Part 1: Normal Distribution Video

While watching the video, answer the following questions

1. What is another name for the Empirical rule?

2. How tall must a woman be to join the Boston Beanstalks Club?

3. How do you calculate a z-score?

4. Based on z-scores, are eligibility requirements to join the Boston Beanstalks more difficult to meet for men or for women?

Part 2: The Standard Normal Distribution

All normal distributions are the same when we measure how many _____ an observation x lies away from the _____, which we calculate as follows:

The standardized value for x, calculated by the formula above, is called its _____. Observations from different normal distributions are best compared by comparing their standardized values, or z-scores. The z-score states how many _____the original observation falls away from the _____ and in which direction. Observations larger than the mean have _____ z-scores, while observations smaller than the mean have _____ z-scores. Converting to standardized values allows us to find proportions that we can't get from the Empirical Rule.

Example 1: Suppose we want to know the percentage of data from a normal distribution with mean $\mu = 8$ and standard deviation $\sigma = 2$ that falls below $x = 9$. We can use the Empirical Rule to learn that the percentage is between 50% and 84%, but that is not a very accurate estimate. Instead, we convert $x = 9$ into a z-score:
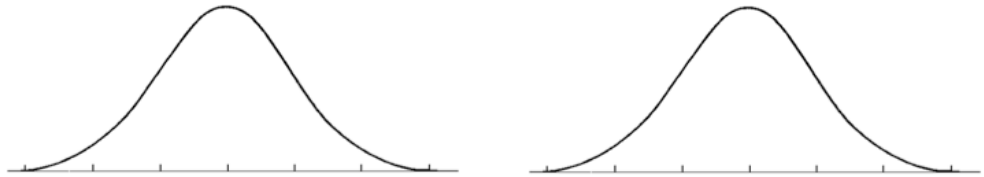
The standard normal graph shows how the value $x = 9$ is _____ standard deviations _____ the mean. This value is in the same position on this graph as it was for the normal distribution graph.

Now, we find the proportion of standard normal data that falls below 0.5. To find this proportion, we use a standard normal table, which gives the proportion of data that falls _____ any value for z. Using the portion of a z-table, we look down the z-column to locate 0.5 and then move to the right under the .00 column. Our answer is _____, or around _____.

	0.00	0.01	0.02	0.03	0.04
0.0	0.5000	0.5040	0.5080	0.5120	0.5160
0.1	0.5398	0.5438	0.5478	0.5517	0.5557
0.2	0.5793	0.5832	0.5871	0.5910	0.5948
0.3	0.6179	0.6217	0.6255	0.6293	0.6331
0.4	0.6554	0.6591	0.6628	0.6664	0.6700
0.5	0.6915	0.6950	0.6985	0.7019	0.7054
0.6	0.7257	0.7291	0.7324	0.7357	0.7389

Standardizing a data value by finding the z-score places it on the _____. This is a special normal distribution with a mean of ____ and a standard deviation of ____.
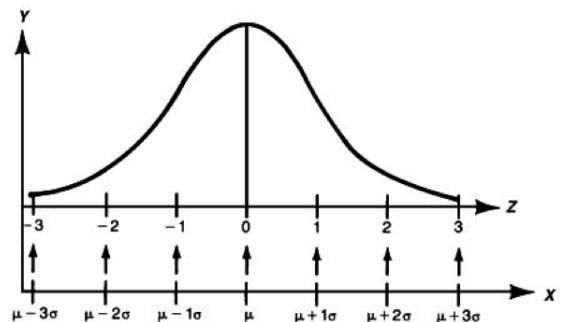
Part 3: Practice Finding and Interpreting Z-Scores

Example 2: Caley scored 84% in her Data Management course, while Lauren, who attends a different Data Management class, scored 83%. If Caley's class average is 74% with a standard deviation of 8, and Lauren's class average is 70% with a standard deviation of 9.8, use z-scores to determine who has the better mark.

Note: z-scores are used to standardize the data so that they can be accurately compared.

Part 4: Area to Left (Percentile)

The proportion (percentage) of data with a _____ z-score is equal to the _____ the value can be categorized by. The areas given on the z-score table are considered percentiles because the z-score table always gives areas to the left of the given z-score.

Percentile: the kth percentile is the least data value that is greater than k% of the population.

For example; If your test result is in the 85th percentile, that means that 85% of tests had a lower score than yours.

Example 3: Perch in a lake have a mean length of 20cm and a standard deviation of 5cm. Find the percent of the population that is less than 22 cm.

|	0.00	0.01	0.0
0.0	0.5000	0.5040	0.50
0.1	0.5398	0.5438	0.54
0.2	0.5793	0.5832	0.58
0.3	0.6179	0.6217	0.62
0.4	0.6554	0.6591	0.6
0.5	0.6915	0.

Example 4: Perch in a lake have a mean length of 20cm and a standard deviation of 5cm. Find the percent of the population that is greater than to 17 cm.

–0.8	0.2119	0.2090	0.2061	0.2033
–0.7	0.2420	0.2389	0.2358	0.2327
–0.6	0.2743	0.2709	0.2676	0.2643
–0.5	0.3085	0.3050	0.3015	0.2981
–0.4	0.3446	0.3409	0.3372	0.3336
–0.3	0.3821	0.3783	0.3745	0.3707
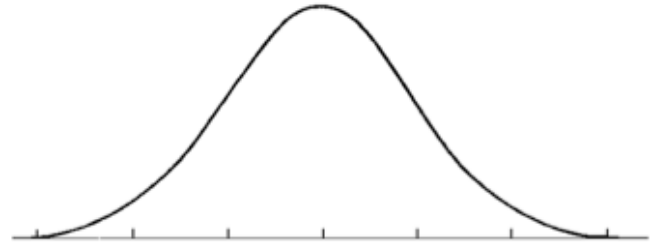
Part 6: Area Between Two Values

Example 5: Using the normal distribution X~N(7, 2.2^2), find the percent of data that is within the interval 3 < x < 6
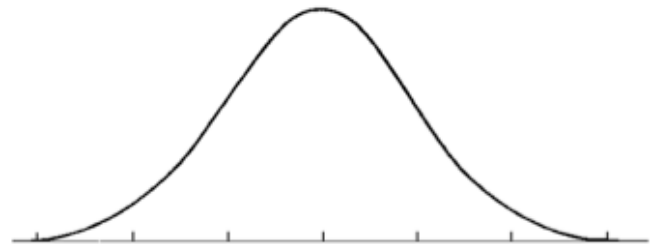
Part 7: Percentile to Scores: z in Reverse

Finding areas from z-scores is the simplest way to work with the normal distribution. But sometimes we start with areas and are asked to work backward to find the corresponding z-score or even the original data value.

Example 6: SAT test scores have a mean of 500 and a standard deviation of 100. Suppose a college says it only admits students who have scores in at least the 90th percentile. How high a score does it take to be eligible?

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

Using the Ti-84

Using ti-83 for a Normal Distribution

Access normalcdf function: 2nd → VARS (DISTR) → NORMALCDF
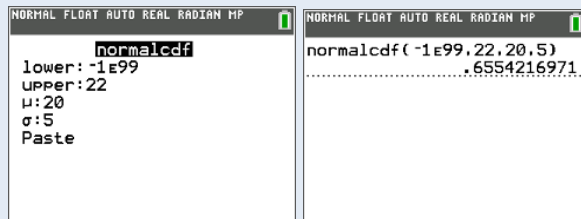
For a normal distribution:
normalcdf(left boundary, right boundary, mean, standard deviation)

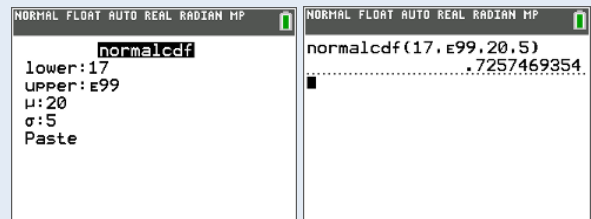**For a standard normal distribution:**
normalcdf(left boundary z-score, right boundary z-score)

Note: Use 1E99 for positive infinity and -1E99 for negative infinity

Example 3: $X \sim N(20, 5^2)$. Find percent less than 22.

```
NORMAL FLOAT AUTO REAL RADIAN MP
     normalcdf
 lower: -1E99
 upper:22
 µ:20
 σ:5
 Paste
```
```
NORMAL FLOAT AUTO REAL RADIAN MP
 normalcdf(-1E99,22,20,5)
                   .6554216971
```

Example 4: $X \sim N(20, 5^2)$. Find percent greater than 17.

```
NORMAL FLOAT AUTO REAL RADIAN MP
     normalcdf
 lower:17
 upper:E99
 µ:20
 σ:5
 Paste
```
```
NORMAL FLOAT AUTO REAL RADIAN MP
 normalcdf(17,E99,20,5)
                   .7257469354
```

Example 5: $X \sim N(7, 2.2^2)$. Find percent between 3 and 6.

```
NORMAL FLOAT AUTO REAL RADIAN MP
     normalcdf
 lower:3
 upper:6
 µ:7
 σ:2.2
 Paste
```
```
NORMAL FLOAT AUTO REAL RADIAN MP
 normalcdf(3,6,7,2.2)
                   .2902000428
```

Example 6: $X \sim N(500, 100^2)$. Score in 90th percentile.

```
NORMAL FLOAT AUTO REAL RADIAN MP
     invNorm
 area:0.9
 µ:500
 σ:100
 Paste
```
```
NORMAL FLOAT AUTO REAL RADIAN MP
 invNorm(0.9,500,100)
                 628.1551567
```