\quad x = 5$

POI #1

$f(x) = 2x - 5$

$f(4) = 2(4) - 5$

$f(4) = 3$

$(4, 3)$

POI #2

$f(x) = 2x - 5$

$f(5) = 2(5) - 5$

$f(5) = 5$

$(5, 5)$

b) $y = 3x^2 - 16x + 37$ and $y = 8x + 1$

$8x+1 = 3x^2-16x+37$

$0 = 3x^2 - 24x + 36$

$0 = 3(x^2 - 8x + 12)$

$0 = x^2 - 8x + 12$

$0 = (x-2)(x-6)$

$x-2=0$ $\quad$ $x-6=0$

$x=2$ $\quad$ $x=6$

POI #1

$f(x) = 8x+1$

$f(2) = 8(2)+1$

$f(2) = 17$

$(2,17)$

POI #2

$f(x) = 8x+1$

$f(6) = 8(6)+1$

$f(6) = 49$

$(6,49)$

c) $y = \frac{1}{2}x^2 - 2x - 3$ and $y = -3x + 1$

$-3x+1 = \frac{1}{2}x^2 - 2x - 3$

$0 = \frac{1}{2}x^2 + 1x - 4$

$0 = \frac{1}{2}(x^2 + 2x - 8)$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x+4=0$ $\quad$ $x-2=0$

$x=-4$ $\quad$ $x=2$

POI #1

$F(x) = -3x+1$

$f(-4) = -3(-4)+1$

$f(-4) = 13$

$(-4,13)$

POI #2

$f(x) = -3x+1$

$F(2) = -3(2)+1$

$f(2) = -5$

$(2,-5)$

3) Determine the value of the y-intercept of a line with the given slope that is a tangent line to the given curve.

a) $y = -2x^2 + 5x + 4$ and a line with a slope of 1 $y = 1x+b$

$1x+b = -2x^2+5x+4$

$0 = -2x^2+4x+(4-b)$

*Set $b^2-4ac=0$ *

$(4)^2 - 4(-2)(4-b) = 0$

$16 - 4(-8+2b) = 0$

$16+32-8b = 0$

$-8b = -48$

$b = 6$

b) $y = -x^2 - 5x - 5$ and a line with a slope of -3 $y=-3x+b$

$-3x+b = -x^2-5x-5$

$0 = -1x^2 - 2x + (-5-b)$

* set $b^2-4ac = 0$ *

$(-2)^2 - 4(-1)(-5-b) = 0$

$4 - 4(5+b) = 0$

$4 - 20 - 4b = 0$

$-4b = 16$

$b = -4$

4) The path of an underground stream is given by the function $y = 4x^2 + 17x - 32$. Two new houses need wells to be dug. On the area plan, these houses lie on a line defined by the equation $y = -15x + 100$. Determine the coordinates where the two new wells should be dug.

$-15x+100 = 4x^2+17x-32$

$0 = 4x^2+32x-132$

$0 = 4(x^2+8x-33)$

$0 = x^2+8x-33$

$0 = (x+11)(x-3)$

$x+11=0$ $\quad x-3=0$

$x=-11$ $\quad x=3$

POI #1

$f(x) = -15x+100$

$f(-11) = -15(-11)+100$

$f(-11) = 265$

$(-11, 265)$

POI #2

$f(3) = -15(3)+100$

$f(3) = 55$

$(3, 55)$

Answers

1) a) do not intersect **b)** once **c)** twice **d)** twice

2) a) (4, 3), (5, 5) **b)** (2, 17), (6, 49) **c)** (-4, 13), (2, -5)

3) a) 6 **b)** -4

4) (-11, 265), (3, 55)

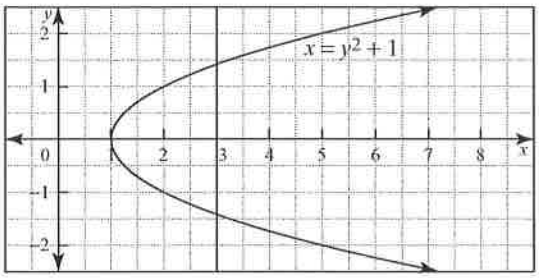
Chapter 1 – Functions – REVIEW
MCR3U
Jensen

SOLUTIONS

Section 1: Functions, Domain, and Range

1) State the domain and range of the following relations and then state if it is a function or not.

a)

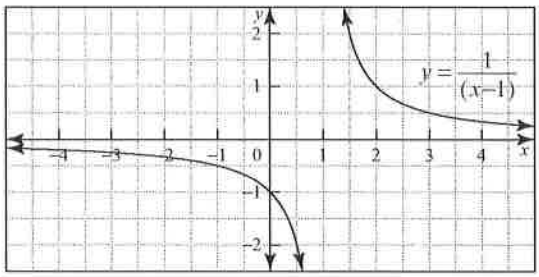


D: $\{x \in \mathbb{R} \mid x \geq 1\}$

R: $\{y \in \mathbb{R}\}$

NOT a function

b)

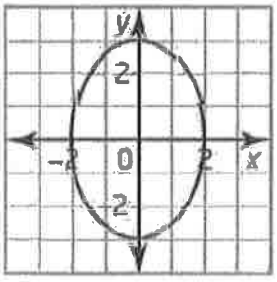


D: $\{x \in \mathbb{R} \mid x \neq 1\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$

function

c)

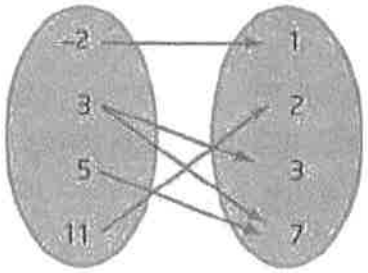


D: $\{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$

R: $\{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$

NOT a function

d)



D: $(-2, 3, 5, 11)$

R: $(1, 2, 3, 7)$

NOT a Function

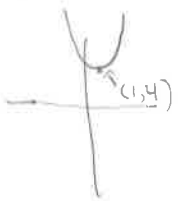
e) $\{(1, 4), (2, 6), (3, 10), (4, 18), (5, 29)\}$

D: $(1, 2, 3, 4, 5)$

R: $(4, 6, 10, 18, 29)$

Function

f) $y = 3(x - 1)^2 + 4$



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 4\}$

Function

Section 2: Max or Min of a Quadratic Function

2) Determine the vertex for each quadratic function by completing the square. State if the vertex is a maximum or a minimum.

a) $f(x) = x^2 - 10x + 7$

$= (x^2 - 10x) + 7$

$= (x^2 - 10x + 25 - 25) + 7$

$= (x^2 - 10x + 25) - 25 + 7$

$= (x - 5)^2 - 18$

Vertex is $(5, -18)$ → Min

b) $f(x) = x^2 + 2x + 6$

$= (x^2 + 2x + 1 - 1) + 6$

$= (x^2 + 2x + 1) - 1 + 6$

$= (x+1)^2 + 5$

Vertex is $(-1, 5)$ → min

c) $f(x) = 3x^2 - 30x + 73$

$= 3(x^2 - 10x) + 73$

$= 3(x^2 - 10x + 25 - 25) + 73$

$= 3(x^2 - 10x + 25) - 75 + 73$

$= 3(x - 5)^2 - 2$

Vertex is $(5, -2)$ → min

d) $f(x) = -2x^2 - 8x + 7$

$= -2(x^2 + 4x) + 7$

$= -2(x^2 + 4x + 4 - 4) + 7$

$= -2(x^2 + 4x + 4) + 8 + 7$

$= -2(x+2)^2 + 15$

Vertex is $(-2, 15)$ → Max

3) Use partial factoring to determine the vertex of each function. State if the vertex is a max or min.

a) $y = -x^2 + 4x + 11$

$11 = -x^2 + 4x + 11$

$0 = -x^2 + 4x$

$0 = -x(x - 4)$

$-1x = 0$ $\quad x - 4 = 0$

$x = 0$ $\quad x = 4$

$x\text{-vertex} = \frac{0+4}{2}$

$= 2$

$y\text{-vertex} = -(2)^2 + 4(2) + 11$

$= 15$

Vertex is $(2, 15)$ → Max

b) $y = 3x^2 - 18x + 14$

$14 = 3x^2 - 18x + 14$

$0 = 3x^2 - 18x$

$0 = 3x(x - 6)$

$3x = 0$ $\quad x - 6 = 0$

$x = 0$ $\quad x = 6$

$x\text{-vertex} = \frac{0+6}{2}$

$= 3$

$y\text{-vertex} = 3(3)^2 - 18(3) + 14$

$= -13$

Vertex is $(3, -13)$ → min

c) $y = 5x^2 + 14x - 21$

$0 = 5x^2 + 14x$

$0 = x(5x + 14)$

$x = 0$ $\quad 5x + 14 = 0$

$x = -14/5$

$x = -2.8$

$x\text{-vertex} = \frac{0+(-2.8)}{2}$

$= -1.4$

$y\text{-vertex} = 5(-1.4)^2 + 14(-1.4) - 21$

$= -30.8$

Vertex is $(-1.4, -30.8)$

Min

d) $y = -2x^2 - 11x + 1$

$0 = -2x^2 - 11x$

$0 = -1x(2x + 11)$

$-1x = 0$ $\quad 2x + 11 = 0$

$x = 0$ $\quad x = -11/2$

$x = -5.5$

$x\text{-vertex} = \frac{0+(-5.5)}{2}$

$= -2.75$

$y\text{-vertex} = -2(-2.75)^2 - 11(-2.75) + 1$

$= 16.125$

Vertex is $(-2.75, 16.125)$

Max

4) A hall charges \$30 per person for a sports banquet when 120 attend. For every 10 extra people that attend, the hall will decrease the price by \$1.50 per person. What number of people will maximize the revenue for the hall?

Revenue = (cost)(# of people)

$0 = (30 - 1.50x)(120 + 10x)$

$30 - 1.5x = 0 \qquad 120 + 10x = 0$

$x = 20 \qquad x = -12$

$x\text{-vertex} = \frac{20 + (-12)}{2}$

$= 4$

# of people $= [120 + 10(4)]$

$= 160$ people.

5) The power, P, in watts, produced by a solar panel is given by the function $P(I) = -5I^2 + 100I$, where I represents the current, in amperes.

a) What value of the current will maximize the power? x-coord of vertex

$0 = -5I^2 + 100I$

$0 = -5I(I - 20)$

$-5I = 0 \qquad I - 20 = 0$

$I = 0 \qquad I = 20$

$x\text{-vertex} = \frac{0 + 20}{2}$

$= 10$

A current of 10 amps maximizes power.

) What is the maximum power?

$P(10) = -5(10)^2 + 100(10)$

$= 500$

The max power is 500 watts.

6) a) Find the vertex of the parabola defined by $f(x) = -\frac{1}{3}x^2 + 2x - 4$.

$f(x) = -\frac{1}{3}(x^2 - 6x) - 4$

$f(x) = -\frac{1}{3}(x^2 - 6x + 9 - 9) - 4$

$f(x) = -\frac{1}{3}(x^2 - 6x + 9) + 3 - 4$

$f(x) = -\frac{1}{3}(x - 3)^2 - 1$

Vertex is $(3, -1)$

b) Is the vertex a minimum or a maximum?

Max

c) Without finding them, how many x-intercepts does the parabola have? Explain.

None. Vertex is below x-axis and parabola opens down.

Section 3: Radicals

7) Perform each radical operation and simplify where needed.

a) $\sqrt{27} - 4\sqrt{3} + \sqrt{243} - 8\sqrt{81} + 2$

$= (\sqrt{9})(\sqrt{3}) - 4\sqrt{3} + (\sqrt{81})(\sqrt{3}) - 8(9) + 2$

$= 3\sqrt{3} - 4\sqrt{3} + 9\sqrt{3} - 72 + 2$

$= 8\sqrt{3} - 70$

b) $-3\sqrt{3}(\sqrt{3} + 5\sqrt{2})$

$= -3\sqrt{9} - 15\sqrt{6}$

$= -3(3) - 15\sqrt{6}$

$= -9 - 15\sqrt{6}$

c) $(5 + \sqrt{3})(5 - \sqrt{3})$ D.O.S.

$= (5)^2 - (\sqrt{3})^2$

$= 25 - 3$

$= 22$

d) $5\sqrt{2}(11 + 2\sqrt{2}) - 4(8 + 3\sqrt{2})$

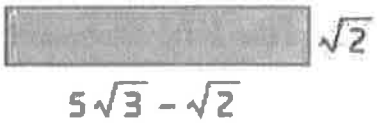
$= 55\sqrt{2} + 10\sqrt{4} - 32 - 12\sqrt{2}$

$= 43\sqrt{2} + 10(2) - 32$

$= 43\sqrt{2} - 12$

8) Find a simplified expression for the area of each shape

a)



$A = (\sqrt{2})(5\sqrt{3} - \sqrt{2})$

$= 5\sqrt{6} - 2$

b)

$A = \frac{\pi r^2}{2}$

$= \frac{\pi(3\sqrt{3})^2}{2}$

$= \frac{\pi(9\sqrt{9})}{2}$

$= \frac{27\pi}{2}$

Section 4: Solve Quadratics by Factoring

9) Solve each of the following quadratic equations by factoring.

a) $x^2 + 4x - 21 = 0$ S: 4 P: -21 (7 and -3)

$(x+7)(x-3) = 0$

$x+7=0 \quad x-3=0$

$x=-7 \quad x=3$

b) $5x^2 - 19x - 4 = 0$ S: -19 P: -20 (-20 and 1)

$5x^2 - 20x + 1x - 4 = 0$

$(5x^2 - 20x) + (1x - 4) = 0$

$5x(x-4) + 1(x-4) = 0$

$(x-4)(5x+1) = 0$

$x-4=0 \quad 5x+1=0$

$x=4 \quad x=-\frac{1}{5}$

c) $4x^2 + 12x = -9$ P.S.T.

$4x^2 + 12x + 9 = 0$

$(2x+3)^2 = 0$

$2x+3=0$

$x = -\frac{3}{2}$

Section 5: Solve Quadratics Using the Quadratic Formula

10) Solve each quadratic equation. Give exact answers.

a) $f(x) = 2x^2 - 9x - 1$

$x = \frac{9 \pm \sqrt{(-9)^2 - 4(2)(-1)}}{2(2)}$

$x = \frac{9 \pm \sqrt{89}}{4}$

$x = \frac{9+\sqrt{89}}{4}$ and $x = \frac{9-\sqrt{89}}{4}$

b) $g(x) = -3x^2 + 4x + 2$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(-3)(2)}}{2(-3)}$

$x = \frac{-4 \pm \sqrt{40}}{-6}$

$x = \frac{-4 \pm 2\sqrt{10}}{-6}$

$x = \frac{2(2 \mp \sqrt{10})}{-6}$

$x = \frac{2 - \sqrt{10}}{3}$ and $x = \frac{2+\sqrt{10}}{3}$

11) Solve each quadratic equation using any method.

a) $3x^2 - 15x = 42$

$3x^2 - 15x - 42 = 0$

$3(x^2 - 5x - 14) = 0$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x - 7 = 0 \quad x + 2 = 0$

$x = 7 \quad x = -2$

b) $5x^2 + 11x + 1 = 0$

$x = \frac{-11 \pm \sqrt{(11)^2 - 4(5)(1)}}{2(5)}$

$x = \frac{-11 \pm \sqrt{101}}{10}$

$x = \frac{-11+\sqrt{101}}{10}$ and $x = \frac{-11-\sqrt{101}}{10}$

12) Use the discriminant to predict the number of real roots of...

a) $f(x) = x^2 + 3x + 3$

$b^2 - 4ac = (3)^2 - 4(1)(3)$

$= -3$

∴ No Roots

b) $f(x) = -2x^2 + 4x - 2$

$b^2 - 4ac = (4)^2 - 4(-2)(-2)$

$= 0$

∴ 1 Root.

c) $f(x) = 2x^2 + 5x - 8$

$b^2 - 4ac = (5)^2 - 4(2)(-8)$

$= 89$

∴ 2 Roots.

Section 6: Determine a Quadratic Equation Given its Roots

13) Determine the equation in standard form for each quadratic function...

a) x-intercepts $\overset{r}{-2}$ and $\overset{s}{5}$, containing the point $(\overset{x}{3}, \overset{y}{5})$

$y = a(x-r)(x-s)$

$5 = a(3+2)(3-5)$

$5 = a(5)(-2)$

$5 = -10a$

$a = -\frac{1}{2}$

$y = -\frac{1}{2}(x+2)(x-5)$

$y = -\frac{1}{2}(x^2-3x-10)$

$y = -\frac{1}{2}x^2 + \frac{3}{2}x + 5$

b) x-intercepts $-2 \pm \sqrt{5}$, containing the point (-4, 5)

$y = a(x-r)(x-s)$

$y = a[x-(-2+\sqrt{5})][x-(-2-\sqrt{5})]$

$y = a(x+2-\sqrt{5})(x+2+\sqrt{5})$ D.O.S.

$y = a[(x+2)^2 - (\sqrt{5})^2]$

$y = a(x^2+4x+4-5)$

$y = a(x^2+4x-1)$

sub in point (-4, 5)

$y = a(x^2+4x-1)$

$5 = a[(-4)^2+4(-4)-1]$

$5 = a(-1)$

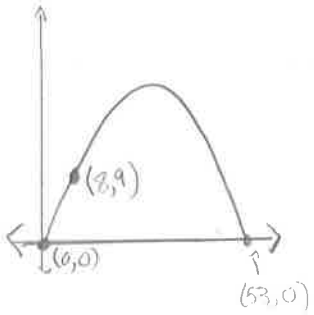
$a = -5$

$y = -5(x^2+4x-1)$

$y = -5x^2 - 20x + 5$

14) A golf ball is hit, and it lands at a point on the same horizontal plane 53 meters away. The path of the ball took it just over a 9 meter tall tree that was 8 meters in front of the golfer.

a) Assume the ball is hit from the origin of a coordinate plane. Find a quadratic function that describes the path of the ball.



$y = a(x-r)(x-s)$

$9 = a(8-0)(8-53)$

$9 = a(-360)$

$a = -\frac{1}{40}$

$y = -\frac{1}{40}(x)(x-53)$

$y = -\frac{1}{40}(x^2-53x)$

$y = -\frac{1}{40}x^2 + \frac{53}{40}x$

b) What is the maximum height of the ball?

$x\text{-vertex} = \frac{0+53}{2}$

$= 26.5$

$y\text{-vertex} = -\frac{1}{40}(26.5)^2 + \frac{53}{40}(26.5)$

≈ 17.6 m

Section 7: Linear-Quadratic Systems

15) Determine the points of intersection of each pair of functions.

a) $f(x) = 4x^2 - 15x + 20$ and $g(x) = 5x - 4$

$5x-4 = 4x^2-15x+20$

$0 = 4x^2-20x+24$

$0 = 4(x^2-5x+6)$

$0 = x^2-5x+6$

$0 = (x-2)(x-3)$

$x-2=0 \quad x-3=0$

$x=2 \quad x=3$

POI #1

$g(2) = 5(2)-4$

$= 6$

$(2,6)$

POI #2

$g(3) = 5(3)-4$

$= 11$

$(3,11)$

b) $f(x) = -2x^2 + 9x + 9$ and $g(x) = -3x - 5$

$-3x-5 = -2x^2+9x+9$

$0 = -2x^2+12x+14$

$0 = -2(x^2-6x-7)$

$0 = x^2-6x-7$

$0 = (x-7)(x+1)$

$x-7=0 \quad x+1=0$

$x=7 \quad x=-1$

POI #1

$g(7) = -3(7)-5$

$= -26$

$(7,-26)$

$g(-1) = -3(-1)-5$

$= -2$

$(-1,-2)$

16) For what value of b will the line $y = -2x + b$ be tangent to the parabola $y = 3x^2 + 4x - 1$

$-2x+b = 3x^2+4x-1$

$0 = 3x^2+6x-1-b$

$0 = 3x^2+6x+(-1-b)$

* set $b^2-4ac=0$ *

$(6)^2-4(3)(-1-b) = 0$

$36-4(-3-3b) = 0$

$36+12+12b = 0$

$48 = -12b$

$b = -4$

$\boxed{b=-4}$

Answers

1) a) domain: $\{X \in \mathbb{R} | x \geq 1\}$ range: $\{Y \in \mathbb{R}\}$ **b)** domain: $\{X \in \mathbb{R} | x \neq 1\}$ range: $\{Y \in \mathbb{R} | y \neq 0\}$

c) domain: $\{X \in \mathbb{R} | -2 \leq x \leq 2\}$ range: $\{Y \in \mathbb{R} | -3 \leq y \leq 3\}$

d) domain: $\{-2, 3, 5, 11\}$, range $\{1, 2, 3, 7\}$ **e)** domain: $\{1, 2, 3, 4, 5\}$, range: $\{4, 6, 10, 18, 29\}$

f) domain:$\{X \in \mathbb{R}\}$, range: $\{Y \in \mathbb{R} | y \geq 4\}$

2) a) (5, -18) min **b)** (-1, 5) min **c)** (5, -2) min **d)** (-2, 15) max

3) a) (2, 15) max **b)** (6, -3) min **c)** (-1.4, 8.4) min **d)** $\left(-\frac{11}{4}, \frac{129}{8}\right)$ max

4) 160

5) a) 10 A **b)** 500 W

6) a) (3, -1) **b)** max **c)** vertex is below x-axis and opens down, therefore no x-intercepts

7) a) $-70 + 8\sqrt{3}$ **b)** $-9 - 15\sqrt{6}$ **c)** 22 **d)** $-12 + 43\sqrt{2}$

8) a) $5\sqrt{6} - 2$ square units **b)** $\frac{27\pi}{2}$ square units

9) a) -7 and 3 **b)** $-\frac{1}{5}$ and 4 **c)** $-\frac{3}{2}$

10) a) $x = \frac{9 \pm \sqrt{89}}{4}$ **b)** $x = \frac{-2 \pm \sqrt{10}}{-3}$

11) a) $x = 7$ and $x = -2$ **b)** $x = \frac{-11 \pm \sqrt{101}}{10}$

12) a) none **b)** 1 **c)** 2

13) a) $f(x) = -\frac{1}{2}x^2 + \frac{3}{2}x + 5$ **b)** $f(x) = -5x^2 - 20x + 5$

14) a) $f(x) = -\frac{1}{40}x^2 + \frac{53}{40}x$ **b)** 17.6 m

15) a) (2, 6), (3, 11) **b)** (-1, -2), (7, -26)

16) $b = -4$