W1 – 2.1 – Long Division of Polynomials and The Remainder Theorem
MHF4U
Jensen

1) Use the remainder theorem to determine the remainder when $2x^3 + 7x^2 - 8x + 3$ is divided by each binomial.

a) $x + 1$

$P(-1) = 2(-1)^3 + 7(-1)^2 - 8(-1) + 3$
$= 16$

b) $x - 2$

$P(2) = 2(2)^3 + 7(2)^2 - 8(2) + 3$
$= 31$

c) $x + 3$

$P(-3) = 2(-3)^3 + 7(-3)^2 - 8(-3) + 3$
$= 36$

2)a) Divide $x^3 + 3x^2 - 2x + 5$ by $x + 1$. Express the result in quotient form.

$$
\begin{array}{r}
x^2 + 2x - 4 \\
x+1 \,\big)\, \overline{x^3 + 3x^2 - 2x + 5} \\
\underline{x^3 + x^2} \\
2x^2 - 2x \\
\underline{2x^2 + 2x} \\
-4x + 5 \\
\underline{-4x - 4} \\
R = 9
\end{array}
$$

$$\boxed{\frac{x^3 + 3x^2 - 2x + 5}{x+1} = x^2 + 2x - 4 + \frac{9}{x+1}}$$

b) Write the corresponding statement that can be used to check the division.

$$x^3 + 3x^2 - 2x + 5 = (x+1)(x^2 + 2x - 4) + 9$$

3) Divide $3x^4 - 4x^3 - 6x^2 + 17x - 8$ by $3x - 4$. Express the result in quotient form.

$$
\begin{array}{r}
x^3 + 0x^2 - 2x + 3 \\
3x-4 \,\big)\, \overline{3x^4 - 4x^3 - 6x^2 + 17x - 8} \\
\underline{3x^4 - 4x^3} \\
0x^3 - 6x^2 \\
\underline{0x^3 - 0x^2} \\
-6x^2 + 17x \\
\underline{-6x^2 + 8x} \\
9x - 8 \\
\underline{9x - 12} \\
R = 4
\end{array}
$$

$$\boxed{\frac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{3x-4} = x^3 - 2x + 3 + \frac{4}{3x-4}}$$

b) Write the corresponding statement that can be used to check the division.

$$3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x-4)(x^3 - 2x + 3) + 4$$

4) Perform each division. Express the result in quotient form.

a) $x^3 + 7x^2 - 3x + 4$ divided by $x + 2$

$x^2 + 5x - 13$

$x+2 \overline{) x^3 + 7x^2 - 3x + 4}$

$x^3 + 2x^2$

$5x^2 - 3x$

$5x^2 + 10x$

$-13x + 4$

$-13x - 26$

$R = 30$

$$\boxed{\frac{x^3 + 7x^2 - 3x + 4}{x+2} = x^2 + 5x - 13 + \frac{30}{x+2}}$$

b) $6x^3 + x^2 - 14x - 6$ divided by $3x + 2$

$2x^2 - 1x - 4$

$3x+2 \overline{) 6x^3 + x^2 - 14x - 6}$

$6x^3 + 4x^2$

$-3x^2 - 14x$

$-3x^2 - 2x$

$-12x - 6$

$-12x - 8$

$R = 2$

$$\boxed{\frac{6x^3 + x^2 - 14x - 6}{3x+2} = 2x^2 - x - 4 + \frac{2}{3x+2}}$$

c) $10x^3 + 11 - 9x^2 - 8x$ divided by $5x - 2$

$2x^2 - 1x - 2$

$5x-2 \overline{) 10x^3 - 9x^2 - 8x + 11}$

$10x^3 - 4x^2$

$-5x^2 - 8x$

$-5x^2 + 2x$

$-10x + 11$

$-10x + 4$

$R = 7$

$$\boxed{\frac{10x^3 - 9x^2 - 8x + 11}{5x - 2} = 2x^2 - x - 2 + \frac{7}{5x-2}}$$

d) $11x - 4x^4 - 7$ divided by $x - 3$

$-4x^3 - 12x^2 - 36x - 97$

$x-3 \overline{) -4x^4 + 0x^3 + 0x^2 + 11x - 7}$

$-4x^4 + 12x^3$

$-12x^3 + 0x^2$

$-12x^3 + 36x^2$

$-36x^2 + 11x$

$-36x^2 + 108x$

$-97x - 7$

$-97x + 291$

$R = -298$

$$\boxed{\frac{-4x^4 + 11x - 7}{x-3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x-3}}$$

e) $6x^3 + x^2 + 7x + 3$ divided by $3x + 2$

$2x^2 - 1x + 3$

$3x+2 \overline{) 6x^3 + x^2 + 7x + 3}$

$6x^3 + 4x^2$

$-3x^2 + 7x$

$-3x^2 - 2x$

$9x + 3$

$9x + 6$

$R = -3$

$$\boxed{\frac{6x^3 + x^2 + 7x + 3}{3x+2} = 2x^2 - 1x + 3 - \frac{3}{3x+2}}$$

f) $8x^3 + 4x^2 - 31$ divided by $2x - 3$

$4x^2 + 8x + 12$

$2x-3 \overline{) 8x^3 + 4x^2 + 0x - 31}$

$8x^3 - 12x^2$

$16x^2 + 0x$

$16x^2 - 24x$

$24x - 31$

$24x - 36$

$R = 5$

$$\boxed{\frac{8x^3 + 4x^2 - 31}{2x-3} = 4x^2 + 8x + 12 + \frac{5}{2x-3}}$$

g) $6x^2 - 6 + 8x^3$ divided by $4x - 3$

$2x^2 + 3x + \frac{9}{4}$

$4x-3 \overline{) 8x^3 + 6x^2 + 0x - 6}$

$8x^3 - 6x^2$

$12x^2 + 0x$

$12x^2 - 9x$

$9x - 6$

$9x - \frac{27}{4}$

$R = 3/4$

$$\boxed{\frac{8x^3 + 6x^2 - 6}{4x-3} = 2x^2 + 3x + \frac{9}{4} + \frac{3}{4(4x-3)}}$$

5) The volume, in cubic cm, of a rectangular box can be modelled by the polynomial expression $2x^3 + 17x^2 + 38x + 15$. Determine possible dimensions of the box if the height, in cm, is given by $x + 5$.

$$\begin{array}{r|l} & 2x^2 + 7x + 3 \\ \hline x+5 & 2x^3 + 17x^2 + 38x + 15 \\ & 2x^3 + 10x^2 \\ \hline & 7x^2 + 38x \\ & 7x^2 + 35x \\ \hline & 3x + 15 \\ & 3x + 15 \\ \hline & R = 0 \end{array}$$

$2x^3 + 17x^2 + 38x + 15 = (x+5)(2x^2 + 7x + 3)$ Factor

$= (x+5)(x+3)(2x+1)$

height length width

P 6, S 7, $\frac{3}{1}$ = $\frac{6}{2}$, $\frac{1}{2}$

6) Determine the value of k such that when $P(x) = kx^3 + 5x^2 - 2x + 3$ is divided by $x + 1$, the remainder is 7.

$P(-1) = k(-1)^3 + 5(-1)^2 - 2(-1) + 3$

$7 = -1k + 5 + 2 + 3$

$7 = -1k + 10$

$-3 = -1k$

$\boxed{k = 3}$

ANSWER KEY

1) a) 16 **b)** 31 **c)** 36

2)a) $\frac{x^3+3x^2-2x+5}{x+1} = x^2 + 2x - 4 + \frac{9}{x+1}$ **b)** $x^3 + 3x^2 - 2x + 5 = (x+1)(x^2 + 2x - 4) + 9$

3)a) $\frac{3x^4-4x^3-6x^2+17x-8}{3x-4} = x^3 - 2x + 3 + \frac{4}{3x-4}$ **b)** $3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x-4)(x^3 - 2x + 3) + 4$

4)a) $\frac{x^3+7x^2-3x+4}{x+2} = x^2 + 5x - 13 + \frac{30}{x+2}$ **b)** $\frac{6x^3+x^2-14x-6}{3x+2} = 2x^2 - x - 4 + \frac{2}{3x+2}$

c) $\frac{10x^3-9x^2-8x+11}{5x-2} = 2x^2 - x - 2 + \frac{7}{5x-2}$ **d)** $\frac{-4x^4+11x-7}{x-3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x-3}$

e) $\frac{6x^3+x^2+7x+3}{3x+2} = 2x^2 - x + 3 - \frac{3}{3x+2}$ **f)** $\frac{8x^3+4x^2-31}{2x-3} = 4x^2 + 8x + 12 + \frac{5}{2x-3}$

g) $\frac{6x^2-6+8x^3}{4x-3} = 2x^2 + 3x + \frac{9}{4} + \frac{3}{4(4x-3)}$

5) $2x^3 + 17x^2 + 38x + 15 = (x+5)(x+3)(2x+1)$

6) $k = 3$