

3.3b - Exponent Laws

Part 1: Do It Now

Simplify and evaluate each of the following expressions:

$$1) (3^2)(3^5) = 3^{2+5} = 3^7 = 2187$$

$$2) \frac{y^{10}}{y} = y^{10-1} = y^9$$

$$3) (y^3)^4 = y^{3 \times 4} = y^{12}$$

$$4) 3xy - 2xy = 1xy$$

Note: #4 is a subtraction question. Simplifying this is called 'collecting like terms'.

Complete the following table:

Product of Powers Rule	$x^a \cdot x^b = x^{a+b}$
Quotient of Powers Rule	$x^a \div x^b = x^{a-b}$
Power of a Power Rule	$(x^a)^b = x^{a \cdot b}$
Power of a Quotient	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Power of a Product	$(ab)^x = a^x \cdot b^x$
Zero Exponent Rule	$x^0 = 1$
Negative Exponent Rule	$x^{-a} = \frac{1}{x^a}$

Part 2: Negative Exponents

Any non-zero number raised to a negative exponent is equal to its RECIPROCAL raised to the opposite positive power

$$1) x^{-3} \\ = \frac{1}{x^3}$$

$$2) 5^{-2} \\ = \frac{1}{5^2} \\ = \frac{1}{25}$$

$$3) \frac{x^3}{x^5} \\ = x^{3-5} \\ = x^{-2} \\ = \frac{1}{x^2}$$

You Try:

a) $x^5 \div x^9$

$$\begin{aligned} &= x^{5-9} \\ &= x^{-4} \\ &= \frac{1}{x^4} \end{aligned}$$

b) $\frac{6x^3}{3x^7}$

$$\begin{aligned} &= \frac{2x^3}{1x^7} \\ &= 2x^{3-7} \\ &= 2x^{-4} \\ &= \frac{2}{x^4} \end{aligned}$$

Part 3: Simplify Expressions Using Exponent Laws

4) $5x^2 \cdot 2x^7$

$$\begin{aligned} &= (5)(2)(x^2)(x^7) \\ &= 10x^{2+7} \\ &= 10x^9 \end{aligned}$$

5) $2a^2b^3 \cdot 3a^6b^4$

$$\begin{aligned} &= (2)(3)(a^2)(a^6)(b^3)(b^4) \\ &= 6a^{2+6}b^{3+4} \\ &= 6a^8b^7 \end{aligned}$$

Hint: start by multiplying coefficients together. Then look for powers with the same base and simplify by writing them as a single power by following the proper exponent laws.

6) $(5x^3)^2$

$$\begin{aligned} &= (5)^2(x^3)^2 \\ &= 25x^{3 \times 2} \\ &= 25x^6 \end{aligned}$$

7) $(x^4y^3)^2$

$$\begin{aligned} &= (x^4)^2(y^3)^2 \\ &= (x^{4 \times 2})(y^{3 \times 2}) \\ &= x^8y^6 \end{aligned}$$

Hint: Use the power of a product rule. The exponent outside of the brackets must be applied to all coefficients and variables inside the brackets using the proper exponent laws.

$$8) \frac{12k^2m^8}{4k^5m^5}$$

$$= \frac{3k^2m^8}{1k^5m^5}$$

$$= \frac{3k^{2-5}m^{8-5}}{1}$$

$$= \frac{3k^{-3}m^3}{1}$$

$$= \frac{3m^3}{k^3}$$

$$9) \frac{-2uv^3 \cdot 8u^2v^2}{(4uv^2)^2}$$

$$= \frac{-16u^{1+2}v^{3+2}}{(4)^2(u)^2(v^2)^2}$$

$$= \frac{-16u^3v^5}{16u^2v^4}$$

$$= -1u^{3-2}v^{5-4}$$

$$= -1uv$$

Hint: start by simplifying the numerator and denominator separately as much as possible using exponent laws. Then reduce the coefficients if possible and use the quotient rule to simplify powers with the same base.

$$10) \frac{(3m^2n)^2}{(2mn)(3m^2n)}$$

$$= \frac{(3)^2(m^2)^2(n)^2}{(2)(3)(m)(m^2)(n)(n)}$$

$$= \frac{3 \cancel{9} m^4 n^2}{2 \cancel{6} m^3 n^2}$$

$$= \frac{3m^{4-3}n^{2-2}}{2}$$

$$= \frac{3m^1n^0}{2}$$

$$= \frac{3m}{2}$$

You try:

$$\begin{aligned} \text{a) } & \frac{5c^3d \cdot 4c^2d^2}{(2c^2d)^2} \\ & = \frac{5(4)(c^3)(c^2)(d)(d^2)}{(2)^2(c^2)^2(d)^2} \\ & = \frac{5\cancel{2}0c^5d^3}{1\cancel{4}c^4d^2} \\ & = 5c^{5-4}d^{3-2} \\ & = 5cd \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{(3xy)^3}{9x^4y^4} \\ & = \frac{(3)^3(x)^3(y)^3}{9x^4y^4} \\ & = \frac{\cancel{3}27x^3y^3}{1\cancel{9}x^4y^4} \\ & = \frac{3x^{3-4}y^{3-4}}{1} \\ & = \frac{3x^{-1}y^{-1}}{1} \\ & = \frac{3}{x^1y^1} \\ & = \frac{3}{xy} \end{aligned}$$