W4 – 2.3 – Solving Polynomial Equations
MHF4U
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1) Determine the solutions of the following polynomials.

a) $(3x+2)(x+9)(x-2)=0$

$3x+2=0$ $\quad x+9=0$ $\quad x-2=0$

$x_1=-\frac{2}{3}$ $\quad x_2=-9$ $\quad x_3=2$

$(-\frac{2}{3},0)$, $(-9,0)$, $(2,0)$

b) $(x^2+1)(x-4)=0$

$x^2+1=0$ $\quad x-4=0$

$x^2=-1$ $\quad x=4$

$\therefore$ No solutions

$(4,0)$

2) Determine the solutions of the following polynomials by factoring. Use the tools you have learned this unit to help you. (remainder theorem, integral zero theorem, division etc.)

a) $x^3-4x^2-3x+18=0$

Possible zeros: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

$f(-2)=0$; $\therefore x+2$ is a factor

$$\begin{array}{c|cccc} -2 & 1 & -4 & -3 & 18 \\ & \downarrow & -2 & 12 & -18 \\ \hline \times & 1 & -6 & 9 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x+2)(x^2-6x+9)=0$

$(x+2)(x-3)^2=0$

$x+2=0$ $\quad x-3=0$

$\boxed{x_1=-2}$ $\quad \boxed{x_2=3}$

Solutions: $(-2,0)$ and $(3,0)$

b) $x^3-3x^2-4x+12=0$

Possible zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$f(2)=0$; $\therefore x-2$ is a factor

$$\begin{array}{c|cccc} 2 & 1 & -3 & -4 & 12 \\ & \downarrow & 2 & -2 & -12 \\ \hline \times & 1 & -1 & -6 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x-2)(x^2-x-6)=0$

$(x-2)(x-3)(x+2)=0$

$x-2=0$ $\quad x-3=0$ $\quad x+2=0$

$\boxed{x_1=2}$ $\quad \boxed{x_2=3}$ $\quad \boxed{x_3=-2}$

Solutions: $(2,0)$, $(3,0)$, and $(-2,0)$

c) $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

$f(-1) = 0$; ∴ $x+1$ is a factor

-1	1	-1	-11	9	18	
	↓	-1	2	9	-18	+
x	1	-2	-9	18	0	
	x^3	x^2	x	#	R	

$(x+1)(x^3 - 2x^2 - 9x + 18) = 0$

$(x+1)[x^2(x-2) - 9(x-2)] = 0$

$(x+1)(x-2)(x^2-9) = 0$

$(x+1)(x-2)(x-3)(x+3) = 0$

$\boxed{x_1 = -1 \quad x_2 = 2 \quad x_3 = 3 \quad x_4 = -3}$

Solutions:
$(-1,0), (2,0), (3,0)$, and $(-3,0)$

d) $x^3 - 64 = 0$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(x-4)(x^2 + 4x + 16) = 0$

↓

$\boxed{x = 4}$

↓ check $b^2 - 4ac = (4)^2 - 4(1)(16)$
$= -48$
No solutions

Solution: $(4,0)$

e) $2x^3 - 7x^2 + 10x - 5 = 0$

Possible zeros: $\pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}$

$f(1) = 0$; ∴ $x-1$ is a factor

1	2	-7	10	-5	
	↓	2	-5	5	+
x	2	-5	5	0	
	x^2	x	#	R	

$(x-1)(2x^2 - 5x + 5) = 0$

↓

$x - 1 = 0$

$\boxed{x = 1}$

↓ check $b^2 - 4ac = (-5)^2 - 4(2)(5)$
$= -15$
∴ No solutions

Solution: $(1,0)$

3) Solve each equation by first factoring the sum or difference of cubes.

a) $x^3 - 8 = 0$

$(x-2)(x^2+2x+4)=0$

$x-2=0$

$\boxed{x=2}$

Check $b^2-4ac=(2)^2-4(1)(4)$

$=-12$

$\therefore$ No solution

$\boxed{\text{Solution: } (2,0)}$

b) $x^3 + 27 = 0$

$(x+3)(x^2-3x+9)=0$

$x+3=0$

$\boxed{x=-3}$

Check $b^2-4ac=(-3)^2-4(1)(9)$

$=-27$

$\therefore$ No solution

$\boxed{\text{Solution: } (-3,0)}$

4) Solve by factoring

$x^3 - 4x^2 - 7x + 10 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 5, \pm 10$

$f(1)=0$; $\therefore$ $x-1$ is a factor

1	1	-4	-7	10
	↓	1	-3	-10 +
x	1	-3	-10	0
	x^2	x	#	R

$(x-1)(x^2-3x-10)=0$

$(x-1)(x-5)(x+2)=0$

$x-1=0$ $\quad$ $x-5=0$ $\quad$ $x+2=0$

$\boxed{x_1=1}$ $\quad$ $\boxed{x_2=5}$ $\quad$ $\boxed{x_3=-2}$

$\boxed{\text{Solutions: } (1,0), (5,0), \text{ and } (-2,0)}$

b) $2x^3 - 11x^2 + 12x + 9 = 0$

Possible zeros: $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

$f(3)=0$; $\therefore$ $x-3$ is a factor

3	2	-11	12	9
	↓	6	-15	-9 +
x	2	-5	-3	0
	x^2	x	#	R

P: -6 ; S: -5 ; $\frac{-6}{2}$, $\frac{-3}{1}$, $\frac{1}{2}$

$(x-3)(2x^2-5x-3)=0$

$(x-3)(x-3)(2x+1)=0$

$(x-3)^2(2x+1)=0$

$x-3=0$ $\quad$ $2x+1=0$

$\boxed{x_1=3}$ $\quad$ $\boxed{x_2=-\frac{1}{2}}$

$\boxed{\text{Solutions: } (3,0) \text{ and } (-\tfrac{1}{2},0)}$

c) $x^4 - x^3 - 2x - 4 = 0$

Possible zeros: $\pm 1, \pm 2, \pm 4$

$f(-1) = 0$; $x+1$ is a factor

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & \downarrow & -1 & 2 & -2 & 4 \\ \hline \times & 1 & -2 & 2 & -4 & 0 \\ & x^3 & x^2 & x & \# & R \end{array} \quad +$$

$(x+1)(x^3 - 2x^2 + 2x - 4) = 0$

$(x+1)\left[x^2(x-2) + 2(x-2)\right] = 0$

$(x+1)(x-2)(x^2+2) = 0$

$x+1=0$ $\quad$ $x-2=0$ $\quad$ $x^2+2=0$

$\boxed{x_1 = -1}$ $\quad$ $\boxed{x_2 = 2}$ $\quad$ No solution

$\boxed{\text{Solutions: } (-1, 0) \text{ and } (2, 0)}$

ANSWER KEY

1a) $\left(-\frac{2}{3}, 0\right), (-9,0), (2,0)$ **b)** (4,0)

2a) (-2,0) and (3,0) **b)** (3,0), (-2,0), (2,0) **c)** (-1,0), (2,0), (-3,0), (3,0) **d)** (4,0) **e)** (1,0)

3)a) (2, 0) **b)** (-3, 0)

4)a) (5, 0), (-2, 0), (1, 0) **b)** (-0.5, 0) and (3, 0) **c)** (-1, 0) and (2, 0)