W2 – 2.1 – Synthetic Division
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SOLUTIONS

1) Calculate each of the following using synthetic division. Express your answer using the statement that could be used to check the division.

a) $x^3 - 7x - 6$ divided by $x - 3$

$$\begin{array}{c|cccc} 3 & 1 & 0 & -7 & -6 \\ & \downarrow & 3 & 9 & 6 \quad + \\ \hline \times & 1 & 3 & 2 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$\boxed{x^3 - 7x - 6 = (x-3)(x^2+3x+2)}$$

b) $2x^3 - 7x^2 - 7x + 19$ divided by $x - 1$

$$\begin{array}{c|cccc} 1 & 2 & -7 & -7 & 19 \\ & \downarrow & 2 & -5 & -12 \quad + \\ \hline \times & 2 & -5 & -12 & 7 \\ & x^2 & x & \# & R \end{array}$$

$$\boxed{2x^3 - 7x^2 - 7x + 19 = (x-1)(2x^2-5x-12)+7}$$

c) $6x^4 + 13x^3 - 34x^2 - 47x + 28$ divided by $x + 3$

$$\begin{array}{c|ccccc} -3 & 6 & 13 & -34 & -47 & 28 \\ & \downarrow & -18 & 15 & 57 & -30 \quad + \\ \hline & 6 & -5 & -19 & 10 & -2 \\ & x^3 & x^2 & x & \# & R \end{array}$$

$$\boxed{\begin{aligned} &6x^4+13x^3-34x^2-47x+28 \\ &= (x+3)(6x^3-5x^2-19x+10)-2 \end{aligned}}$$

d) $2x^3 + x^2 - 22x + 20$ divided by $2x - 3$ $= 2(x - \frac{3}{2})$

$$\begin{array}{c|cccc} 3/2 & 2 & 1 & -22 & 20 \\ & \downarrow & 3 & 6 & -24 \quad + \\ \hline \times & 2 & 4 & -16 & -4 \\ & & & & R \end{array}$$

$\underbrace{2 \quad 4 \quad -16}_{\div 2}$

$$\begin{array}{ccc} 1 & 2 & -8 \\ x^2 & x & \# \end{array}$$

$$\boxed{\begin{aligned} &2x^3+x^2-22x+20 \\ &= (2x-3)(x^2+2x-8)-4 \end{aligned}}$$

e) $12x^4 - 56x^3 + 59x^2 + 9x - 18$ divided by $2x + 1$ $= 2(x + \frac{1}{2})$

$$\begin{array}{c|ccccc} -1/2 & 12 & -56 & 59 & 9 & -18 \\ & \downarrow & -6 & 31 & -45 & 18 \\ \hline & 12 & -62 & 90 & -36 & 0 \\ & & & & & R \end{array}$$

$\underbrace{12 \quad -62 \quad 90 \quad -36}_{\div 2}$

$$\begin{array}{cccc} 6 & -31 & 45 & -18 \end{array}$$

$$\boxed{12x^4-56x^3+59x^2+9x-18 = (2x+1)(6x^3-31x^2+45x-18)}$$

f) $6x^3 - 15x^2 - 2x + 5$ divided by $2x - 5$ $= 2(x - \frac{5}{2})$

$$\begin{array}{c|cccc} 5/2 & 6 & -15 & -2 & 5 \\ & \downarrow & 15 & 0 & -5 \quad + \\ \hline \times & 6 & 0 & -2 & 0 \\ & & & & R \end{array}$$

$\underbrace{6 \quad 0 \quad -2}_{\div 2}$

$$\begin{array}{ccc} 3 & 0 & -1 \\ x^2 & x & \# \end{array}$$

$$\boxed{6x^3-15x^2-2x+5 = (2x-5)(3x^2-1)}$$

g) $x^3 - 2x + 1$ divided by $x - 4$

4	1	0	-2	1	
	↓	4	16	56	+
x	1	4	14	57	
	x^2	x	#	R	

$$\boxed{x^3 - 2x + 1 = (x-4)(x^2+4x+14)+57}$$

h) $x^3 + 2x^2 - 6x + 1$ divided by $x + 2$

-2	1	2	-6	1	
	↓	-2	0	12	+
x	1	0	-6	13	
	x^2	x	#	R	

$$\boxed{x^3+2x^2-6x+1 = (x+2)(x^2-6)+13}$$

2) Divide $x^4 - 16x^3 + 4x^2 + 10x - 11$ by each of the following binomials...

a) $x - 2$

2	1	-16	4	10	-11	
	↓	2	-28	-48	-76	+
x	1	-14	-24	-38	-87	
	x^3	x^2	x	#	R	

$$\boxed{\begin{aligned} &x^4-16x^3+4x^2+10x-11 \\ &= (x-2)(x^3-14x^2-24x-38)-87 \end{aligned}}$$

b) $x + 4$

-4	1	-16	4	10	-11	
	↓	-4	80	-336	1304	+
x	1	-20	84	-326	1293	
	x^3	x^2	x	#	R	

$$\boxed{\begin{aligned} &x^4-16x^3+4x^2+10x-11 \\ &= (x+4)(x^3-20x^2+84x-326)+1293 \end{aligned}}$$

3) Are either of the binomials in question #2 factors of $x^4 - 16x^3 + 4x^2 + 10x - 11$? Explain.

No, because there is a non-zero Remainder for each.

ANSWER KEY

1)a) $x^3 - 7x - 6 = (x - 3)(x^2 + 3x + 2)$ **b)** $2x^3 - 7x^2 - 7x + 19 = (x - 1)(2x^2 - 5x - 12) + 7$

c) $6x^4 + 13x^3 - 34x^2 - 47x + 28 = (x + 3)(6x^3 - 5x^2 - 19x + 10) - 2$

d) $2x^3 + x^2 - 22x + 20 = (2x - 3)(x^2 + 2x - 8) - 4$

e) $12x^4 - 56x^3 + 59x^2 + 9x - 18 = (2x + 1)(6x^3 - 31x^2 + 45x - 18)$ **f)** $6x^3 - 15x^2 - 2x + 5 = (2x - 5)(3x^2 - 1)$

g) $x^3 - 2x + 1 = (x - 4)(x^2 + 4x + 14) + 57$ **h)** $x^3 + 2x^2 - 6x + 1 = (x + 2)(x^2 - 6) + 13$

2)a) $x^4 - 16x^3 + 4x^2 + 10x - 11 = (x - 2)(x^3 - 14x^2 - 24x - 38) - 87$
b) $x^4 - 16x^3 + 4x^2 + 10x - 11 = (x + 4)(x^3 - 20x^2 + 84x - 326) + 1293$

3) No, because for each division problem, there is a remainder.