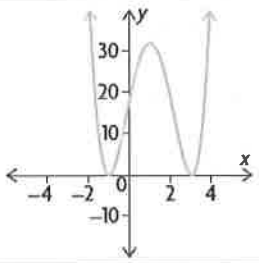
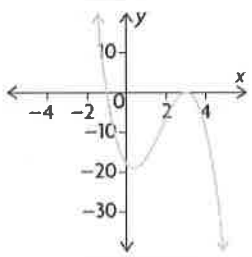
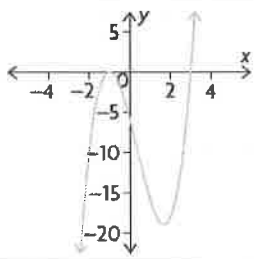
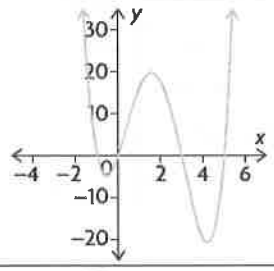
W3 – 1.3 – Factored Form Polynomial Functions
MHF4U
Jensen

ANSWERS

1) Match each equation with the most suitable graph. Write the letter of the equation beneath the matching graph.

A) $f(x) = 2(x+1)^2(x-3)$ **B)** $f(x) = (x+1)^2(x-3)^2$

C) $f(x) = -2(x+1)(x-3)^2$ **D)** $f(x) = x(x+1)(x-3)(x-5)$

			
B	C	A	D

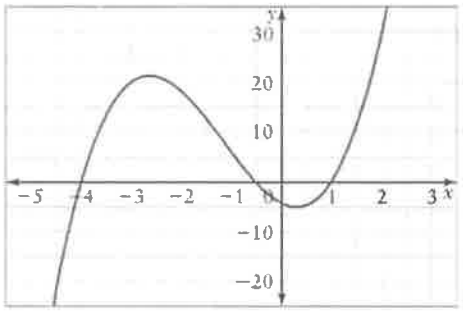
2) Complete the table

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x-4)(x+3)(2x-1)$	$(x)(x)(x) = x^3$ (3)	$(1)(1)(2) = 2$	Q3 to Q1	$(4,0)$ $(-3,0)$ $(\frac{1}{2},0)$
$g(x) = -2(x+2)(x-2)(1+x)(x-1)$	$(x)(x)(x)(x) = x^4$ (4)	$-2(1)(1)(1)(1) = -2$	Q3 to Q4	$(-2,0)$ $(-1,0)$ $(2,0)$ $(1,0)$
$h(x) = (3x+2)^2(x-4)(x+1)(2x-3)$	$(x^2)(x)(x)(x) = x^5$ (5)	$(3^2)(1)(1)(2) = 18$	Q3 to Q1	$(-\frac{2}{3},0)$ $(\frac{3}{2},0)$ $(4,0)$ $(-1,0)$
$p(x) = -(x+5)^3(x-5)^3$	$(x^3)(x^3) = x^6$ (6)	$-1(1^3)(1^3) = -1$	Q3 to Q4	$(-5,0)$ $(5,0)$

3) For each graph, state...

- **i)** the least possible degree and the sign of the leading coefficient
- **ii)** the x-intercepts (specify order of zero) and the factors of the function
- **iii)** the intervals where the function is positive/negative

a)



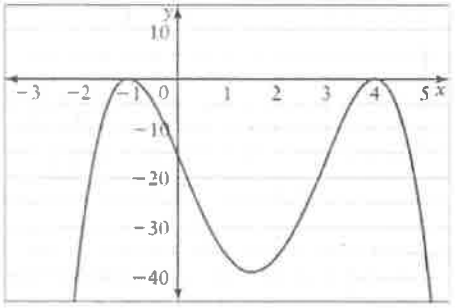
i) degree: 3
leading coefficient: Positive

ii) x-intercepts: $-4, -\frac{1}{2}, 1$
factors: $(x+4), (2x+1), (x-1)$

iii)

Interval	$(-\infty, -4)$	$(-4, -\frac{1}{2})$	$(-\frac{1}{2}, 1)$	$(1, \infty)$
Sign	−	+	−	+

b)



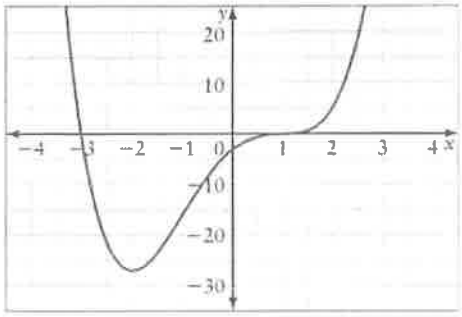
i) degree: 4
leading coefficient: Negative

ii) x-intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2, (x-4)^2$

iii)

Interval	$(-\infty, -1)$	$(-1, 4)$	$(4, \infty)$
Sign	−	−	−

c)



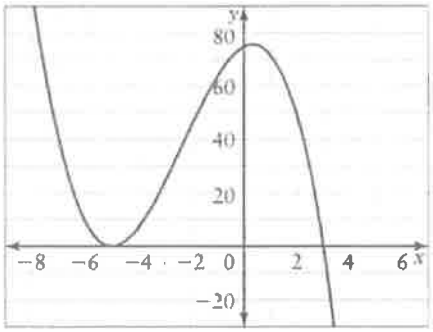
i) degree: 4
leading coefficient: Positive

ii) x-intercepts: -3, 1 (order 3)
factors: $(x+3), (x-1)^3$

iii)

Interval	$(-\infty, -3)$	$(-3, 1)$	$(1, \infty)$
Sign	+	−	+

d)



i) degree: 3
leading coefficient: Negative

ii) x-intercepts: -5 (order 2), 3
factors: $(x+5)^2, (x-3)$

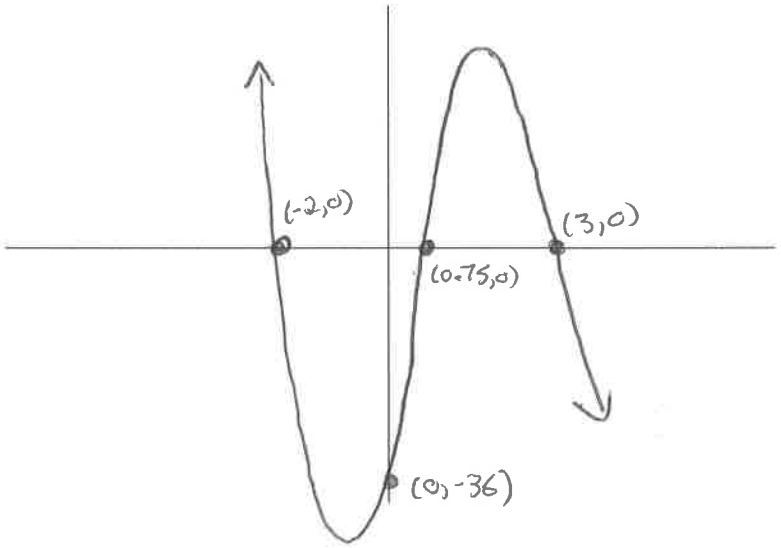
iii)

Interval	$(-\infty, -5)$	$(-5, 3)$	$(3, \infty)$
Sign	+	+	−

4) For each function, complete the chart and sketch a possible graph of the function labelling key points.

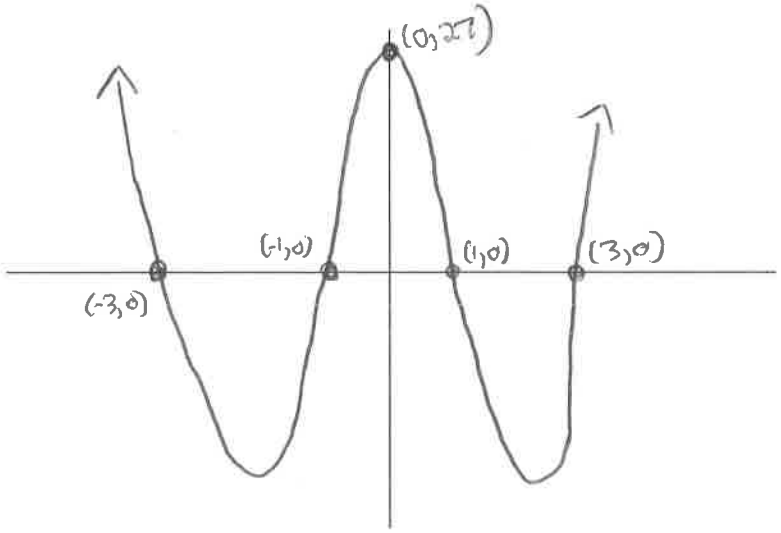
a) $f(x) = -2(x-3)(x+2)(4x-3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)$ $= x^3$ Degree 3	$-2(1)(1)(4)$ $= -8$	Q2 to Q4	$(3,0)$ $(-2,0)$ $(\frac{3}{4},0)$	$f(0) = -2(-3)(2)(-3)$ $= -36$ $(0,-36)$



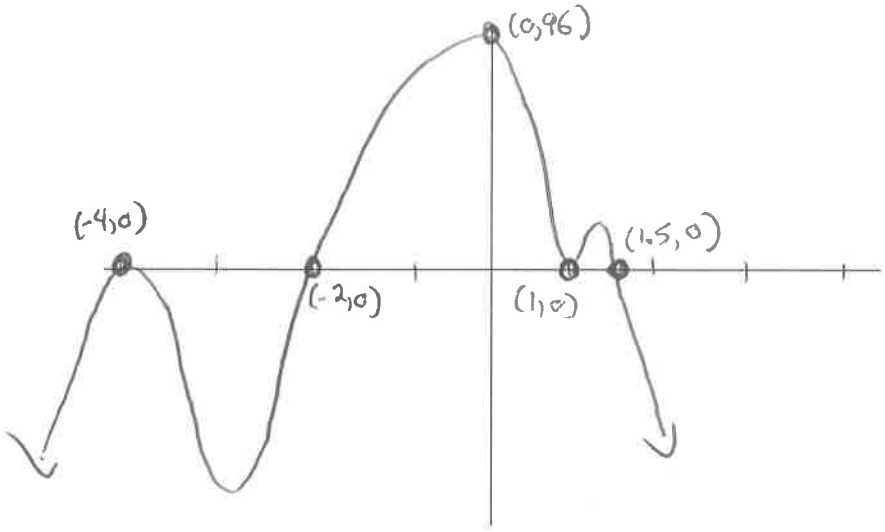
b) $g(x) = (x-1)(x+3)(1+x)(3x-9)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x)(x)(x)$ $= x^4$ Degree 4	$(1)(1)(1)(3)$ $= 3$	Q2 to Q1	$(1,0)$ $(3,0)$ $(-3,0)$ $(-1,0)$	$g(0) = (-1)(3)(1)(-9)$ $= 27$



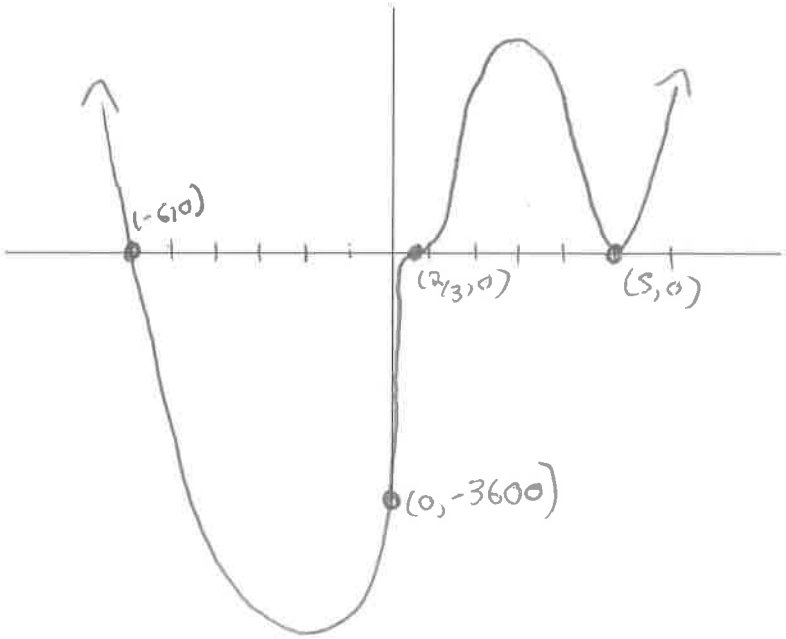
c) $h(x) = -(x+4)^2(x-1)^2(x+2)(2x-3)$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x^2)(x^2)(x)(x)$ $= x^6$ Degree 6	$-1(1^2)(1^2)(1)(2)$ $= -2$	Q3 to Q4	$(-4,0)$ order 2 $(1,0)$ order 2 $(-2,0)$ $(3/2,0)$	$h(0) = -1(4)^2(-1)^2(2)(-3)$ $= 96$



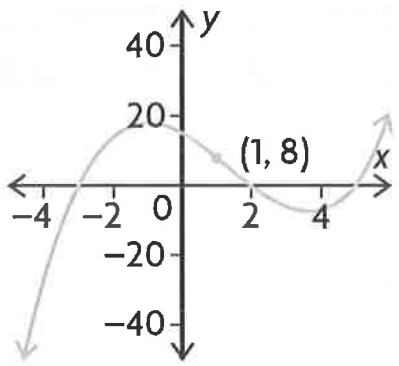
d) $p(x) = 3(x+6)(x-5)^2(3x-2)^3$

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
$(x)(x^2)(x^3)$ $= x^6$ Degree 6	$3(1)(1)^2(3)^3$ $= 81$	Q2 to Q1	$(-6,0)$ $(5,0)$ order 2 $(2/3,0)$ order 3	$p(0) = 3(6)(-5)^2(-2)^3$ $= -3600$



5) Write the equation of each function

a)



$f(x) = k(x+3)(x-2)(x-5)$

$8 = k(1+3)(1-2)(1-5)$

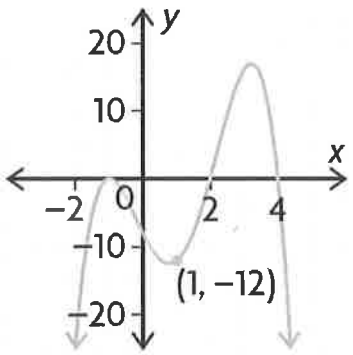
$8 = k(4)(-1)(-4)$

$8 = 16k$

$k = \frac{1}{2}$

$f(x) = \frac{1}{2}(x+3)(x-2)(x-5)$

b)



$g(x) = k(x+1)^2(x-2)(x-4)$

$-12 = k(1+1)^2(1-2)(1-4)$

$-12 = k(4)(-1)(-3)$

$-12 = 12k$

$k = -1$

$g(x) = -(x+1)^2(x-2)(x-4)$

6) Determine an equation for a quintic function with zeros -1 (order 3) and 3 (order 2) that passes through the point (-2, 50)

$h(x) = k(x+1)^3(x-3)^2$

$50 = k(-2+1)^3(-2-3)^2$

$50 = k(-1)^3(-5)^2$

$50 = k(-1)(25)$

$50 = -25k$

$k = -2$

$h(x) = -2(x+1)^3(x-3)^2$

7) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$

$2x^2 - x - 1$ p: -2, s: -1 → -2 and 1

$= 2x^2 - 2x + 1x - 1$

$= 2x(x-1) + 1(x-1)$

$= (x-1)(2x+1)$

$x^2 - 3x - 4$ p: -4, s: -3 → -4 and 1

$= (x-4)(x+1)$

$f(x) = (x-1)(2x+1)(x-4)(x+1)$

$0 = (x-1)(2x+1)(x-4)(x+1)$

$x_1 = 1 \quad x_2 = -\frac{1}{2} \quad x_3 = 4 \quad x_4 = -1$

Answer Key

1) B C A D

Equation	Degree	Leading Coefficient	End Behaviour	x-intercepts
$f(x) = (x-4)(x+3)(2x-1)$	3	2	Q3 → Q1	(4, 0) (-3, 0) $\left(\frac{1}{2}, 0\right)$
$g(x) = -2(x+2)(x-2)(1+x)(x-1)$	4	-2	Q3 → Q4	(-2, 0) (-1, 0) (1, 0) (2, 0)
$h(x) = (3x+2)^2(x-4)(x+1)(2x-3)$	5	18	Q3 → Q1	(4, 0) (-1, 0) $\left(-\frac{2}{3}, 0\right)$ $\left(\frac{3}{2}, 0\right)$
$p(x) = -(x+5)^3(x-5)^3$	6	-1	Q3 → Q4	(-5, 0) (5, 0)

3) a) i) degree: 3
leading coefficient: positive

ii) x-intercepts: -4, -0.5, 1
factors: $(x+4)$, $(2x+1)$, and $(x-1)$

iii)

Interval	$(-\infty,-4)$	$(-4,-0.5)$	$(-0.5,1)$	$(1,\infty)$
Sign	-	+	-	+

b) i) degree: 4
leading coefficient: negative

ii) x-intercepts: -1 (order 2), 4 (order 2)
factors: $(x+1)^2$, and $(x-4)^2$

iii)

Interval	$(-\infty,-1)$	$(-1,4)$	$(4,\infty)$
Sign	-	-	-

c) i) degree: 4
leading coefficient: positive

ii) x-intercepts: -3, 1 (order 3)
factors: $(x+3)$, and $(x-1)^3$

iii)

Interval	$(-\infty,-3)$	$(-3,1)$	$(1,\infty)$
Sign	+	-	+

d) i) degree: 3
leading coefficient: negative

ii) x-intercepts: -5 (order 2), 3
factors: $(x+5)^2$, and $(x-3)$

iii)

Interval	$(-\infty,-5)$	$(-5,3)$	$(3,\infty)$
Sign	+	+	-

4) a)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
3	-8	Q2 → Q4	(3, 0) (-2, 0) $\left(\frac{3}{4}, 0\right)$	(0, -36)

b)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
4	3	Q2 → Q1	(1, 0) (-3, 0) (-1, 0) (3,0)	(0, 27)

c)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	-2	Q3 → Q4	(-4, 0) order 2 (1, 0) order 2 (-2, 0) (1.5,0)	(0, 96)

d)

Degree	Leading Coefficient	End Behaviour	x-intercepts	y-intercept
6	81	Q2 → Q1	(-6, 0) (5, 0) order 2 $\left(\frac{2}{3}, 0\right)$ order 3	(0, -3600)

5) a) $y = 0.5(x+3)(x-2)(x-5)$ **b)** $y = -(x+1)^2(x-2)(x-4)$

$y = -2(x+1)^3(x-3)^2$

7) 4, 1, -1, and -0.5