

W3 - 2.2 - Factor Theorem

MHF4U

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Determine if $x + 3$ is a factor of each polynomial:

a) $x^3 + x^2 - x + 6$

$$f(-3) = (-3)^3 + (-3)^2 - (-3) + 6$$

$$= -27 + 9 + 3 + 6$$

$$= -9$$

∴ Not a factor

b) $2x^3 + 9x^2 + 10x + 3$

$$f(-3) = 2(-3)^3 + 9(-3)^2 + 10(-3) + 3$$

$$= -54 + 81 - 30 + 3$$

$$= 0$$

∴ a factor

c) $x^3 + 27$

$$f(-3) = (-3)^3 + 27$$

$$= 0$$

∴ a factor

2) Find possible factors of the following polynomials using integral zero theorem. Then, factor the polynomial.

a) $x^3 + 3x^2 - 6x - 8$

Possible factors: $\pm 1, \pm 2, \pm 4, \pm 8$

$f(-1) = 0$; ∴ $x+1$ is a factor

-1	1	3	-6	-8	
	↓	-1	-2	8	+
⊗	1	2	-8	0	
	x^2	x	#	R	

$$x^3 + 3x^2 - 6x - 8 = (x+1)(x^2 + 2x - 8)$$

$$= (x+1)(x+4)(x-2)$$

b) $x^3 + 4x^2 - 15x - 18$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$f(-1) = 0$; ∴ $x+1$ is a factor

-1	1	4	-15	-18	
	↓	-1	-3	18	+
x	1	3	-18	0	
	x^2	x	#	R	

$$x^3 + 4x^2 - 15x - 18 = (x+1)(x^2 + 3x - 18)$$

$$= (x+1)(x+6)(x-3)$$

c) $x^3 - 3x^2 - 10x + 24$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$f(2) = 0$; ∴ $x-2$ is a factor

2	1	-3	-10	24	
	↓	2	-2	-24	+
x	1	-1	-12	0	
	x^2	x	#	R	

$$x^3 - 3x^2 - 10x + 24 = (x-2)(x^2 - x - 12)$$

$$= (x-2)(x-4)(x+3)$$

3) Factor by grouping:

a) $x^3 + x^2 - 9x - 9$

$$= (x^3 + x^2) + (-9x - 9)$$

$$= x^2(x+1) - 9(x+1)$$

$$= (x+1)(x^2 - 9)$$

$$= (x+1)(x-3)(x+3)$$

b) $2x^3 - x^2 - 72x + 36$

$$= (2x^3 - x^2) + (-72x + 36)$$

$$= x^2(2x-1) - 36(2x-1)$$

$$= (2x-1)(x^2 - 36)$$

$$= (2x-1)(x-6)(x+6)$$

4) Determine a value of k so that $x+2$ is a factor of $x^3 - 2kx^2 + 6x - 4$.

$$f(-2) = (-2)^3 - 2(k)(-2)^2 + 6(-2) - 4$$

$$0 = -8 - 8k - 12 - 4$$

$$0 = -24 - 8k$$

$$24 = -8k$$

$$k = -3$$

5) Find possible factors of the following polynomials using integral zero theorem. Then, factor the polynomial.

a) $3x^3 + x^2 - 22x - 24$

Possible factors: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm 4, \pm \frac{4}{3}, \pm 6, \pm 8, \pm \frac{8}{3}, \pm 12, \pm 24$

$f(-2) = 0$; $\therefore x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 3 & 1 & -22 & -24 \\ & \downarrow & & & \\ x & 3 & -5 & -12 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$3x^3 + x^2 - 22x - 24 = (x+2)(3x^2 - 5x - 12)$$

$$= (x+2)(x-3)(3x+4)$$

$$\begin{array}{r} P \\ -36 \\ \hline \frac{-3}{1} = \frac{-9}{3} \times \frac{4}{3} \\ -5 \\ \hline 5 \end{array}$$

b) $2x^3 - 9x^2 + 10x - 3$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}$

$f(1) = 0$; $\therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 10 & -3 \\ & \downarrow & & & \\ x & 2 & -7 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$2x^3 - 9x^2 + 10x - 3 = (x-1)(2x^2 - 7x + 3)$$

$$= (x-1)(x-3)(2x-1)$$

$$\begin{array}{r} P \\ 6 \\ \hline \frac{-3}{1} = \frac{6}{2} \times \frac{-1}{2} \\ -7 \\ \hline 5 \end{array}$$

c) $6x^3 - 11x^2 - 26x + 15$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{3}, \pm \frac{5}{6}, \dots$

$f(3) = 0$; $\therefore x-3$ is a factor

$$\begin{array}{r|rrrr} 3 & 6 & -11 & -26 & 15 \\ & \downarrow & & & \\ x & 6 & -7 & -5 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$6x^3 - 11x^2 - 26x + 15 = (x-3)(6x^2 + 7x - 5)$$

$$= (x-3)(3x+5)(2x-1)$$

$$\begin{array}{r} P \\ -36 \\ \hline \frac{5}{3} = \frac{10}{6} \times \frac{-3}{6} \\ 7 \\ \hline 5 \end{array}$$

d) $4x^3 + 3x^2 - 4x - 3$

Possible factors: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$f(1) = 0$; $\therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 4 & 3 & -4 & -3 \\ & \downarrow & & & \\ x & 4 & 7 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$4x^3 + 3x^2 - 4x - 3 = (x-1)(4x^2 + 7x + 3)$$

$$= (x-1)(x+1)(4x+3)$$

$$\begin{array}{r} P \\ 12 \\ \hline \frac{1}{1} = \frac{4}{4} \times \frac{3}{4} \\ 7 \\ \hline 5 \end{array}$$

6) Factor each polynomial

a) $2x^3 + 5x^2 - x - 6$

Possible Factors: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

$f(1) = 0$; $\therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -1 & -6 \\ & \downarrow & & & \\ x & 2 & 7 & 6 & 0 \\ & x^2 & x & \# & R \end{array}$$

P
 $\frac{3}{1} = \frac{4}{2} = \frac{3}{2}$

$$2x^3 + 5x^2 - x - 6 = (x-1)(2x^2 + 7x + 6)$$

$$= (x-1)(x+2)(2x+3)$$

b) $4x^3 - 7x - 3$

Possible Factors: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$f(-1) = 0$; $\therefore x+1$ is a factor

$$\begin{array}{r|rrrr} -1 & 4 & 0 & -7 & -3 \\ & \downarrow & & & \\ x & 4 & -4 & -3 & 0 \\ & x^2 & x & \# & R \end{array}$$

P
 $\frac{-3}{2} = \frac{-6}{4} = \frac{2}{4} = \frac{1}{2}$

$$4x^3 - 7x - 3 = (x+1)(4x^2 - 4x - 3)$$

$$= (x+1)(2x-3)(2x+1)$$

c) $x^4 - 15x^2 - 10x + 24$

Possible factors: $\pm 1, \pm 2, \pm 4$

$f(1) = 0$; $\therefore x-1$ is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -15 & -10 & 24 \\ & \downarrow & & & & \\ x & 1 & 1 & -14 & -24 & 0 \\ & x^3 & x^2 & x & \# & R \end{array}$$

$$x^4 - 15x^2 - 10x + 24 = (x-1)(x^3 + x^2 - 14x - 24)$$

$$= (x-1)(x+2)(x^2 - x - 12)$$

$$= (x-1)(x+2)(x-4)(x+3)$$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$f(-2) = 0$; $\therefore x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & \downarrow & & & \\ x & 1 & -1 & -12 & 0 \\ & x^2 & x & \# & R \end{array}$$

ANSWER KEY

1) **a)** No **b)** Yes **c)** Yes

2) **a)** $(x - 2)(x + 1)(x + 4)$ **b)** $(x - 3)(x + 1)(x + 6)$ **c)** $(x - 4)(x - 2)(x + 3)$

3) **a)** $(x - 3)(x + 1)(x + 3)$ **b)** $(x - 6)(x + 6)(2x - 1)$

4) $k = -3$

5) **a)** $(x - 3)(x + 2)(3x + 4)$ **b)** $(x - 3)(x - 1)(2x - 1)$ **c)** $(x - 3)(2x - 1)(3x + 5)$ **d)** $(x - 1)(x + 1)(4x + 3)$

6) **a)** $(x - 1)(x + 2)(2x + 3)$ **b)** $(x + 1)(2x - 3)(2x + 1)$ **c)** $(x - 4)(x - 1)(x + 2)(x + 3)$