

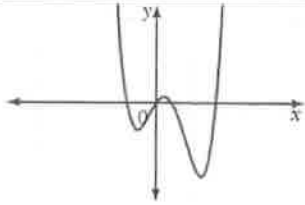
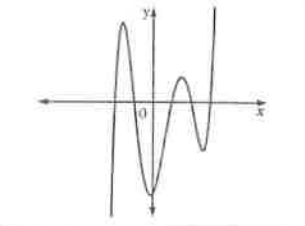
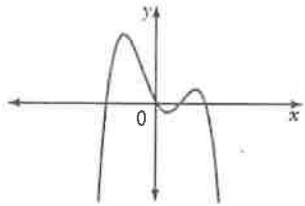
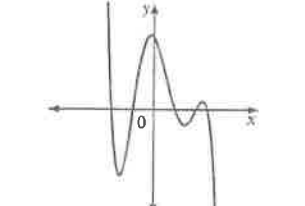
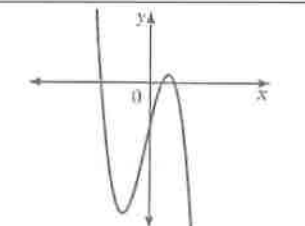
W2 – 1.2 – Characteristics of Polynomial Functions

MHF4U

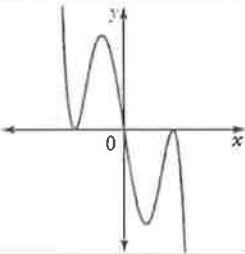
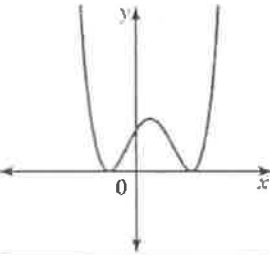
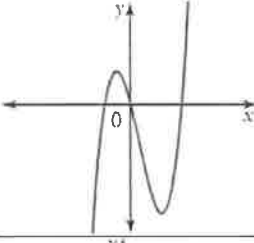
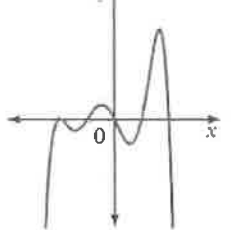
Jensen

ANSWERS

1) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	+	Even	Q2 to Q1	None	3	4	4
	+	Odd	Q3 to Q1	None	4	5	5
	-	Even	Q3 to Q4	None	3	4	4
	-	Odd	Q2 to Q4	None	4	5	5
	-	Odd	Q2 to Q4	Point	2	3	3

2) Complete the following table

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	-	Odd	Q2 to Q4	Point	4	3	5
	+	Even	Q2 to Q1	Line	3	2	4
	+	Odd	Q3 to Q1	Point	2	3	3
	-	Even	Q3 to Q4	None	5	5	6

3) Complete the following table

Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	-	Even	Q3 to Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	+	Odd	Q3 to Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	-	Odd	Q2 to Q4	2, 0	3, 2, 1
$h(x) = 2x(x - 5)(3x + 2)(4x - 3)$	4	+	Even	Q2 to Q1	3, 1	4, 3, 2, 1, 0

4) Use end behaviours, turning points, and zeros to match each equation with the most likely graph. Write the letter of the equation beneath the graph.

$$A) y = 2x^3 - 4x^2 + 3x + 2$$

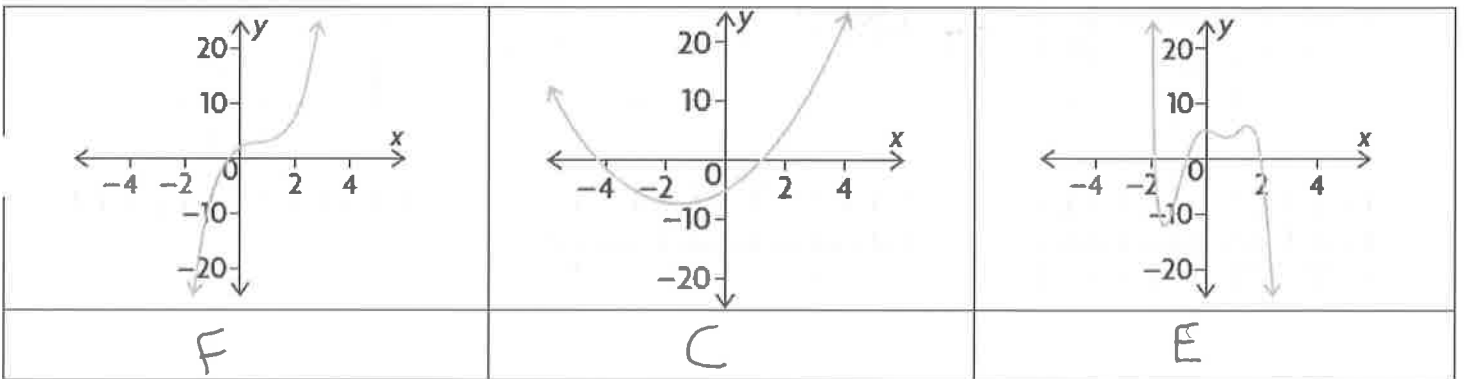
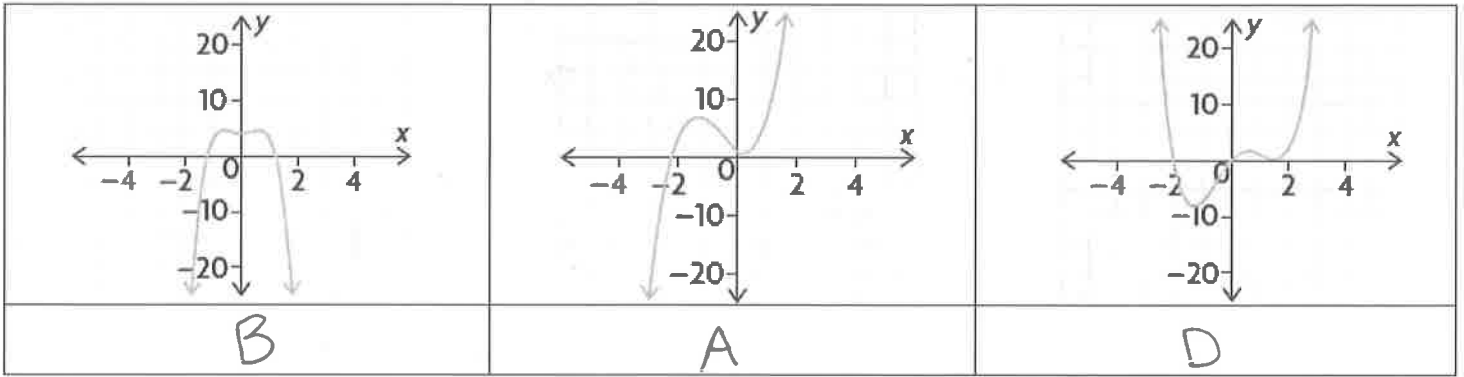
$$B) y = -4x^4 + 3x^2 + 4$$

$$C) y = x^2 + 3x - 5$$

$$D) y = x^4 - x^3 - 4x^2 + 5x$$

$$E) y = -2x^5 + 3x^4 + 6x^3 - 10x^2 + 2x + 5$$

$$F) y = 3x^3 + 5x^2 - 3x + 1$$



5) State the degree of the polynomial function that corresponds to each constant finite difference. Then determine the value of the leading coefficient for each polynomial function.

a) ^{degree 2}
second differences = -8

$$-8 = a(2!)$$

$$-8 = 2a$$

$$\boxed{-4 = a}$$

b) ^{degree 4}
fourth differences = 24

$$24 = a(4!)$$

$$24 = 24a$$

$$\boxed{1 = a}$$

6) Use finite differences to determine the degree and value of the leading coefficient for each polynomial function.

a)

x	y
-3	-45
-2	-16
-1	-3
0	0
1	-1
2	0
3	9
4	32

1st 2nd 3rd

29 }
13 }
3 }
-1 }
1 }
9 }
23 }

-16 }
-10 }
-4 }
2 }
8 }
14 }

6 }
6 }
6 }
6 }
6 }

Degree = 3

$6 = a(3!)$

$6 = 6a$

$1 = a$

b)

x	y
-2	-40
-1	12
0	20
1	26
2	48
3	80
4	92
5	30

1st 2nd 3rd 4th

52 }
8 }
6 }
22 }
32 }
12 }
-62 }

-44 }
-2 }
16 }
10 }
-20 }
~~-24~~ }

42 }
18 }
-6 }
-30 }
~~54~~ }

-24 }
-24 }
-24 }
-24 }

Degree = 4

$-24 = a(4!)$

$-24 = 24a$

$-1 = a$

7) By analyzing the impact of growing economic conditions, a demographer establishes that the predicted population, P , of a town t years from now can be modelled by the function

$$P(t) = 6t^4 - 5t^3 + 200t + 12000$$

a) What is the value of the constant finite differences

$$\begin{aligned} \text{Finite Differences} &= a(n!) \\ &= 6(4!) \\ &= 144 \end{aligned}$$

b) What is the current population of the town $P(0) = 6(0)^4 - 5(0)^3 + 200(0) + 12000$

$$= 12000$$

c) What will the population of the town be 10 years from now

$$\begin{aligned} P(10) &= 6(10)^4 - 5(10)^3 + 200(10) + 12000 \\ &= 69000 \end{aligned}$$

ANSWER KEY

1)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	POS	EVEN	Q2 to Q1	NONE	3	4	4
	POS	ODD	Q3 to Q1	NONE	4	5	5
	NEG	EVEN	Q3 to Q4	NONE	3	4	4
	NEG	ODD	Q2 to Q4	NONE	4	5	5
	NEG	ODD	Q2 to Q4	POINT	2	3	3

2)

Graph	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Symmetry	Number of turning points	Number of x-intercepts	Least Possible Degree
	NEG	ODD	Q2 to Q4	Point	4	3	5
	POS	EVEN	Q2 to Q1	Line	3	2	4
	POS	ODD	Q3 to Q1	Point	2	3	3
	NEG	EVEN	Q3 to Q4	None	5	5	6

3)

Equation	Degree	Sign of Leading Coefficient	Even or Odd Degree?	End Behaviour	Possible number of turning points	Possible number of x-intercepts
$f(x) = -4x^4 + 3x^2 - 15x + 5$	4	NEG	EVEN	Q3 → Q4	3, 1	4, 3, 2, 1, 0
$g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$	5	POS	ODD	Q3 → Q1	4, 2, 0	5, 4, 3, 2, 1
$p(x) = 4 - 5x + 4x^2 - 3x^3$	3	NEG	ODD	Q2 → Q4	2, 0	3, 2, 1
$h(x) = 2x(x - 5)(3x + 2)(4x - 3)$	4	POS	EVEN	Q2 → Q1	3, 1	4, 3, 2, 1, 0

4) B F D
A C E

5) a) degree 2, $a = -4$ b) degree 4, $a = 1$

6) a) degree 3, $a = 1$ b) degree 4, $a = -1$

7) a) 144 b) 12 000 c) 69 000