Section 4.3 – Probability Using Sets

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Part 1: Warm-Up

When rolling a die with sides numbered from 1 to 20, what is the probability of rolling a number divisible by 5?

$$P(divisible\ by\ 5) = \frac{n(divisible\ by\ 5)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$

Part 2: New Terms

1. Simple Event

- an event that consists of exactly one outcome

2. Compound event

- consists of two or more simple events

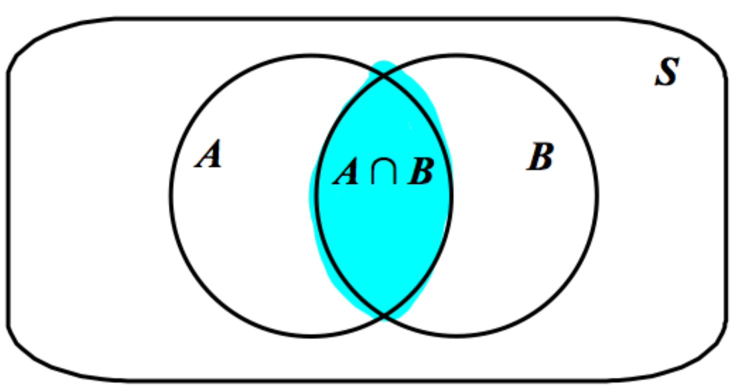
3. Subsets

- sets that exist as a set within a larger set

Part 3: Intersection of Sets

Given two sets, A and B, the set of common elements is called the intersection of A and B, and is written as $A \cap B$. These common elements are members of set A and are also elements of set B.

$A \cap B$ = {elements in both A AND B}

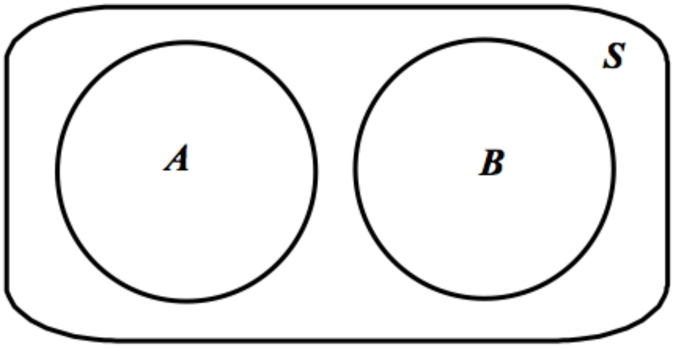


Note: A and B are both subsets of S

Disjoint Sets

If A and B have no elements in common they are said to be disjoint or mutually exclusive and their 'intersection' is the empty set ∅.

$$\mathbf{A} \cap \mathbf{B} = \emptyset$$

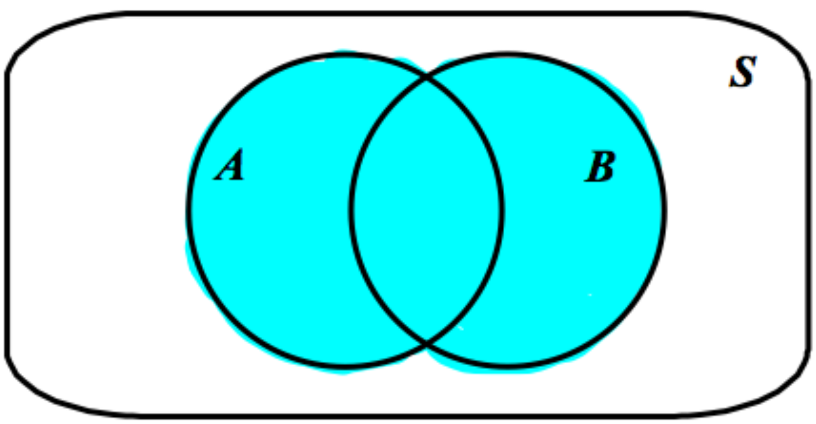


Note: n($A \cap B$) = 0

Part 4: Union of Sets

The set formed by combining the elements of A with those in B is called the union of A and B, and is written as $A \cup B$. The elements in $A \cup B$ are elements of A or they are elements of B.

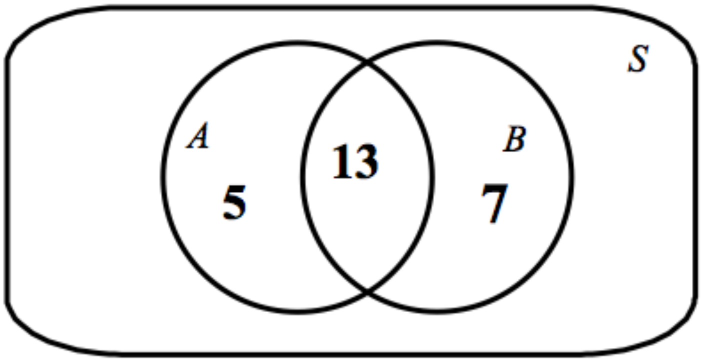
$A \cup B$ = {elements in A OR B}



The set $A \cup B$ is represented by the shaded region in the Venn diagram.

Example 1

Using the following Venn diagram, determine:



a) $n(A) = 18$

b) $n(B) = 20$

c) $n(A \cap B) = 13$

d) $n(A \cup B) = 25$

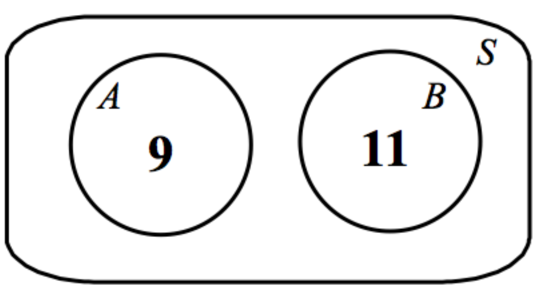
e) Show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$n(A \cup B) = 18 + 20 - 13$$

$$= 25$$

Example 2:

If A and B are disjoint sets:



a) $n(A \cap B) = 0$

b) $n(A \cup B) = 20$

Part 5: Additive Principle for the Union of Two Sets

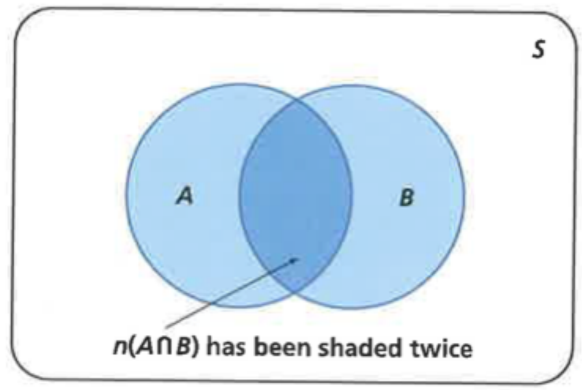
Given two sets, A and B, the number of elements in $A \cup B$ can be found by totaling the number of elements in both sets and then subtracting the number that have been counted twice. The double counted elements will be found in the intersection of the two sets ($A \cap B$).

Number of elements in set A or B:

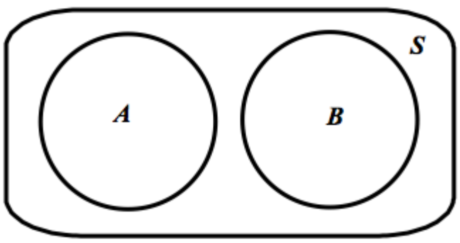
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Probability of the event that A or B occurs is:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Additive Principle for Mutually Exclusive Events



Note: n($A \cap B$) = 0

If events A and B are mutually exclusive (they can't occur at the same time); the rule can be simplified:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = n(A) + n(B) - 0$

$n(A \cup B) = n(A) + n(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - 0$

$P(A \cup B) = P(A) + P(B)$

Example 3:

The Cav's are sending their basketball and hockey teams to OFSAA on the same bus. There are 10 students on the basketball team and 17 students on the hockey team. Three students play on both teams. How many students must the bus taking the teams be able to hold?

$n(basketball \cup hockey) = n(basketball) + n(hockey) - n(basketball \cap hockey)$

$n(basketball \cup hockey) = 10 + 17 - 3$

$= 24$

Example 4:

When rolling a 6 sided die, what is the probability you roll an even number **or** a number less than 3?

$P(even \cup less\ than\ 3) = P(even) + P(less\ than\ 3) - P(even \cap less\ than\ 3)$

$P(even \cup less\ than\ 3) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$

$= \frac{4}{6}$

$= \frac{2}{3}$

Example 5:

A sporting goods store has 22 Bauer hockey sticks (14 right, 8 left) and 38 Easton hockey sticks (20 right, 18 left). If the sales representative randomly grabs a stick to make a sales pitch to you, what is the probability that it is a Bauer stick or a left handed stick?

$P(Bauer \cup left\ handed) = P(Bauer) + P(left\ handed) - P(Bauer \cap left\ handed)$

$P(Bauer \cup left\ handed) = \frac{22}{60} + \frac{26}{60} - \frac{8}{60}$

$= \frac{40}{60}$

$= \frac{2}{3}$

Example 6:

A blood bank catalogs the types of blood, including positive or negative Rh-factor, given by donors during the last five days. The umber of donors who gave each blood type is shown in the table. A donor is selected at random.

		Blood Type				
		O	A	B	AB	Total
Rh-factor	Positive	156	139	37	12	344
	Negative	28	25	8	4	65
	Total	184	164	45	16	409

a) Find the probability the donor has type O or type A blood.

$P(type\ O \cup type\ A) = P(type\ O) + P(type\ A)$

$= \frac{184}{409} + \frac{164}{409}$

$= \frac{348}{409}$

Note: *the events are mutually exclusive because a donor can not have type O and type A blood.*

b) Find the probability the donor has type B blood or is Rh-negative.

$$P(type\ B \cup Rh\ negative) = P(type\ B) + P(Rh\ negative) - P(type\ B \cap Rh\ negative)$$

$$P(type\ B \cup Rh\ negative) = \frac{45}{409} + \frac{65}{409} - \frac{8}{409}$$

$$= \frac{102}{409}$$

***Note:** the events <u>are not</u> mutually exclusive because a donor can have type B blood and be Rh-negative.*

Example 7:

If two dice are rolled, one red and one green, find the probability that you roll:

a) a total of 2 or 12

$$P(2 \cup 12) = P(2) + P(12)$$

$$= \frac{1}{36} + \frac{1}{36}$$

$$= \frac{2}{36}$$

$$= \frac{1}{18}$$

D_2 \ D_1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Mutually Exclusive

b) a total of 4 or a pair will occur

Let A be the event of rolling a total of 4 and B be the event of rolling a pair.

$$P(4 \cup pair) = P(4) + P(pair) - P(4 \cap pair)$$

$$= \frac{3}{36} + \frac{6}{36} - \frac{1}{36}$$

$$= \frac{8}{36}$$

$$= \frac{2}{9}$$

D_2 \ D_1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

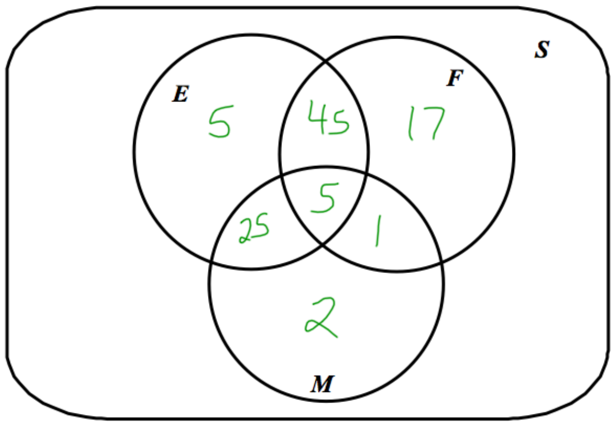
Not Mutually Exclusive

Example 8 (Using Venn Diagrams):

A survey of 100 grade 12 students in a local high school produced the following results:

a) Create a Venn Diagram of the information

Course Taken	No. of students
English	80
Mathematics	33
French	68
English and Mathematics	30
French and Mathematics	6
English and French	50
All three courses	5



b) How many students study English only? $n(english\ only) = 5$

c) How many students study French only? $n(french\ only) = 17$

d) How many students study Math only? $n(math\ only) = 2$

If you were asked to randomly select a student from the group of students described, what is the probability that:

e) the student selected is enrolled only in Math?

$$P(math\ only) = \frac{2}{100} = \frac{1}{50}$$

f) the student is enrolled in french or math?

$$P(french \cup math) = P(french) + P(math) - P(french \cap math)$$

$$= \frac{68}{100} + \frac{33}{100} - \frac{6}{100}$$
$$= \frac{95}{100}$$

$$= \frac{19}{20}$$

g) the student is enrolled in french and math?

$$P(french \cap math) = \frac{6}{100} = \frac{3}{50}$$