L2 – 2.1 – Synthetic Division Lesson
MHF4U
Jensen

In this section you will learn how to use synthetic division as an alternate method to dividing a polynomial by a binomial. Synthetic division is an efficient way to divide a polynomial by a binomial of the form $x - b$.

IMPORTANT: When using Polynomial OR Synthetic division…

- Terms must be arranged in descending order of degree, in both the divisor and the dividend.
- Zero must be used as the coefficient of any missing powers of the variable in both the divisor and the dividend.

Part 1: Synthetic division when the binomial is of the form $x - b$

Divide $3x^3 - 5x^2 - 7x - 1$ by $x - 3$. In this question, $b = 3$.

coefficients of dividend

zero of divisor

$$
\begin{array}{c|cccc}
3 & 3 & -5 & -7 & -1 \\
 & \downarrow & 9 & 12 & 15 \quad + \\
\hline
\times & 3 & 4 & 5 & 14 \\
 & x^2 & x & \# & R
\end{array}
$$

$\times 3$ $\times 3$ $\times 3$

List the coefficients of the dividend in the first row. To the left, write the b value (the zero of the divisor). Place a + sign above the horizontal line to represent addition and a × sign below the horizontal line to indicate multiplication of the divisor and the terms of the quotient.

Bring the first term down, this is the coefficient of the first term of the quotient. Multiply it by the b value and write this product below the second term of the dividend.

Now add the terms together.

Multiply this sum by the b value and write the product below the third term of the dividend. Repeat this process until you have a completed the chart.

The last number below the chart is the remainder. The first numbers are the coefficients of the quotient, starting with degree that is one less than the dividend.

Don't forget that the answer can be written in two ways…

$$\frac{3x^3 - 5x^2 - 7x - 1}{x - 3} = \mathbf{3x^2 + 4x + 5 + \frac{14}{x - 3}}$$

OR

$$3x^3 - 5x^2 - 7x - 1 = \mathbf{(x - 3)(3x^2 + 4x + 5) + 14}$$

Example 1: Use synthetic division to divide. Then write the multiplication statement that could be used to check the division.

a) $(x^4 - 2x^3 + 13x - 6) \div (x + 2)$

$= x^4 - 2x^3 + 0x^2 + 13x - 6$

$$\begin{array}{c|ccccc}
-2 & 1 & -2 & 0 & 13 & -6 \\
 & \downarrow & -2 & 8 & -16 & 6 \quad + \\
\hline
\times & 1 & -4 & 8 & -3 & 0 \\
 & x^3 & x^2 & x & \# & R
\end{array}$$

$x^4 - 2x^3 + 13x - 6 = (x+2)(x^3 - 4x^2 + 8x - 3)$

Note: *since the remainder is zero, both the quotient and divisor are* ***factors*** *of the dividend.*

b) $(2x^3 - 5x^2 + 8x + 4) \div (x - 3)$

$$\begin{array}{c|cccc}
3 & 2 & -5 & 8 & 4 \\
 & \downarrow & 6 & 3 & 33 \quad + \\
\hline
\times & 2 & 1 & 11 & 37 \\
 & x^2 & x & \# & R
\end{array}$$

$2x^3 - 5x^2 + 8x + 4 = (x-3)(2x^2 + x + 11) + 37$

Part 2: Synthetic division when the binomial is of the form $ax - b$

Divide $6x^3 + 5x^2 - 16x - 15$ by $2x + 3$

To use synthetic division, the divisor must be in the form $x - b$. Re-write the divisor by factoring out the coefficient of the x.

$(2x+3) = 2\left(x + \frac{3}{2}\right) \rightarrow b = -\frac{3}{2}$

We can now divide $6x^3 + 5x^2 - 16x - 15$ by $\left(x + \frac{3}{2}\right)$ using synthetic division as long as you remember to divide the quotient by 2 after.

$$
\begin{array}{r|rrrr}
-3/2 & 6 & 5 & -16 & -15 \\
 & \downarrow & -9 & 6 & 15 \quad + \\
\hline
\times & 6 & -4 & -10 & 0 \\
 & & \div 2 & & R
\end{array}
$$

$$
\begin{array}{ccc}
\div\ 3 & -2 & 5 \\
x^2 & x & \#
\end{array}
$$

$6x^3 + 5x^2 - 16x - 15 = (2x+3)(3x^2 - 2x - 5)$

Check answer using long division

$$
\begin{array}{r|l}
 & 3x^2 - 2x - 5 \\
\hline
2x+3 & 6x^3 + 5x^2 - 16x - 15 \\
 & \underline{6x^3 + 9x^2} \\
 & \quad -4x^2 - 16x \\
 & \quad \underline{-4x^2 - 6x} \\
 & \qquad -10x - 15 \\
 & \qquad \underline{-10x - 15} \\
 & \qquad R = 0
\end{array}
$$

$6x^3 + 5x^2 - 16x - 15 = (2x+3)(3x^2 - 2x - 5)$

Note: Synthetic division can only be used with a linear divisor. It is most useful with a divisor of the form $x - b$. If the divisor is $ax - b$, it can be used but long division may be easier.

Example 2: Find each quotient by choosing an appropriate strategy.

a) Divide $x^3 - 4x^2 + 2x + 3$ by $x - 3$

$$\begin{array}{c|cccc} 3 & 1 & -4 & 2 & 3 \\ & \downarrow & 3 & -3 & -3 \\ \hline & 1 & -1 & -1 & 0 \\ & x^2 & x & \# & R \end{array}$$

use synthetic division because we have a linear divisor of the form $x-b$

$$x^3 - 4x^2 + 2x + 3 = (x-3)(x^2 - x - 1)$$

b) Divide $12x^4 - 56x^3 + 59x^2 + 9x - 18$ by $2x + 1$

$$\begin{array}{r} 6x^3 - 31x^2 + 45x - 18 \\ 2x+1 \overline{\smash{)}\, 12x^4 - 56x^3 + 59x^2 + 9x - 18} \\ \underline{12x^4 + 6x^3} \qquad\qquad\qquad\qquad \\ -62x^3 + 59x^2 \qquad\qquad\quad \\ \underline{-62x^3 - 31x^2} \qquad\qquad\quad \\ 90x^2 + 9x \qquad\quad \\ \underline{90x^2 + 45x} \qquad\quad \\ -36x - 18 \\ \underline{-36x - 18} \\ R = 0 \end{array}$$

$$12x^4 - 56x^3 + 59x^2 + 9x - 18 = (2x+1)(6x^3 - 31x^2 + 45x - 18)$$

c) Divide $x^4 - 2x^3 + 5x + 3$ by $x^2 + 2x + 1$

$$x^2 - 4x + 7$$

$$x^2+2x+1 \overline{\big)\, x^4 - 2x^3 + 0x^2 + 5x + 3}$$

$$\underline{x^4 + 2x^3 + x^2}$$

$$-4x^3 - 1x^2 + 5x$$

$$\underline{-4x^3 - 8x^2 - 4x}$$

$$7x^2 + 9x + 3$$

$$\underline{7x^2 + 14x + 7}$$

$$R = -5x - 4$$

use long division since it is a non-linear divisor

$$x^4 - 2x^3 + 5x + 3 = (x^2+2x+1)(x^2-4x+7) - 5x - 4$$

d) Divide $x^4 - x^3 - x^2 + 2x + 1$ by $x^2 + 2$

$$x^2 - 1x - 3$$

$$x^2+0x+2 \overline{\big)\, x^4 - x^3 - x^2 + 2x + 1}$$

$$\underline{x^4 + 0x^3 + 2x^2}$$

$$-1x^3 - 3x^2 + 2x$$

$$\underline{-1x^3 + 0x^2 - 2x}$$

$$-3x^2 + 4x + 1$$

$$\underline{-3x^2 + 0x - 6}$$

$$4x + 7$$

use long division since it is a non-linear divisor

$$x^4 - x^3 - x^2 + 2x + 1 = (x^2+2)(x^2 - x - 3) + 4x + 7$$