Section 4.4 – Conditional Probability

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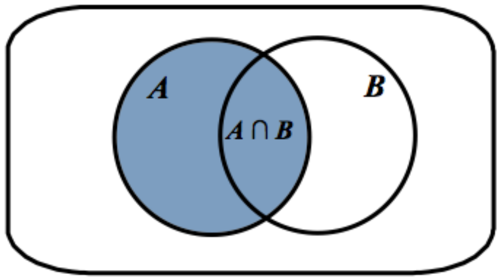
Part 1: Conditional Probability

A conditional probability is the probability of an event occurring, given that another event has already occurred.



formula(s):

$$P(B|A) = \frac{n(B \cap A)}{n(A)} \quad \text{OR} \quad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Note: P(B|A) is read as "probability of B, given A."

Example 1a: The table shows the results of a study in which researchers examined a child's IQ and the presence of a specific ten in the child. Find the probability that a child has a high IQ given that the child has the gene.

$$P(high\ IQ|gene\ present) = \frac{n(high\ IQ \cap gene\ present)}{n(gene\ present)}$$

$$= \frac{33}{72}$$

	Blood Type		
	Gene Present	Gene Not Present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

Example 1b: Find the probability that a child does not have the gene, given that the child has a normal IQ.

$$P(gene\ not\ present|normal\ IQ) = \frac{n(gene\ not\ present \cap normal\ IQ)}{n(normal\ IQ)}$$

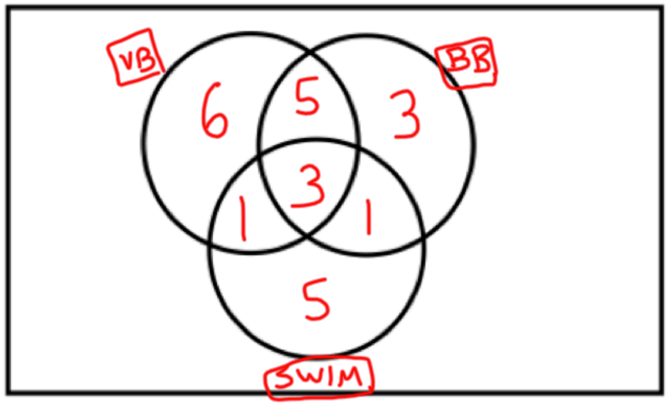
$$= \frac{11}{50}$$

	Blood Type		
	Gene Present	Gene Not Present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

Example 2:

Draw a Venn Diagram
(24 students total)

Team	Number of Students
Volleyball	15
Basketball	12
Swimming	10
VB and BB	8
VB and Swim	4
BB and Swim	4
All three	3



There are often restrictions placed on probabilities. Consider the question from yesterdays homework. This time you are asked to determine the probability of selecting a swim team member given they are on the volleyball team.

$$P(swim|volleyball) = \frac{n(swim \cap volleyball)}{n(volleyball)}$$

$$= \frac{4}{15}$$

Example 3: The probability that Crosby gets a goal and an assist in a game is 20%. Crosby gets an assist in 75% of the games he plays. Determine the probability that Crosby gets a goal given that he gets an assist.

$$P(goal|assist) = \frac{P(goal \cap assist)}{P(assist)}$$

$$= \frac{0.2}{0.75}$$

$$= \frac{4}{15}$$

Part 2: Multiplication Law for Conditional Probability

The probability of events A and B occurring:

$$P(A \cap B) = P(A) \times P(B|A)$$

Note: this formula is used when events are *dependent*

Example 4: What is the probability of drawing two aces in a row from a well-shuffled deck of 52 playing cards? The first card drawn is not replaced.

$P(first\ ace \cap second\ ace) = P(first\ ace) \times P(second\ ace|first\ ace)$

$= \frac{4}{52} \times \frac{3}{51}$

$= \frac{12}{2652}$

$= \frac{1}{221}$

Example 5: Two cards are selected from a standard deck without replacement. Find the probability that they are both hearts.

$P(first\ heart \cap second\ heart) = P(first\ heart) \times P(second\ heart|first\ heart)$

$= \frac{13}{52} \times \frac{12}{51}$

$= \frac{156}{2652}$

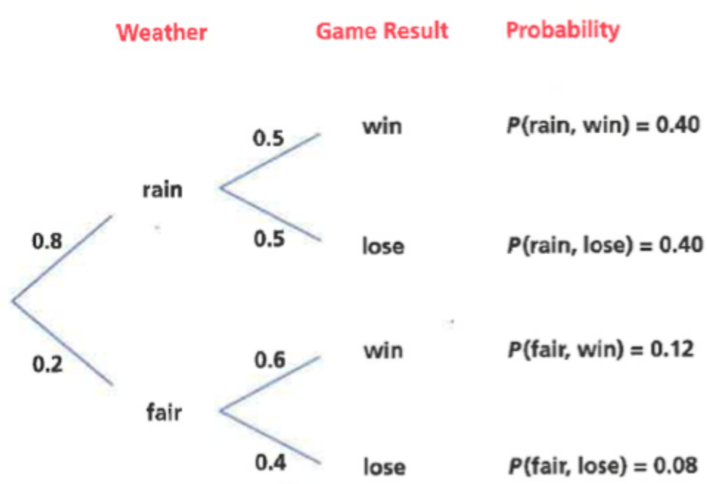
$= \frac{1}{17}$

Part 3: Tree Diagrams and Conditional Probability

Tree diagrams can be a helpful way of organizing outcomes in order to identify probabilities.

Example 6: Your soccer teams wins 50% of it's games when it rains and 60% of it's games on clear days. The weather for your next game calls for an 80% chance of rain. What is the probability that you team will win?

Start with a tree diagram:



There are two scenarios that result in a win: (rain, win) OR (fair, win)

P(win) = P(rain, win) + P(fair, win)

$P(win) = (0.8)(0.5) + (0.2)(0.6)$

$P(win) = 0.4 + 0.12$

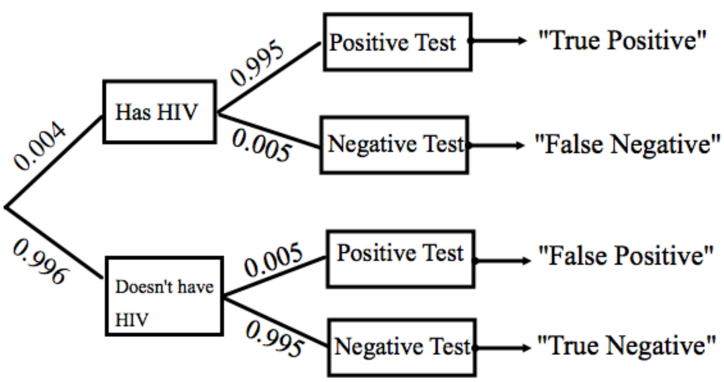
$P(win) = 0.52$

There is a 52% chance your team will win.

Example 7: In the population of the USA, estimates suggest that 1 in every 250 people (0.4%) has been infected with HIV. Tests that are given to detect HIV in people are 99.5% accurate. Chris is randomly selected from the population of the USA for an HIV test and his test is "positive". What is the chance that Chris is infected with HIV?

Start by making a guess: _________

Next make a tree diagram to help visualize the problem:



We want to calculate $P(has\ HIV \mid positive\ test)$. Using the conditional probability formula,

$$P(has\ HIV \mid positive\ test) = \frac{P(has\ HIV \cap positive\ test)}{P(positive\ test)}$$

Numerator:

$$P(has\ HIV \cap positive\ test) = P(has\ HIV) \times P(positive\ test \mid has\ HIV)$$

$= (0.004)(0.995)$

$= 0.00398$

Denominator:

to find the $P(positive\ test)$ we must consider both scenarios. (1) a would could have HIV and test positive OR (2) a would could not have HIV and test positive.

$P(positive\ test) = (0.004)(0.995) + (0.996)(0.005)$

$= 0.00896$

Final Calculation:

$P(has\ HIV \mid positive\ test) = \frac{P(has\ HIV \cap positive\ test)}{P(positive\ test)}$

$= \frac{0.00398}{0.00896}$

$= 0.4442$

Therefore there is only a 44.42% chance that Chris has HIV even though he tested positive.