Section 3.6 – Confidence Intervals

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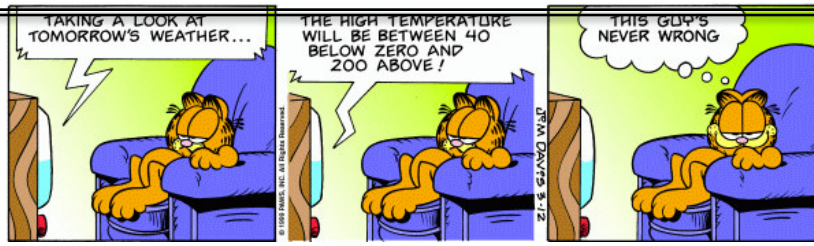


Part 1: Intro to Confidence Intervals

A confidence interval uses sample data to estimate an unknown population parameter with an indication of how precise the estimate is and of how confident we are that the result is correct.

When we take a sample, we do our best to try and obtain values that accurately represent the true values for the population.

For example, if we took a simple random sample of 500 from a population of a town with 10,000 people and found that in the upcoming election, 285 plan to got for Candidate Y, then our best guess for the proportion of the town that will vote for Candidate Y is 285/500 or 57%.

But we don't know if the TRUE proportion of people who will vote for Candidate Y is 57%. For this reason, we report a margin of error. From this information, we can create a confidence interval.

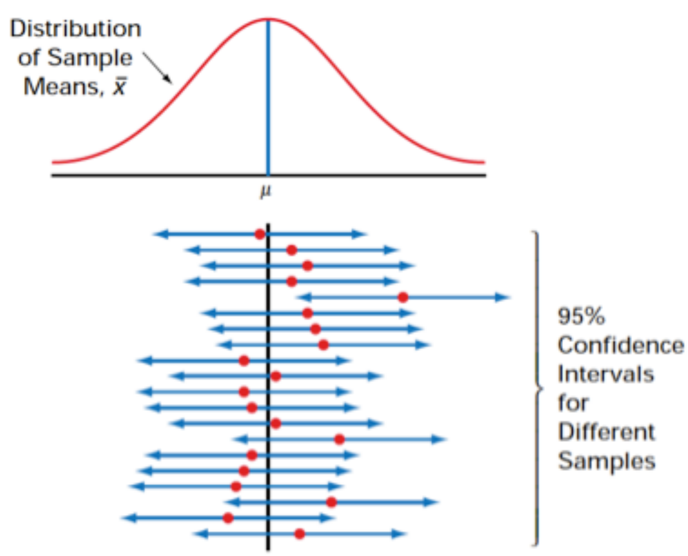
The confidence interval will tell us with a certain percentage of confidence (usually 95% is used) that the population mean will fall between some lower limit and some upper limit.

To interpret a confidence interval we say:

"We are C% confident that the interval from ____ to ____ captures the [parameter in context]."

A computed confidence interval shows the proportion of confidence intervals of size *n* that actually contain the population mean.

In this example, the 95% confidence intervals for various samples are shown. Only 1 of 20 does not contain the population mean, meaning that there is a 95% chance the population mean is within the confidence interval given by different sample means.



Part 2: Confidence Intervals for Population Means

Formula:

$$C.I. = sample\ mean \pm margin\ of\ error$$

$$C.I. = \bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$$

$\bar{x}$ = sample mean

σ = population standard deviation

n = sample size

z^* = critical value for confidence level

This table gives a list of common confidence levels and their associated critical values:

Confidence Level	Tail Size	z^*
90%	0.05	1.645
95%	0.025	1.960
99%	0.005	2.576

Example 1: A paint manufacturer knows from experience that drying times for latex paints have a standard deviations of 10.5 minutes. The manufacturer wants to use the slogan: "Dries in *T* minutes" on its advertising. Twenty test areas of equal size are painted and the mean drying time is found to be 75.4 minutes. Find a 95% confidence interval for the actual mean drying time of the paint.

$C.I. = \bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$

$= 75.4 \pm 1.96 \left(\frac{10.5}{\sqrt{20}}\right)$

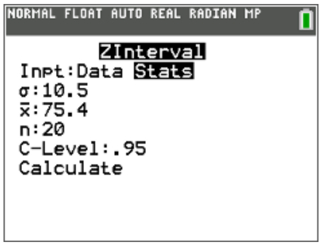
$= 75.4 \pm 4.60183$

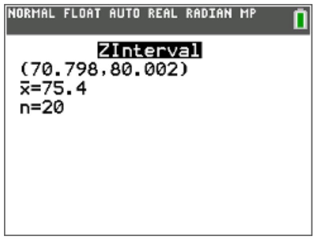
$= (70.8, 80.0)$

We are 95% confident that the interval from 70.8 to 80.0 minutes captures the true mean drying time of the paint.

Using the Ti-84 for example 1:

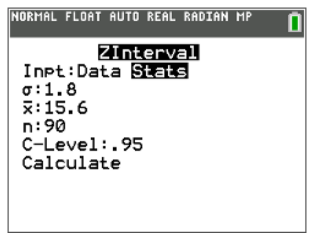
STAT → TESTS → ZINTERVAL

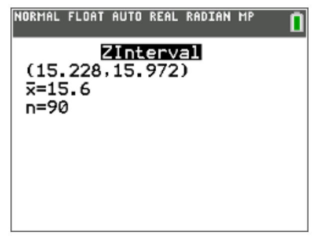




Example 2: Julia enjoys jogging. She has been jogging over a period of several years, during which time her physical condition has remained constantly good. Usually, she jogs 2 miles per day. The standard deviation of her times is 1.8 minutes. During the past year, Julia has recorded her times to run 2 miles. She has a random sample of 90 of these times. For these 90 times, the mean was 15.6 minutes. Find a 95% confidence interval for the mean jogging time for the entire distribution of Julia's 2 -mile running times.

$\sigma = 1.8$
$\bar{x} = 15.6$
$n = 90$
C-Level: 0.95





C.I. = (15.228, 15.972)

We are 95% confident that the interval 15.228 to 15.972 minutes captures the true mean running time for Julia.

Part 2: Confidence Intervals for Population Proportions

Formula:

$$C.I. = sample\ mean \pm margin\ of\ error$$

$$C.I. = \hat{p} \pm z^* \left(\frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}\right)$$

$\hat{p}$ = sample proportion

n = sample size

z^* = critical value for confidence level

Example 3: Voter turnout in municipal elections is often very low. In a recent election, the mayor got 53% of the voters, but only about 1500 voters turned out. Construct a 90% confidence interval for the proportion of people who support the mayor.

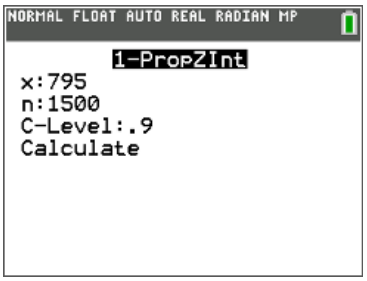
$$C.I. = 0.53 \pm 1.645\left(\frac{\sqrt{0.53(1-0.53)}}{\sqrt{1500}}\right)$$

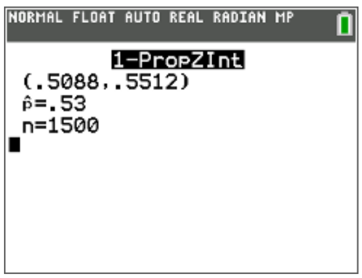
$$= (0.51, 0.55)$$

We are 90% confident that the interval from 0.51 to 0.55 captures the true proportion of voters who support the mayor.

Using the Ti-84 for example 3:

STAT → TESTS → 1-PropZInt



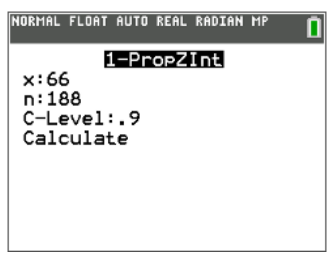


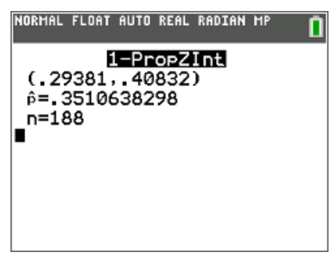
Note: x was calculated by doing $\hat{p} \times n$

Example 4: A random sample of 188 books purchased at a local bookstore showed that 66 of the books were murder mysteries. Find a 90% confidence interval for the proportion of books sold by this store that are murder mysteries.

$x = 66$
$n = 188$
C-Level = 0.9

$C.I. = (0.29381, 40832)$





We are 90% confident that the interval from 0.29 to 0.41 captures the true proportion of books sold by the store that are murder mysteries.

THE ECONOMIC FORECAST IS DIFFICULT FOR AN AMATEUR TO GET RIGHT...BUT THE PROFESSIONALS ARE REASONABLY CONFIDENT THAT THE MARKETS WILL GO EITHER UP OR DOWN IN THE LONG OR SHORT TERM