In this section, you will learn the meaning of a polynomial inequality and examine methods for solving polynomial inequalities.

Part 1: Intro to Inequalities

Task: Read the following on your own

Examine the graph of $y = x^2 + 4x - 12$.

The x-intercepts are 6 and -2. These correspond to the zeros of the function $y = x^2 + 4x - 12$. Note that the factored form version of the function is $y = (x + 6)(x - 2)$. By moving from left to right along the x-axis, we can make the following observations:

- The function is positive when $x < -6$ since the y-values are positive
- The function is negative when $-6 < x < 2$ since the y-values are negative
- The function is positive when $x > 2$ since the y-values are positive.
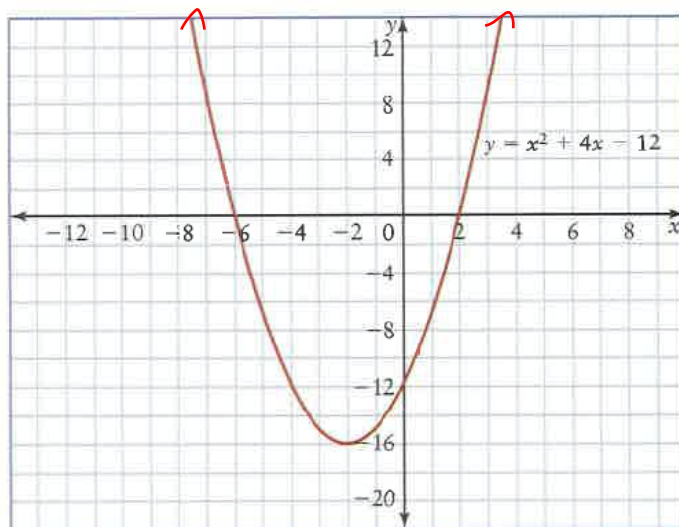
The zeros -6 and 2 divide the x-axis into three intervals. In each interval, the function is either positive or negative. The information can be summarized in a table:

| Interval	$x < -6$	$-6 < x < 2$	$x > 2$
Sign of Function	$+$	$-$	$+$
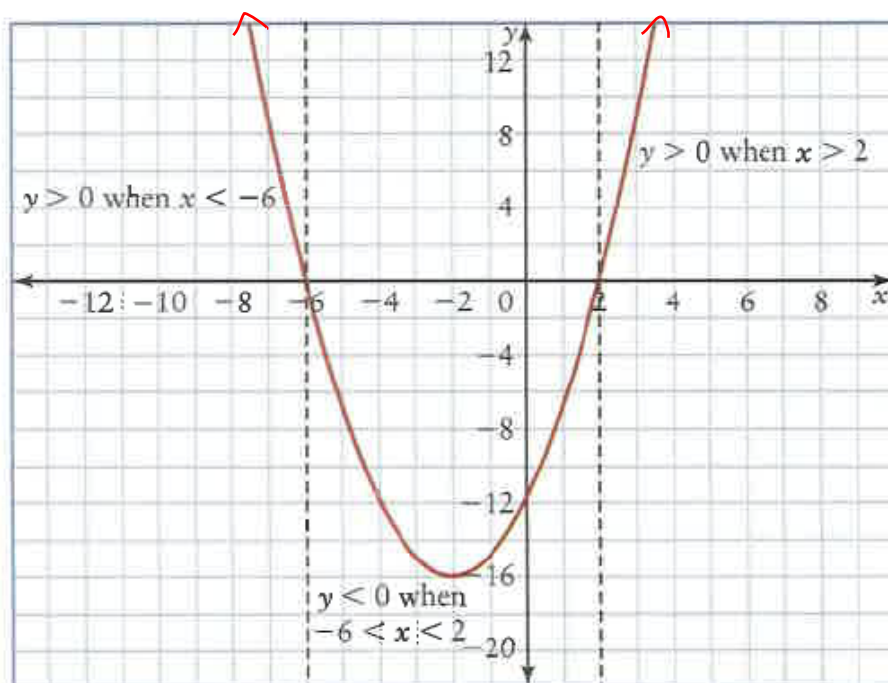
Polynomial Inequalities

A polynomial inequality results when the equal sign in a polynomial equation is replaced with an inequality symbol.

The real zeros of a polynomial function, or x-intercepts of the corresponding graph, divide the x-axis into intervals that can be used to solve a polynomial inequality.

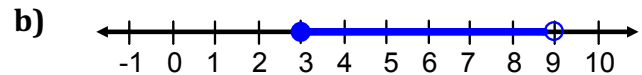## Part 1: Inequalities and Number Lines

Example 1: Write an inequality that corresponds to the values of x shown on each number line

a)

$$x \leq 4$$

OR

$$(-\infty, 4]$$

b)
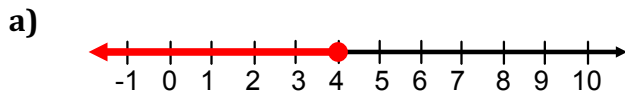
$$3 \leq x < 9$$

OR

$$[3, 9)$$

Part 2: Solve an Inequality given the Graph

Example 2: Use the graph of the function $f(x)$ to answer the following inequalities…

$$f(x) = 0.1(x - 1)(x + 3)(x - 4)$$
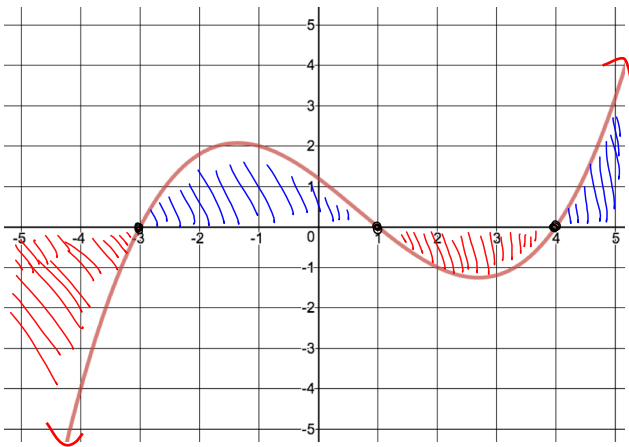

a) $f(x) < 0$

$f(x) < 0$ when: $x < -3$ or $1 < x < 4$

$$(-\infty, -3) \cup (1, 4)$$

b) $f(x) \geq 0$

$f(x) \geq 0$ when: $-3 \leq x \leq 1$ or $x \geq 4$

$$[-3, 1] \cup [4, \infty)$$

Part 2: Solve Linear Inequalities

Note: Solving linear <u>inequalities</u> is the same as solving linear <u>equations</u>. However, when both sides of an inequality are multiplied or divided by a <u>negative</u> number, the inequality sign must be <u>reversed</u>.

Example 3: Solve each inequality

a) $x - 8 \geq 3$

$$x - 8 \geq 3$$
$$x \geq 3 + 8$$
$$x \geq 11$$

b) $-4 - 2x < 12$

$$-4 - 2x < 12$$
$$\frac{-2x}{-2} < \frac{16}{-2}$$

Reverse inequality when dividing by a negative

$$x > -8$$

Part 2: Solve Inequalities of Degree 2 and Higher

> **Steps for solving polynomial inequalities algebraically:**
>
> 1) Use inverse operations to move all terms to one side of the inequality
> 2) Factor the polynomial to determine the zeros of the corresponding equation
> 3) Find the interval(s) where the function is positive or negative by either:
>   a. Graphing the function using the zeros, leading coefficient, and degree
>   b. Make a factor table and test values in each interval

Example 4: Solve each polynomial inequality algebraically

a) $2x^2 + 3x - 9 > 0$ ⟵ when is it above the x-axis?

Method 1: Graph the inequality

$$2x^2 + 3x - 9 > 0 \qquad \begin{array}{l} P: -18 \\ S: 3 \end{array} \quad \boxed{6 \text{ and } -3}$$

$$(2x^2 + 6x) + (-3x - 9) > 0$$
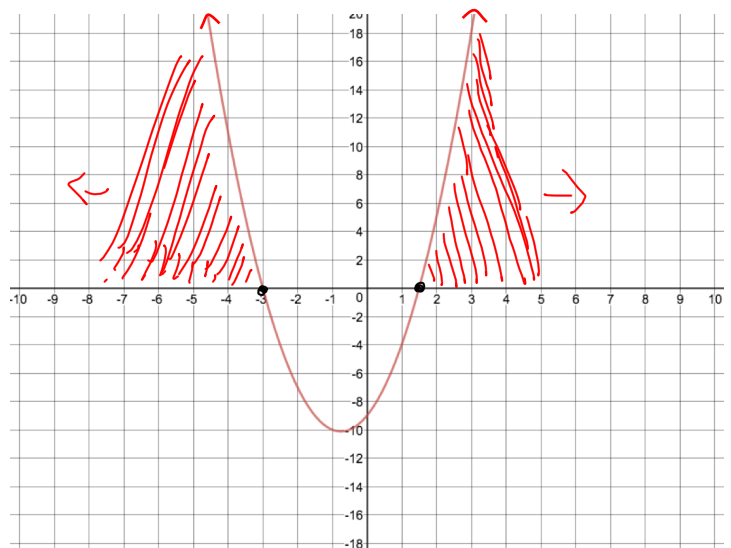
$$2x(x + 3) - 3(x + 3) > 0$$

$$(x + 3)(2x - 3) > 0$$

x-int at -3 and 1.5

$2x^2 + 3x - 9 > 0$ when... $x < -3$ or $x > 1.5$

$$(-\infty, -3) \cup (1.5, \infty)$$

Method 2: Factor Table (sign chart)

To make a factor table:

- Use x-intercepts and vertical asymptotes to divide in to intervals
- Use a test point within each interval to find the sign of each factor
- Determine the overall sign of the product by multiplying signs of each factor within each interval.

	$-\infty$	-3	1.5	∞
		-4	0	2
$x+3$		$-$	$+$	$+$
$2x-3$		$-$	$-$	$+$
overall sign		$+$ *	$-$	$+$ *

$\therefore\ 2x^2+3x-9>0$ when $x<-3$ or $x>1.5$

$$(-\infty,-3)\cup(1.5,\infty)$$

b) $-2x^3 - 6x^2 + 12x \leq -16$

Method 1: Graph the inequality

$$-2x^3 - 6x^2 + 12x + 16 \leq 0$$

$$\frac{-2(x^3 + 3x^2 - 6x - 8)}{-2} \leq \frac{0}{-2}$$

common factor

reverse inequality

$$x^3 + 3x^2 - 6x - 8 \geq 0$$

factors of -8 are: $\pm 1, \pm 2,$ and ± 3

$f(-1) = 0$, so $x+1$ is a factor.

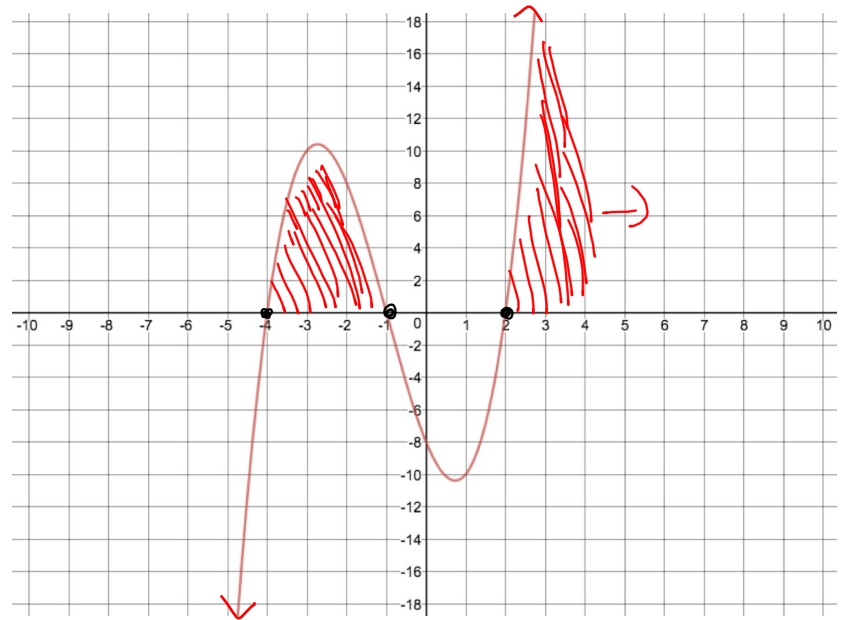| -1	1	3	-6	-8
	↓	-1	-2	8
⊗	1	2	-8	0
	x^2	x	#	R

$(x+1)(x^2 + 2x - 8) \geq 0$

$(x+1)(x+4)(x-2) \geq 0$

x-int at $-1, -4,$ and 2

Positive leading coefficient

Degree 3

Solution: $-4 \leq x \leq -1$ or $x \geq 2$

$[-4, -1] \cup [2, \infty)$

Method 2: Factor Table (sign chart)

	-5	-2	0	3
$x+1$	⊖	⊖	⊕	⊕
$x+4$	⊖	⊕	⊕	⊕
$x-2$	⊖	⊖	⊖	⊕
overall sign	⊖	⊕	⊖	⊕
		*		*

Solution: $[-4, -1] \cup [2, \infty)$

c) $x^3 + 4x^2 + 6x < -24$

$$x^3 + 4x^2 + 6x + 24 < 0$$

$$x^2(x+4) + 6(x+4) < 0$$ factor by grouping

$$(x+4)(x^2+6) < 0$$

x-int at -4   no real solutions

	$-\infty$ -5	-4 0 ∞
$x+4$	$\ominus$	$\oplus$
x^2+6	$\oplus$	$\oplus$
sign	$\ominus$	$\oplus$
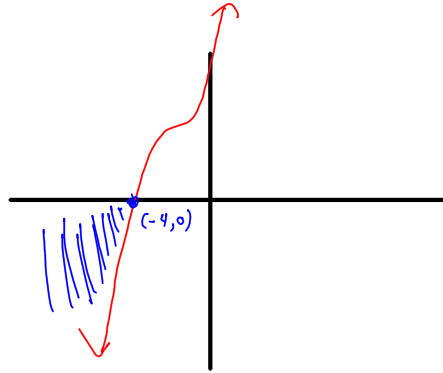
Solution: $x < -4$ OR $(-\infty, -4)$

Part 2: Applications of Inequalities

3) The price, p, in dollars, of a stock t years after 1999 can be modeled by the function $p(t) = 0.5t^3 - 5.5t^2 + 14t$. When will the price of the stock be more than $90?

$$0.5t^3 - 5.5t^2 + 14t > 90$$

$$0.5t^3 - 5.5t^2 + 14t - 90 > 0$$
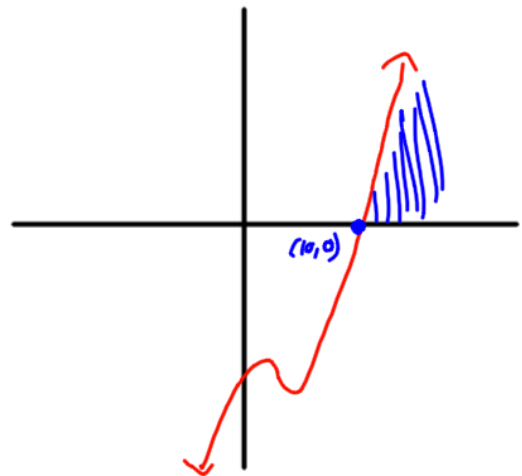
$$0.5(t^3 - 11t^2 + 28t - 180) > 0$$

$$t^3 - 11t^2 + 28t - 180 > 0$$

$f(10) = 0$; so $x-10$ is a factor

```
10 | 1   -11    28   -180
   |      10   -10    180
   -----------------------
     1    -1    18     0
     t²    t    #      R
```

$$(t-10)(t^2-t+18) > 0$$

x-int at $(10,0)$

$b^2 - 4ac = (-1)^2 - 4(1)(18)$
$= -71$

∴ no solutions

solution: $t > 10$

& the price of the stock will be above $90 after year 2009