Section 4.5 – Multiplication of Independent Events

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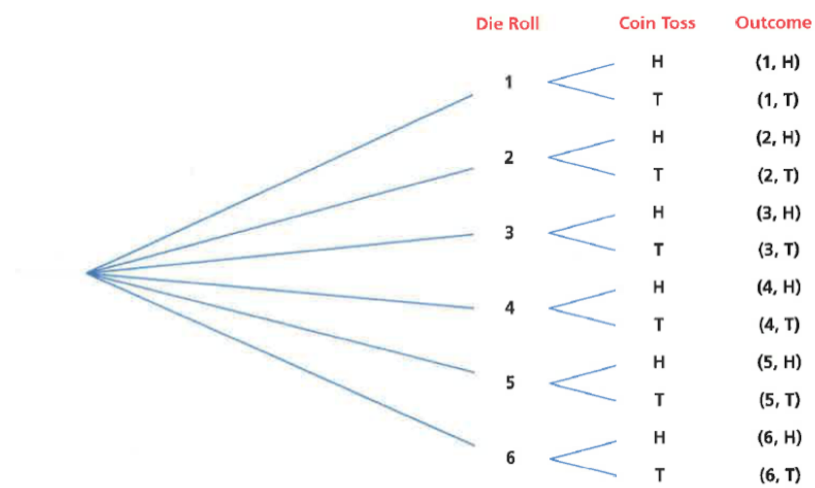
Example 1: Consider an experiment in which you first roll a six-sided die and then flip a coin.

a) Create a tree diagram to show the possible results



b) What is the probability of tossing tails and rolling an even number?

$P(tails, even) = \frac{n(tails, even)}{n(S)}$

$= \frac{3}{12}$

$= \frac{1}{4}$

Remember: $P(A) = \frac{n(A)}{n(S)}$

Part 1: Multiplicative Principle for Counting Ordered Pairs

If the outcome for each experiment (flipping coins and rolling dice) has no influence on the outcome of any other experiment we can use ordered pairs to write the outcomes. Ordered pairs are used for independent events.

The total number of outcomes is the product of the possible outcomes at each step in the sequence.

We can count these ordered pairs where: $n(a, b) = n(a) \times n(b)$

Another way to think about it….

- If one event can occur in *m* ways and a second event can occur in *n* ways, the number of ways the two events can occur in sequence is $m \times n$

Example 2: How many outcomes are possible in the die roll/coin toss experiment?

$n(die\ roll, coin\ toss) = n(die\ roll) \times n(coin\ toss)$

$= 6 \times 2$

$= 12$

Example 3: How many outcomes are possible in an experiment where you draw a random card from a standard deck and then roll a six-sided die.

$n(card, die) = n(card) \times n(die)$

$= 52 \times 6$

$= 312$

Example 4: You are going to the school dance tonight and can't decide what to wear. You've narrowed it down to 4 possible pairs of pants, 6 shirts, and 2 pairs of shoes. How many possible outfits can you make?

$n(outfits) = n(pants) \times n(shirts) \times n(shoes)$

$= 4 \times 6 \times 2$

$= 48$

Part 2: Independent Events

Consider the experiment where you roll a 6 sided die and then flip a coin.

1. What is the probability tossing the coin and getting tails if you know in advance that the die will show an even number?

$P(tails \mid even) = \dfrac{n(tails \cap even)}{n(even)}$

$= \dfrac{3}{6}$

$= \dfrac{1}{2}$

2. What is the probability of getting tails when you flip a coin?

$$P(A) = \frac{n(A)}{n(S)}$$

$= \frac{1}{2}$

3. Make a conclusion about your answers to 1&2

$P(tails \mid even) = P(tails)$ because they are independent events.

The general rule is:

If A and B are independent events, then $P(B \mid A) = P(B)$

Part 3: Multiplicative Principle for Probabilities of Independent Events

If A and B are independent events, then:

$P(A \cap B) = P(A) \times P(B)$

Example 5: In the die roll/coin toss experiment; use the multiplicative law for probabilities of independent events to determine the probability of rolling an even number and then flipping tails.

$P(even, tails) = P(even) \times P(tails)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

> ***note:*** *sequences of independent events can be written as ordered pairs*
>
> *P(even roll, tails) = P(even roll ∩ tails)*

Example 6: A coin is tossed and a die is rolled. Find the probability of flipping heads and then rolling a 6.

$P(heads \cap 6) = P(heads) \times P(6)$

$= \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{12}$

Example 7: The probability that Crosby gets a point in a game is 85%

a) Find the probability that he gets a point in three consecutive games

$P(point, point, point) = P(point) \times P(point) \times P(point)$

$= (0.85)(0.85)(0.85)$

$= 61.4\%$

b) Find the probability that he gets no points in three consecutive games

$P(no\ point, no\ point, no\ point) = P(no\ point) \times P(no\ point) \times P(no\ point)$

$= (0.15)(0.15)(0.15)$

$= 0.3\%$

c) Find the probability that he gets a point in at least one of the three games.

The phrase "at least one" means one or more. The complement to the event "at least one" is the event "none."

$P(point\ in\ at\ least\ 1\ game) = 1 - P(point\ in\ no\ games)$

$= 1 - 0.003$

$= 0.997$

$= 99.7\%$