L4 – 1.4 – Transformations Lesson
MHF4U
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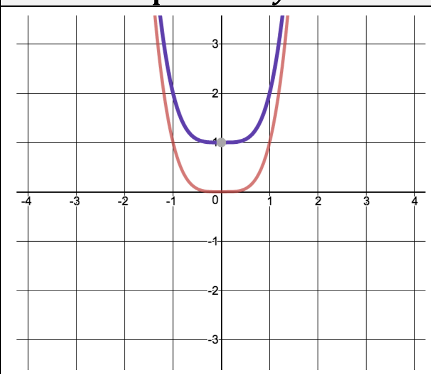
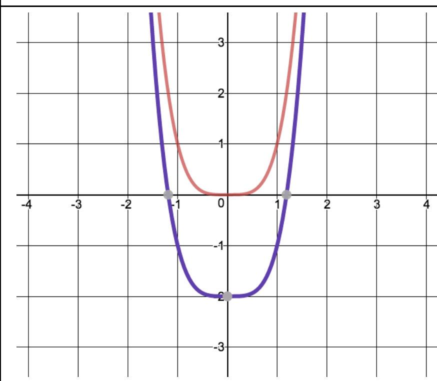
In this section, you will investigate the roles of the parameters a, k, d, and c in polynomial functions of the form $\boldsymbol{f(x) = a[k(x-d)]^n + c}$. You will apply transformations to the graphs of basic power functions to sketch the graph of its transformed function.

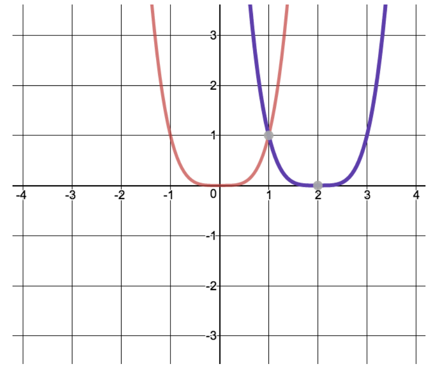
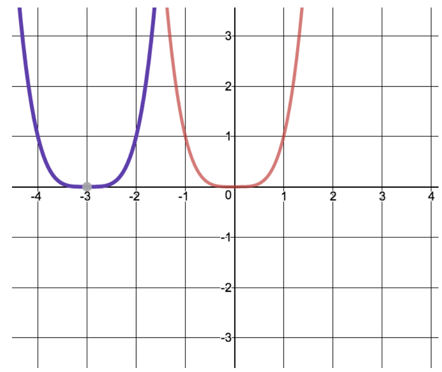
Part 1: Transformations Investigation

In this investigation, you will be looking at transformations of the power function $y = x^4$. Complete the following table using graphing technology to help. The graph of $y = x^4$ is given on each set of axes; sketch the graph of the transformed function on the same set of axes. Then comment on how the value of the parameter a, k, d, or c transforms the parent function.

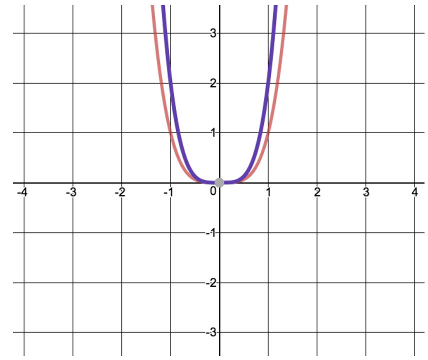
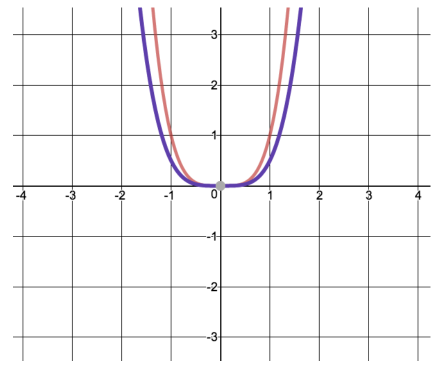
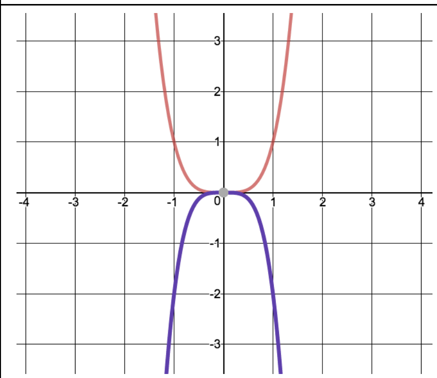
Effects of c on $y = x^4 + c$

Transformed Function	Value of c	Transformations to $y = x^4$	Graph of transformed function compared to $y = x^4$
$y = x^4 + 1$	$c = 1$	Shift up 1 unit	
$y = x^4 - 2$	$c = -2$	Shift down 2 units	

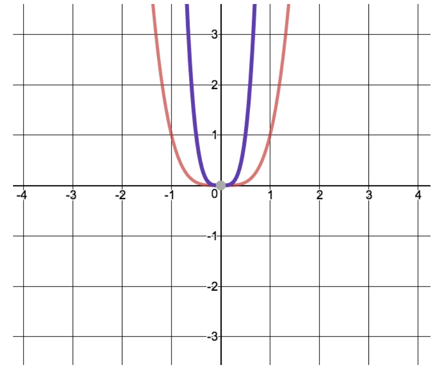
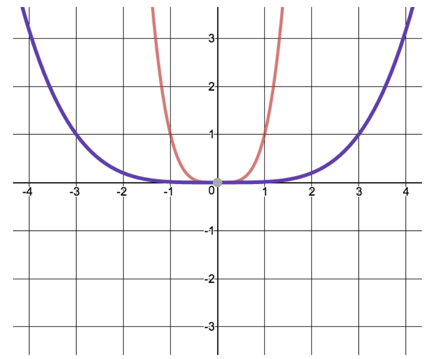
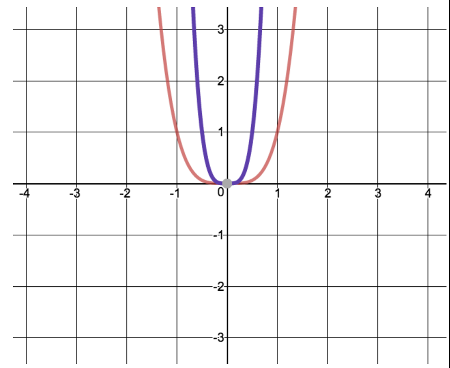
Effects of d on $y = (x - d)^4$

Transformed Function	**Value of d**	**Transformations to $y = x^4$**	**Graph of transformed function compared to $y = x^4$**
$y = (x - 2)^4$	$d = 2$	Shift right 2 units	
$y = (x + 3)^4$	$d = -3$	Shift left 3 units	

Effects of a on $y = ax^4$

Transformed Function	**Value of a**	**Transformations to $y = x^4$**	**Graph of transformed function compared to $y = x^4$**
$y = 2x^4$	$a = 2$	Vertical stretch by a factor of 2	
$y = \frac{1}{2}x^4$	$a = \frac{1}{2}$	Vertical compression by a factor of $\frac{1}{2}$	
$y = -2x^4$	$a = -2$	Vertical stretch by a factor of 2 and a vertical reflection.	

Effects of k on $y = (kx)^4$

Transformed Function	Value of k	Transformations to $y = x^4$	Graph of transformed function compared to $y = x^4$
$y = (2x)^4$	$k = 2$	Horizontal compression by a factor of $\frac{1}{2}$	
$y = \left(\frac{1}{3}x\right)^4$	$k = \frac{1}{3}$	Horizontal stretch by a factor of 3	
$y = (-2x)^4$	$k = -2$	Horizontal compression by a factor of $\frac{1}{2}$ and a horizontal reflection	

Summary of effects of a, k, d, and c in polynomial functions of the form $f(x) = a[k(x-d)]^n + c$

Value of c in $f(x) = a[k(x-d)]^n + c$	
$c > 0$	Shift c units up
$c < 0$	Shift c units down

Value of d in $f(x) = a[k(x-d)]^n + c$	
$d > 0$	Shift d units right
$d < 0$	Shift $\lvert d\rvert$ units left

Value of a in $f(x) = a[k(x-d)]^n + c$	
$a > 1$ or $a < -1$	Vertical stretch by a factor of $\lvert a\rvert$
$-1 < a < 1$	Vertical compression by a factor of $\lvert a\rvert$
$a < 0$	Vertical reflection (reflection in the x-axis)

Value of k in $f(x) = a[k(x-d)]^n + c$	
$k > 1$ or $k < -1$	Horizontal compression by a factor of $\frac{1}{\lvert k\rvert}$
$-1 < k < 1$	Horizontal stretch by a factor of $\frac{1}{\lvert k\rvert}$
$k < 0$	Horizontal reflection (reflection in the y-axis)

Note:

a and c cause **VERTICAL** transformations and therefore effect the y-coordinates of the function.

k and d cause **HORIZONTAL** transformations and therefore effect the x-coordinates of the function.

When applying transformations to a parent function, make sure to apply the transformations represented by a and k BEFORE the transformations represented by d and c.

Part 2: Describing Transformations from an Equation

Example 1: Describe the transformations that must be applied to the graph of each power function, $f(x)$, to obtain the transformed function, $g(x)$. Then, write the corresponding equation of the transformed function. Then, state the domain and range of the transformed function.

a) $f(x) = x^4, g(x) = 2f\left[\frac{1}{3}(x-5)\right]$

$a = 2$; vertical stretch by a factor of 2 $(2y)$

$k = \frac{1}{3}$; horizontal stretch by a factor of 3 $(3x)$

$d = 5$; shift 5 units right $(x + 5)$

$$g(x) = 2\left[\frac{1}{3}(x-5)\right]^4$$

b) $f(x) = x^5, g(x) = \frac{1}{4}f[-2(x-3)] + 4$

$a = \frac{1}{4}$; vertical compression by a factor of $\frac{1}{4}$ $\left(\frac{y}{4}\right)$

$k = -2$; horizontal compression by a factor of $\frac{1}{2}$ and a horizontal reflection $\left(\frac{x}{-2}\right)$

$d = 3$; shift right 3 units $(x + 3)$

$c = 4$; shift 4 units up $(y + 4)$

$$g(x) = \frac{1}{4}[-2(x-3)]^5 + 4$$

Part 3: Applying Transformations to Sketch a Graph

Example 2: The graph of $f(x) = x^3$ is transformed to obtain the graph of $g(x) = 3[-2(x+1)]^3 + 5$.

a) State the parameters and describe the corresponding transformations

$a = 3$; vertical stretch by a factor of 3 $(3y)$

$k = -2$; horizontal compression by a factor of $\frac{1}{2}$ and a horizontal reflection $\left(\frac{x}{-2}\right)$

$d = -1$; shift left 1 unit $(x - 1)$

$c = 5$; shift up 5 units $(y + 5)$

b) Make a table of values for the parent function and then use the transformations described in part a) to make a table of values for the transformed function.

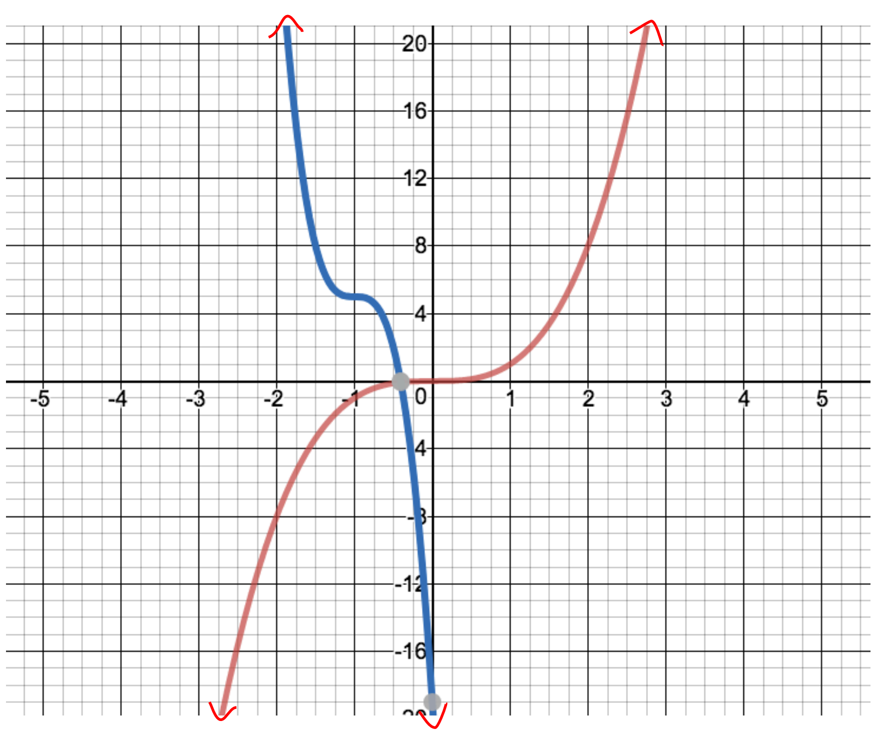
Note: When choosing key points for the parent function, always choose x-values between -2 and 2 and calculate the corresponding values of y.

$f(x) = x^3$	
x	y
-2	-8
-1	-1
0	0
1	1
2	8

→

$g(x) = 3[-2(x+1)]^3 + 5$	
$\frac{x}{-2} - 1$	$3y + 5$
0	-19
-0.5	2
-1	5
-1.5	8
-2	29

c) Graph the parent function and the transformed function on the same grid.



Example 3: The graph of $f(x) = x^4$ is transformed to obtain the graph of $g(x) = -\left(\frac{1}{3}x + 2\right)^4 - 1$.

a) State the parameters and describe the corresponding transformations

Note: k value must be factored out in to the form $[k(x+d)]$

$g(x) = -\left[\frac{1}{3}(x+6)\right]^4 - 1$

$a = -1$; vertical reflection $(-1y)$

$k = \frac{1}{3}$; horizontal stretch by a factor of 3 $(3x)$

$d = -6$; shift left 6 units $(x - 6)$

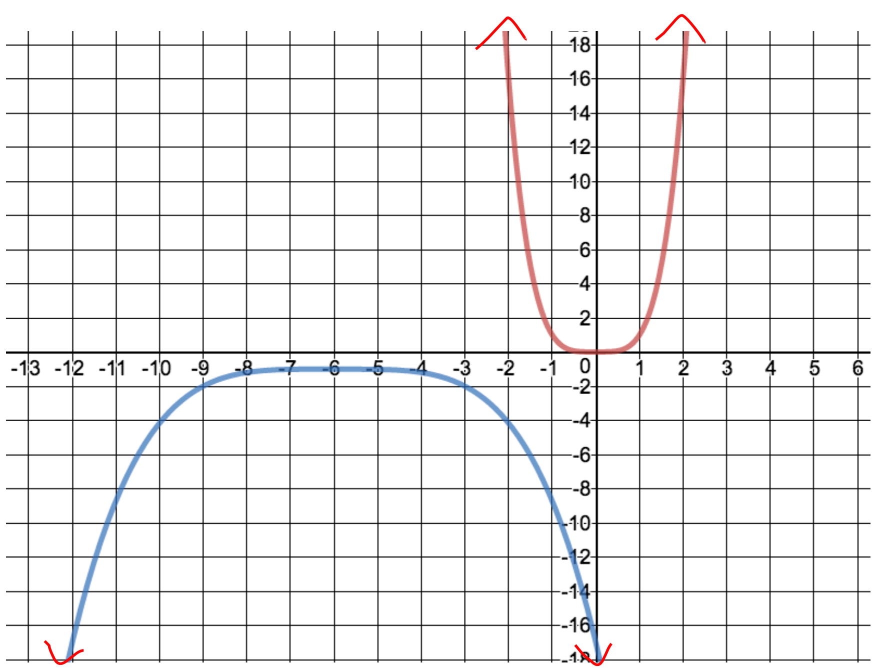
$c = -1$; shift down 1 unit $(y - 1)$

b) Make a table of values for the parent function and then use the transformations described in part a) to make a table of values for the transformed function.

$f(x) = x^4$	
x	y
-2	16
-1	1
0	0
1	1
2	16

$g(x) = -\left[\frac{1}{3}(x+6)\right]^4 - 1$	
$3x - 6$	$-y - 1$
-12	-17
-9	-2
-6	-1
-3	-2
0	-17

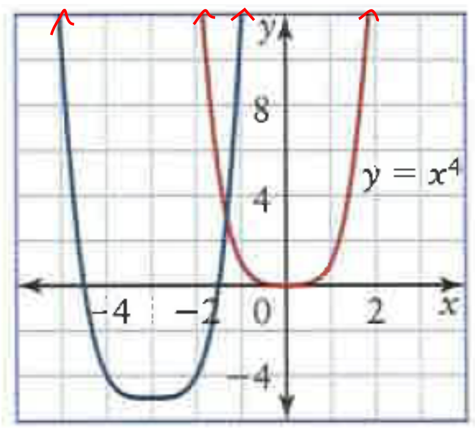
c) Graph the parent function and the transformed function on the same grid.



Part 4: Determining an Equation Given the Graph of a Transformed Function

Example 4: Transformations are applied to each power function to obtain the resulting graph. Determine an equation for the transformed function. Then state the domain and range of the transformed function.

a)



Notice the transformed function is the same shape as the parent function. Therefore, it has not been stretched or compressed.

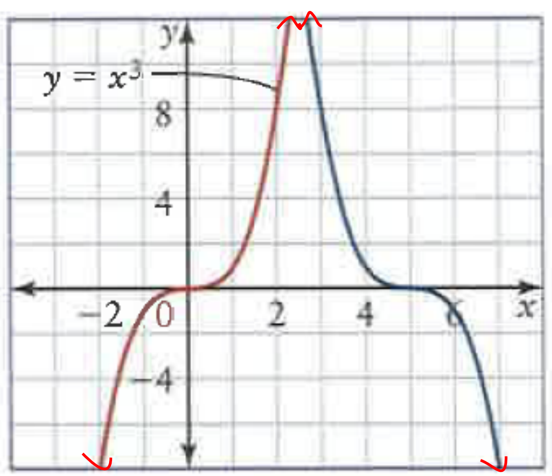
$d = -3$; it has been shifted left 3 units

$c = -5$; it has been shifted down 5 units

$\boldsymbol{g(x) = (x+3)^4 - 5}$

Domain: $(-\infty, \infty)$ **Range:** $[\boldsymbol{-5}, \infty)$

b)



Notice the transformed function is the same shape as the parent function. Therefore, it has not been stretched or compressed.

$a = -1$; it has been reflected vertically

$d = 5$; it has been shifted right 5 units

$\boldsymbol{g(x) = -(x-5)^3}$

Domain: $(-\infty, \infty)$ **Range:** $(-\infty, \infty)$