Section 3.5b Worksheet – Applying the Normal Distribution

MDM4U
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1) Copy and complete the chart below, assuming a normal distribution for each situation.

Mean, μ	Standard Deviation, σ	Probability
12	3	$P(X < 9) = normalcdf(lower = -E99, upper = 9, \mu = 12, \sigma = 3) = 0.1587$
30	5	$P(X < 25) = normalcdf(lower = -E99, upper = 25, \mu = 30, \sigma = 5) = 0.1587$
5	2.2	$P(X > 6) = normalcdf(lower = 6, upper = E99, \mu = 5, \sigma = 2.2) = 0.3247$
245	18	$P(233 < X < 242) = normalcdf(lower = 233, upper = 242, \mu = 245, \sigma = 18) = 0.1813$

2) There have been some outstanding hitters in baseball. In 1911, Ty Cobb's batting average was 0.420. In 1941, Ted Williams batted 0.406. George Brett's 0.390 average in 1980 was one of the highest since Ted Williams. Batting averages have historically been approximately normally distributed with means and standard deviations as shown below. Compute z-scores for each of these three hitters. Can you rank the three hitters? Explain.

Decade	Mean, μ	Standard Deviation, σ
1910's	0.266	0.0371
1940's	0.267	0.0326
1970s-1980s	0.261	0.0317

Ty Cobb: $z_{0.42} = \frac{x-\mu}{\sigma} = \frac{0.42-0.266}{0.0371} = 4.15$

Ted Williams: $z_{0.406} = \frac{x-\mu}{\sigma} = \frac{0.406-0.267}{0.0326} = 4.26$

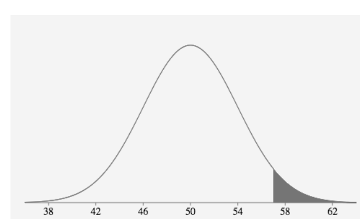
George Brett: $z_{0.39} = \frac{x-\mu}{\sigma} = \frac{0.39-0.261}{0.0317} = 4.07$

Based on z-scores, which tell us how many standard deviations a players batting average is above the mean, Ted Williams has the best average, then Ty Cobb, then George Brett.

3) The amount of annual rainfall in a certain region is known to be a normally distributed random variable with a mean of 50 inches and a standard deviation of 4 inches. If the rainfall exceeds 57 inches during the year, it leads to floods. Find the probability that during a randomly selected year there will be floods.

$P(rainfall > 57) = normalcdf(lower = 57, upper = E99, \mu = 50, \sigma = 4) = 0.0401$

There is about a 4.01% chance of floods happening in a randomly selected year.

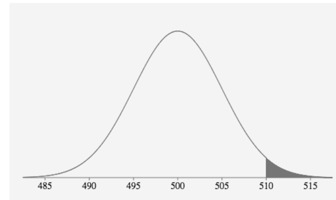


4) The weight of food packed in certain containers is a normally distributed random variable with a mean weight of 500 pounds and a standard deviation of 5 pounds. If a container is picked at random, find the probability that it contains:

a) more than 510 pounds

$P(pounds > 510) = normalcdf(lower = 510, upper = E99, \mu = 500, \sigma = 5) = 0.0228$

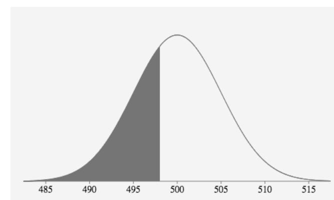
There is about a 2.285% chance a randomly picked container has more than 510 pounds of food.



b) less than 498 pounds

$P(pounds < 498) = normalcdf(lower = -E99, upper = 498, \mu = 500, \sigma = 5) = 0.3446$

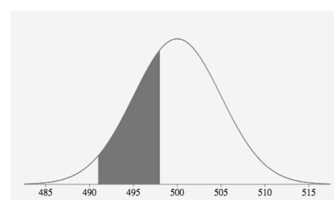
There is about a 34.46% chance a randomly picked container has less than 498 pounds of food.



c) between 491 and 498 pounds

$P(491 < pounds < 498) = normalcdf(lower = 491, upper = 498, \mu = 500, \sigma = 5) = 0.3086$

There is about a 30.86% percent chance a randomly picked container has between 491 and 498 pounds of food.

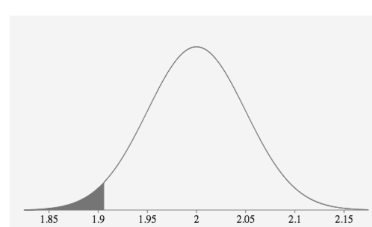


5) The diameter of a lead shot has a normal distribution with a mean diameter equal to 2 inches and a standard deviation equal to 0.05 inches. Find what diameter a circular hole should be so that only 3 percent of the lead shots can pass through it.

We are looking for the size of a lead shot in the 3rd percentile. A lead shot in the 3rd percentile is bigger than 3% of all lead shots. This means 3% of lead shots will be smaller and therefore be able to fit through a hole of that size.

3rd percentile = $invnorm(area = 0.03, \mu = 2, \sigma = 0.05) = 1.906$ inches

The hole should be about 1.906 inches in diameter so that only 3% of the lead shots can pass through it.

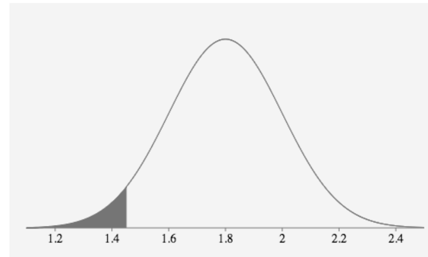


6) The nicotine content in a brand of king-size cigarettes has a normal distribution with a mean content of 1.8 mg and a standard deviation of 0.2 mg. Find the probability that the nicotine content of a randomly selected cigarette of this brand will be:

a) less than 1.45 mg

$P(nicotine < 1.45) = normalcdf(lower = -E99, upper = 1.45, \mu = 1.8, \sigma = 0.2) = 0.0401$

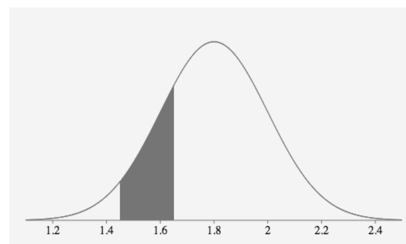
There is about a 4.01% chance a randomly selected cigarette has less than 1.45 mg of nicotine.



b) between 1.45 and 1.65 mg

$P(1.45 < nicotine < 1.65) = normalcdf(lower = 1.45, upper = 1.65, \mu = 1.8, \sigma = 0.2) = 0.1866$

There is about an 18.66% chance a randomly selected cigarette has between 1.45 and 1.65 mg of nicotine.

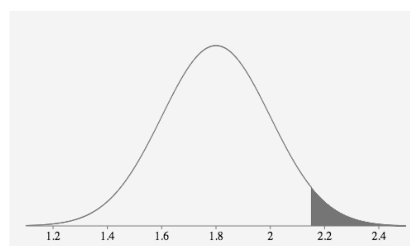


c) more than 2.15 mg.

$P(nicotine > 2.15) = normalcdf(lower = 2.15, upper = E99, \mu = 1.8, \sigma = 0.2) = 0.0401$

There is about a 4.01% chance a randomly selected cigarette has more than 2.15 mg of nicotine.

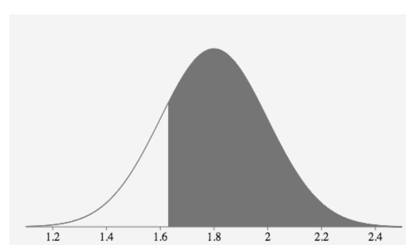
Note: We could have used the property that normal distributions are symmetrical and the answer from part a) to determine this probability.



d) What value is needed so that 80 percent of the cigarettes will exceed it in their nicotine content?

20^{th} percentile of nicotine content = $invnorm(area = 0.2, \mu = 1.8, \sigma = 0.2) = 1.63$

80% of cigarettes have more than about 1.63 mg of nicotine.

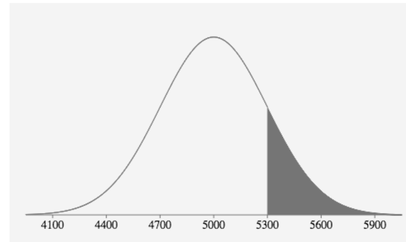


7) The demand for meat at a grocery store during any week is approximately normally distributed with a mean demand of 5000 pounds and a standard deviation of 300 pounds.

a) If the store has 5300 pounds of meat, what is the probability that they will run out during a random week?

$P(meat > 5300) = normalcdf(lower = 5300, upper = E99, \mu = 5000, \sigma = 300) = 0.1587$

There is about a 15.87% chance the grocery store will run out of meat.



b) How much meat should the store have in stock per week so as to only run short 10 percent of the time?

90^{th} percentile for amount of meat sold = $invnorm(area = 0.9, \mu = 5000, \sigma = 300) = 5384.5$

They should stock about 5384.5 pounds of meat in order to only run short 10% of the time.