

W6 - 2.5 - Solving Inequalities

MHF4U

Jensen

→ Solve each linear inequality

a)  $x + 3 \leq 5$

$x \leq 2$

b)  $7x < 4 + 3x$

$4x < 4$

$x < 1$

2) Solve each inequality by graphing

a)  $(x + 3)(x - 2) > 0$

Factor Table

	$-\infty$	$-3$	$2$	$\infty$
		$-4$	$0$	$3$
$x+3$		-	+	+
$x-2$		-	-	+
overall		(+)	-	(+)

b)  $(x + 2)(3 - x)(x + 1) < 0$

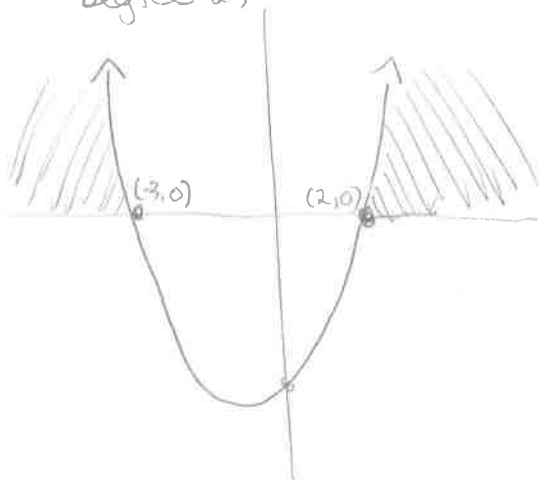
Factor Table:

	$-\infty$	$-2$	$-1$	$3$	$\infty$
		$-3$	$-1.5$	$0$	$4$
$x+2$		-	+	+	+
$3-x$		+	+	+	-
$x+1$		-	-	+	+
overall		+	(-)	+	(-)

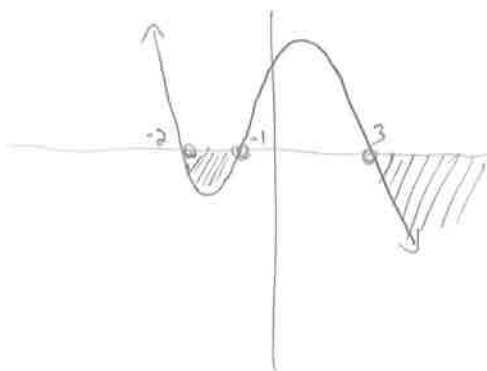
Solution:  $x < -3$  or  $x > 2$   
 $x \in (-\infty, -3) \cup (2, \infty)$

Solution:  $-2 < x < -1$  or  $x > 3$   
 $x \in (-2, -1) \cup (3, \infty)$

Graph: Degree 2; Positive L.C.



Graph: Degree 3; Negative L.C.



3) Solve each of the following polynomial inequalities

a)  $x^2 - 7x + 10 \geq 0$

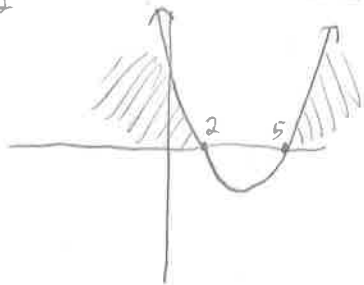
$(x-2)(x-5) \geq 0$

	$-\infty$	0	2	3	5	6	$\infty$
$x-2$		-	+	+			
$x-5$		-	-	-	+		
overall		(+)	-	(+)			

Solution:  $x \leq 2$  or  $x \geq 5$

$x \in (-\infty, 2] \cup [5, \infty)$

Degree 2  
+ L.C.



b)  $x^3 + 6x^2 - 16x > 0$

$x(x^2 + 6x - 16) > 0$

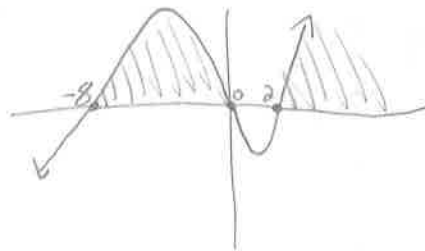
$x(x+8)(x-2) > 0$

	$-\infty$	-8	-1	0	1	2	3	$\infty$
$x$		-	-	+	+	+		
$x+8$		-	+	+	+			
$x-2$		-	-	-	-	+		
overall		-	(+)	-	-	(+)		

Solution:  $-8 < x < 0$  or  $x > 2$

$x \in (-8, 0) \cup (2, \infty)$

Degree 3  
+ L.C.



c)  $-x^2 + 36 \geq 0$

$-1(x^2 - 36) \geq 0$

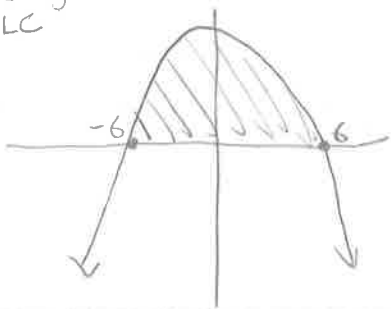
$-1(x-6)(x+6) \geq 0$

	$-\infty$	-7	-6	0	6	7	$\infty$
-1		-	-	-	-	-	
$x-6$		-	-	-	+		
$x+6$		-	-	+	+		
overall		-	(+)	-	-		

Solution:  $-6 \leq x \leq 6$

$x \in [-6, 6]$

Even deg.  
- L.C.



d)  $x^4 - 26x^2 + 25 > 0$

$(x^2 - 25)(x^2 - 1) > 0$

$(x-5)(x+5)(x-1)(x+1) > 0$

	$-\infty$	-5	-2	0	1	2	5	6	$\infty$
$x-5$		-	-	-	-	-	+		
$x+5$		-	+	+	+	+	+		
$x-1$		-	-	-	+	+	+		
$x+1$		-	-	+	+	+	+		
overall		(+)	-	(+)	-	-	(+)		

Solution:  $x < -5$  or  $-1 < x < 1$  or  $x > 5$

$x \in (-\infty, -5) \cup (-1, 1) \cup (5, \infty)$

e)  $x^3 - 3x^2 \geq 25x - 75$

$x^3 - 3x^2 - 25x + 75 \geq 0$

$[x^2(x-3) - 25(x-3)] \geq 0$

$(x-3)(x^2-25) \geq 0$

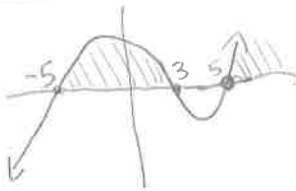
$(x-3)(x-5)(x+5) \geq 0$

	$-\infty$	$-5$	$0$	$3$	$5$	$6$	$\infty$
$x-3$	-	-	-	+	+		
$x-5$	-	-	-	-	+		
$x+5$	-	+	+	+	+		
overall	-	(+)	-	(+)			

Solution:  $-5 \leq x \leq 3$  or  $x \geq 5$

$x \in [-5, 3] \cup [5, \infty)$

odd deg.  
+ LC



$f(-2)=0$ ;  $x+2$  is a factor

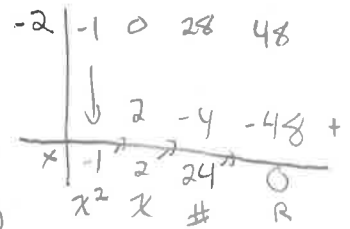
f)  $-x^3 + 28x + 48 \geq 0$

$(x+2)(-x^2+2x+24) \geq 0$

$-1(x+2)(x^2-2x-24) \geq 0$

$-1(x+2)(x-6)(x+4) \geq 0$

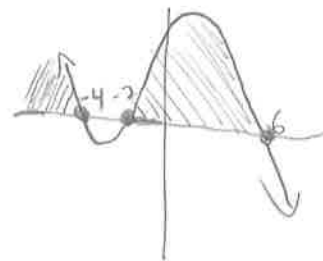
	$-\infty$	$-4$	$-2$	$6$	$7$	$\infty$
$x+2$	-	-	+	+		
$x-6$	-	-	-	+		
$x+4$	-	+	+	+		
overall	(+)	-	(+)	-		



Solution:  $x \leq -4$  or  $-2 \leq x \leq 6$

$x \in (-\infty, -4] \cup [-2, 6]$

odd deg.  
- LC



g)  $x^3 - 2x^2 - 5x + 6 < 0$

$f(1)=0$ ;  $x-1$  is a factor

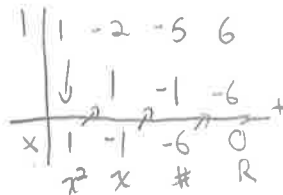
h)  $5x^3 - 12x^2 - 11x + 6 \leq 0$

$f(-1)=0$ ;  $x+1$  is a factor

$(x-1)(x^2-x-6) < 0$

$(x-1)(x-3)(x+2) < 0$

	$-\infty$	$-2$	$1$	$3$	$6$	$\infty$
$x-1$	-	-	+	+		
$x-3$	-	-	-	+		
$x+2$	-	+	+	+		
overall	(-)	+	(-)	+		

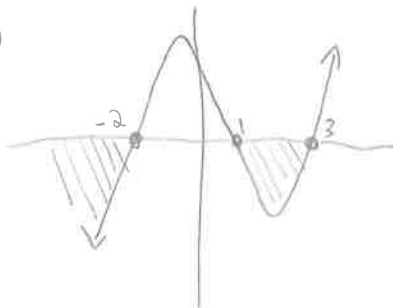


	$-\infty$	$-2$	$1$	$3$	$\infty$
$x-1$	-	-	+	+	
$x-3$	-	-	-	+	
$x+2$	-	+	+	+	
overall	(-)	+	(-)	+	

Solution:  $x < -2$  or  $1 < x < 3$

$x \in (-\infty, -2) \cup (1, 3)$

odd deg.  
+ LC



$(x+1)(5x^2-17x+6) \leq 0$

$(x+1)(5x-2)(x-3) \leq 0$

	$-\infty$	$-1$	$2/5$	$3$	$\infty$
$x+1$	-	+	+	+	
$5x-2$	-	-	+	+	
$x-3$	-	-	-	+	
overall	(-)	+	(-)	+	

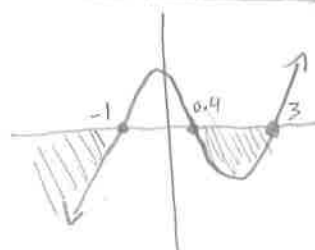
	$-\infty$	$-1$	$2/5$	$3$	$\infty$
$x+1$	-	+	+	+	
$5x-2$	-	-	+	+	
$x-3$	-	-	-	+	
overall	(-)	+	(-)	+	



Solution:  $x \leq -1$  or  $2/5 \leq x \leq 3$

$x \in (-\infty, -1] \cup [2/5, 3]$

odd deg.  
+ LC

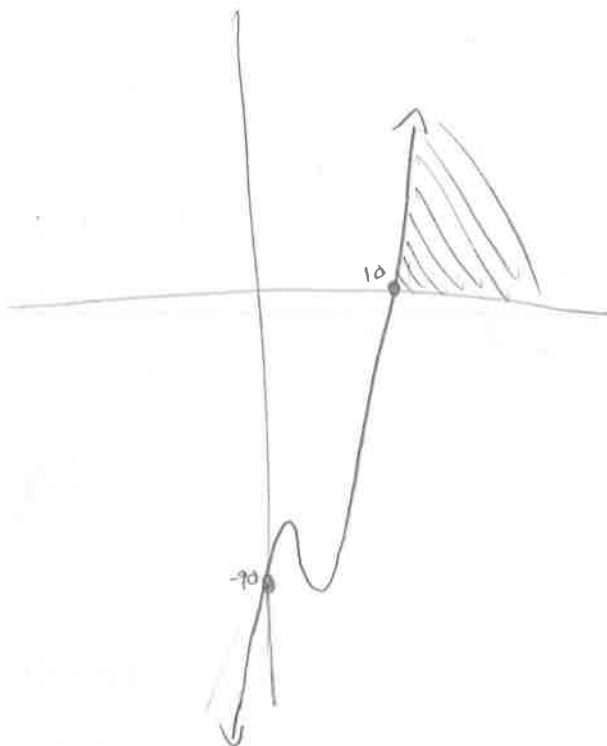


4) The price,  $p$ , in dollars, of a stock  $t$  years after 1999 can be modelled by the function  $p(t) = 0.5t^3 - 5.5t^2 + 14t$ . When will the stock be more than \$90? You may use technology to help you determine the solution.

$$0.5t^3 - 5.5t^2 + 14t > 90$$

$$0.5t^3 - 5.5t^2 + 14t - 90 > 0$$

using Desmos:



Solution:  $t > 10$

$$t \in (10, \infty)$$

∴ The stock will be more than \$90 after 10 years.

#### ANSWER KEY

1) a)  $x \leq 2$  b)  $x < 1$

2) a)  $x < -3$  or  $x > 2$  b)  $-2 < x < -1$  or  $x > 3$

3) a)  $x \leq 2$  or  $x \geq 5$  b)  $-8 < x < 0$  or  $x > 2$  c)  $-6 \leq x \leq 6$  d)  $x < -5$  or  $-1 < x < 1$  or  $x > 5$

e)  $-5 \leq x \leq 3$  or  $x \geq 5$  f)  $x \leq -4$  or  $-2 \leq x \leq 6$  g)  $x < -2$  or  $1 < x < 3$

h)  $x \leq -1$  or  $\frac{2}{5} < x < 3$

4) after 10 years (2009)