

W5 - 1.3 - Symmetry in Polynomial Functions

MHF4U

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ANSWERS

1) Determine whether each function is even, odd, or neither. Does it have line symmetry about the y-axis, point symmetry about the origin, or neither?

a)  $y = x^4 - x^2$

Even

Line symmetry about y-axis

b)  $y = -2x^3 + 5x$

Odd

Point symmetry about origin

c)  $y = -4x^5 + 2x^2$

Neither

d)  $y = x(2x + 1)^2(x - 4)$

$$\begin{aligned} &= x(4x^2 + 4x + 1)(x - 4) \\ &= (4x^3 + 4x^2 + x)(x - 4) \\ &= 4x^4 - 16x^3 + 4x^3 - 16x^2 + x^2 - 4x \\ &= 4x^4 - 12x^3 - 15x^2 - 4x \end{aligned}$$

Neither

e)  $y = -2x^6 + x^4 + 8$

Even

Line symmetry about the y-axis

2) State whether each function is even or odd. Verify algebraically.

a)  $f(x) = x^4 - 13x^2 + 36$  Even

$$\begin{aligned} f(-x) &= (-x)^4 - 13(-x)^2 + 36 \\ &= (-1)^4(x)^4 - 13(-1)^2(x)^2 + 36 \\ &= x^4 - 13x^2 + 36 \end{aligned}$$

∴  $f(x) = f(-x)$

b)  $g(x) = 6x^5 - 7x^3 - 3x$  ODD

$$\begin{aligned} -g(x) &= -1[6x^5 - 7x^3 - 3x] \\ &= -6x^5 + 7x^3 + 3x \end{aligned} \quad \& \quad -g(x) = g(-x)$$

$$\begin{aligned} g(-x) &= 6(-x)^5 - 7(-x)^3 - 3(-x) \\ &= 6(-1)^5(x)^5 - 7(-1)^3(x)^3 - 3(-1)(x) \\ &= -6x^5 + 7x^3 + 3x \end{aligned}$$

3) Use the given graph to state:

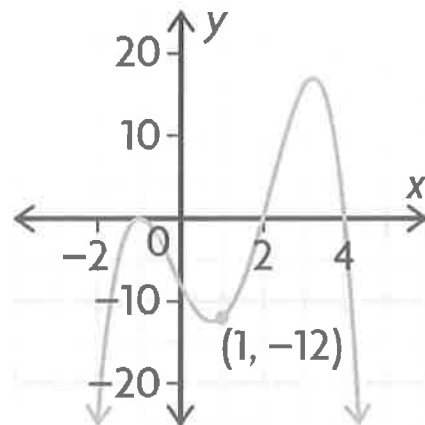
a) x-intercepts  $-1$  (order 2),  $2$ , and  $4$

b) number of turning points  $3$

c) least possible degree  $4$

d) any symmetry present; even or odd function?

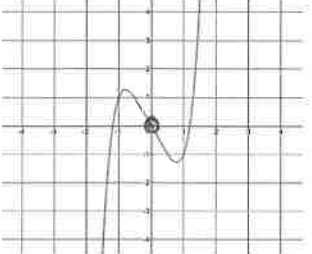
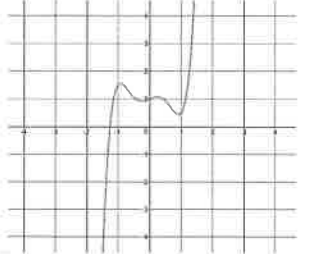
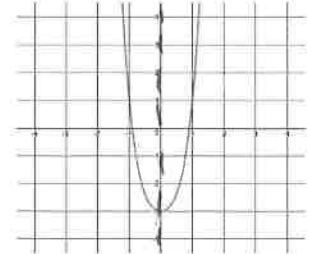
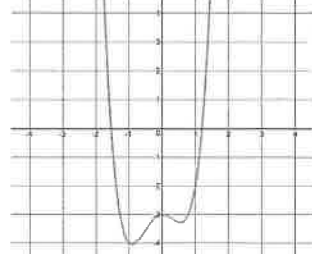
None, Neither



the intervals where  $f(x) < 0$

$$f(x) < 0 \text{ when } x \in (-\infty, -1) \cup (-1, 2) \cup (4, \infty)$$

4) Label each function as even, odd, or neither

			
ODD	NEITHER	EVEN	NEITHER

**ANSWER KEY**

1) a) even, line symmetry about y-axis    b) odd, point symmetry about origin    c) neither

d) neither    e) even, line symmetry about y-axis

2) a) even,  $f(-x) = f(x)$     b) odd,  $f(-x) = -f(x)$

3) a) -1 (order 2), 2, and 4    b) 3    c) 4    d) no symmetry, neither    e)  $X \in (-\infty, -1) \cup (-1, 2) \cup (4, \infty)$

4) odd, neither, even, neither