

4.4 Modelling With Formulas

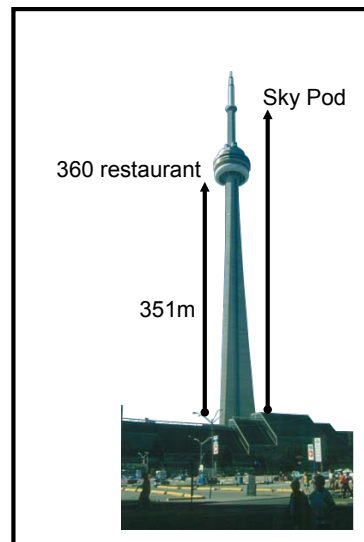
$\Delta \mathbf{x} = \mathbf{x}_f - \mathbf{x}_i$ $\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ $v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$ $\theta = \tan^{-1}(\frac{v_y}{v_x})$
 $\bar{\mathbf{v}} = \frac{\Delta \bar{\mathbf{r}}}{\Delta t}$ $\bar{\mathbf{a}} = \frac{\Delta \bar{\mathbf{v}}}{\Delta t}$ $\theta = \cos^{-1}(\frac{v_x}{v})$ $\theta = \sin^{-1}(\frac{v_y}{v})$ $\mathbf{x} = r\theta$ $\mathbf{v} = \omega \mathbf{r}$ $\omega = \frac{\Delta \theta}{\Delta t}$ $\alpha = \frac{\Delta \omega}{\Delta t}$
 $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2$ $v_x = v \cos(\theta)$ $\mathbf{x} \rightarrow x, y$ $\mathbf{x}_0 \rightarrow x_0, y_0$ $\mathbf{v} \rightarrow v_x, v_y$ $\mathbf{v}_0 \rightarrow v_{0x}, v_{0y}$ $\mathbf{a} \rightarrow a_x, a_y$ $\mathbf{v} = \sqrt{\frac{T}{\rho}}$ $I = \sum_i m_i r_i^2$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$ $\omega = \omega_0 + \alpha t$ $f = \frac{1}{T}$ $\omega = 2\pi f$ $\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$
 $\mathbf{F}_{tot} = m \mathbf{a}$ $E = K + U$ $\Delta Q = (\text{quant.}) C_{\text{const.}} \Delta T$ $\Delta S \geq 0$ $\sum_i \bar{\mathbf{F}}_i = 0$ $\sum_i \bar{\tau}_i = 0$
 $W = \mathbf{F} \cdot d_{\parallel} = F_{\parallel} d$ $E_i = E_f$ $\Delta Q_{\text{into}} = \Delta W_{\text{by}} + \Delta E$ $\Delta Q = 1 \Delta(\text{quant.})$ $PV = nRT$
 $W_{\text{tot}} = \Delta(\text{KE})$ $\frac{1}{2} m v^2$ $\frac{RT}{2} |_{\text{deg. freedom}}$ $C_p = C_v + R$ $e = \frac{\Delta W}{\Delta Q}$ $e = 1 - \frac{T_L}{T_H}$ $P = \frac{F}{A}$
 $\Delta U = -W_{\text{if}}$ $x = A \cos(\omega t)$ $\{ \text{or} \} A \sin(\omega t)$ $M = \rho V$ $P_1 = P_2$
 $\frac{1}{2} k x^2$ $\omega = \sqrt{\frac{k}{m}}$ $v = A \omega \sin(\omega t)$ $\{ \text{or} \} A \omega \cos(\omega t)$ $\Delta P = \rho g \Delta h$
 $p = m v$ $a = A \omega^2 \cos(\omega t)$ $\{ \text{or} \} -A \omega^2 \sin(\omega t)$ $B = \rho_{\text{liq}} V_{\text{disp}} g$
 $\bar{P}_{\text{init}} = \bar{P}_{\text{final}}$ $\frac{GM_e}{R_e} = g R_e$ $\frac{GMm}{r^2}$ $A_1 v_1 = A_2 v_2$
 $\left(\sum_j m_j \bar{\mathbf{v}}_j \right)_{\text{init}} = \left(\sum_j m_j \bar{\mathbf{v}}_j \right)_{\text{final}}$ $M_e = 5.97(10)^{24} \text{ Kg}$ $R_e = 6.37(10)^6 \text{ m}$ $\frac{GMm}{r}$ $P + \frac{1}{2} \rho v^2 = \text{const}$
 $G = 6.67(10)^{-11} \text{ N m}^2/\text{Kg}^2$

Part 1: Do It Now

The relationship between how high you are and how far you can see is given by the formula $d = 2\sqrt{3.2h}$, where h is your height in metres above the ground, and d is your distance in kilometres to the horizon.

Using this information,

- How far can you see from the 360 restaurant?
- from the Sky Pod the horizon is 75.64 km away. How high up is the Sky Pod to the nearest meter?



a) $d = 2\sqrt{3.2h}$

$$d = 2\sqrt{3.2h}$$

$$d = 2\sqrt{3.2(351)}$$

$$d = 2\sqrt{1123.2}$$

$$d \approx 67.03 \text{ km}$$

The horizon is about 67 km away.

b) $d = 2\sqrt{3.2h}$

$$d = 2\sqrt{3.2h}$$

$$\frac{75.64}{2} = \frac{2\sqrt{3.2h}}{2}$$

$$37.82 = \sqrt{3.2h}$$

$$(37.82)^2 = 3.2h$$

$$\frac{1430.3524}{3.2} = \frac{3.2h}{3.2}$$

$$h \approx 446.99 \text{ m}$$

The Sky Pod is about 447 m tall.

c) Which was easier to solve? Why?

a) was easier because the unknown variable was already isolated.

Learning Goals: In this lesson you will learn how rearrange formulas to isolate different variables.

When working with a formula

The known values for variables can be substituted first and then this equation can be solved for the required variable.

OR

The formula can be rearranged to isolate the required variable and then the known values can be substituted into the formula.

Part 2: Rearranging Formulas

To rearrange a formula, isolating one variable, think of the other variables as constants and isolate the variable in the same way that you would solve an equation.

For each of the following examples, rearrange each formula to isolate the variable indicated.

1) $d = a + b$ isolate for a
 $d - b = a$
 $a = d - b$

$$2) \quad \frac{C}{2\pi} = \frac{2\pi r}{2\pi} \quad \text{isolate for } r$$

$$\frac{C}{2\pi} = r$$

$$r = \frac{C}{2\pi}$$

$$3) \quad y = x^2 \quad \text{isolate for } x$$

$$\sqrt{y} = \sqrt{x^2}$$

$$\sqrt{y} = x$$

$$x = \sqrt{y}$$

$$4) \quad y = mx + b \quad \text{isolate for } x$$

$$\frac{y-b}{m} = \frac{mx}{m}$$

$$\frac{y-b}{m} = x$$

$$x = \frac{y-b}{m}$$

First isolate the **term** that contains the variable we want to isolate, then isolate the variable.

5) $V = \frac{W}{q}$ isolate for W

$$q(V) = q \left(\frac{W}{q} \right)$$

$$qV = W$$

$$W = qV$$

6) $k = \frac{1}{2}mv^2$ isolate for v

$$2k = 2 \left(\frac{1}{2} \right) mv^2$$

$$\frac{2k}{m} = \frac{mv^2}{m}$$

$$\frac{2k}{m} = v^2$$

$$\sqrt{\frac{2k}{m}} = \sqrt{v^2}$$

$$v = \sqrt{\frac{2k}{m}}$$

$$7) \quad C = \frac{5}{9}(F - 32) \quad \text{isolate for } f$$

$$9(C) = \cancel{9}\left(\frac{\cancel{5}}{\cancel{9}}\right)(F-32)$$

$$\frac{9C}{\cancel{9}} = \frac{\cancel{5}(F-32)}{\cancel{9}}$$

$$\frac{9C}{9} = F - 32$$

$$\frac{9C}{9} + 32 = F$$

$$F = \frac{9C}{9} + 32$$

Part 3: Application

The area, A , of a square is related to its side length, l , by the formula $A = l^2$. Find the length, to the nearest tenth of a centimeter, of a side of a square with an area of 32 cm^2 .

$$A = l^2$$

$$\sqrt{A} = l$$

$$l = \sqrt{A}$$

$$l = \sqrt{32}$$

$$l = 5.66 \text{ cm}$$

Homework: Worksheet