Section 4.6 – Permutations

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1) Evaluate each of the following

a) 5!

$= 5\times4\times3\times2\times1$
$= 120$

b) ${}_5P_3$

$= \frac{5!}{(5-3)!}$

$= \frac{5!}{2!}$

$= \frac{5\times4\times3\times\cancel{2}\times\cancel{1}}{\cancel{2}\times\cancel{1}}$

$= 5\times4\times3$

$= 60$

c) $\frac{5!}{4!}$

$= \frac{5\times\cancel{4!}}{\cancel{4!}}$

$= 5$

d) P(10, 3)

$= \frac{10!}{(10-3)!}$

$= \frac{10!}{7!}$

$= \frac{10\times9\times8\times\cancel{7!}}{\cancel{7!}}$

$= 10\times9\times8$

$= 720$

2) Simplify each of the following

a) $\frac{n!}{(n-1)!}$

$= \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}}$

$= n$

b) $\frac{(3n)!}{(3n-1)!}$

$= \frac{(3n)\cancel{(3n-1)!}}{\cancel{(3n-1)!}}$

$= 3n$

3) Express the following using factorials

a) $5\times4\times3\times2\times1$

$= 5!$

b) $8\times7\times6$

$= \frac{8!}{5!}$

c) $\frac{30\times29\times28}{3\times2\times1}$

$= \frac{30!}{27!\,3!}$

4) Ten students are to line up for a photograph.

a) In how many ways can the 10 students standing in a line be arranged?

$n(arrangements) = 10! = 3\,628\,800$

b) In how many ways can the 10 students standing in a line be arranged if Jill must be first?

$n(arrangements\ with\ Jill\ first) = 9! = 362\,880$

c) In how many ways can 10 students standing in a line be arranged if Jill must be first and Meera last?

$n(arrangements\ with\ Jill\ first\ and\ Meera\ last) = 8! = 40\,320$

5) The senior choir has rehearsed five songs for an upcoming assembly. In how many different orders can the choir perform the songs?

$n(orders\ of\ songs) = 5! = 120$

6) In how many ways is it possible to elect a president, a vice-president, and a secretary for a club consisting of 15 members?

$= P(15, 3)$

$= \frac{15!}{(15-3)!}$

$= \frac{15!}{12!}$

$= 15 \times 14 \times 13$

$= 2\ 730$

7) In how many ways can the letters of the word MONDAY be arranged?

$= 6!$

$= 720$

8) In how many different ways can the letters of the word MISSISSAUGA be arranged?

$= \frac{11!}{(4!\,2!\,2!)}$

$= 415\ 800$

9) Forty-three race cars started the 2004 Daytona 500. How many ways can the cars finish first, second, and third?

$= P(43, 3)$

$= \frac{43!}{(43-3)!}$

$= \frac{43!}{40!}$

$= 43 \times 42 \times 41$

$= 74\ 046$

10) There are 12 people entered in a swimming race. Assuming that there are no ties, in how many different ways can these people finish first, second, and third?

$= P(12, 3)$

$= \frac{12!}{(12-3)!}$

$= \frac{12!}{9!}$

$= 12 \times 11 \times 10$

$= 1\ 320$

11) A landscaper wants to plant four oak trees, eight maple trees, and six poplar trees along the border of a lawn. The trees are to be spaced evenly apart. In how many distinguishable ways can they be planted?

$= \frac{18!}{(4!\,8!\,6!)}$

$= 9\ 189\ 180$

12) There are ten questions on a test.

a) In how many ways can these questions be arranged?

$= 10!$

$= 3\ 628\ 800$

b) In how many ways can these questions be arranged if the easiest question and the hardest question are side-by-side.

Treat the easiest question and the hardest question as a single question unit making nine questions that are to be arranged. Also, don't forget that the two questions can be arranged in 2! Ways within their unit.

$n(arrangements) = 9! \times 2! = 725\ 760$

13) In how many ways can the 12 members of a volleyball team line up, if the captain and assistant captain must remain together?

Treat he captain and assistant captain as a single unit making there be 11 members to be arranged. Don't forget that the captain and assistant can be arranged in 2! ways within their unit.

$n(arrangements) = 11! \times 2! = 79\ 833\ 600$

14) A combination lock opens when the right combination of three numbers from 00 to 99 is entered. The same number may be used more than once.

a) What is the probability of getting the correct combination by chance?

$$P(correct) = \frac{n(correct)}{n(combinations)} = \frac{1}{100\times100\times100} = \frac{1}{1\,000\,000}$$

b) What is the probability of getting the right combination if you already know the first digit?

$$P(correct) = \frac{n(correct)}{n(combinations)} = \frac{1}{100\times100} = \frac{1}{10\,000}$$

c) How many possible combinations would there be if numbers could NOT be re-used?

$$n(combinations) = P(100, 3) = 100\times99\times98 = 970\,200$$

15) You are taking a chemistry test and are asked to list the first 10 elements of the periodic table in order as they appear in the table. You know the first 10 elements but not the order. What is the probability of you guessing the correct answer?

$$P(correct) = \frac{n(correct)}{n(orders)} = \frac{1}{10!} = \frac{1}{3\,628\,800}$$

16) Plates issued by the Motor Vehicle License Office now use four letters followed by three numbers. How many such plates are possible?

$$n(plates) = 26\times26\times26\times26\times10\times10\times10 = 456\,976\,000$$

17) Solve for n: $\frac{(n-1)!}{(n-3)!} = 20$

$$\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 20$$

$$n^2 - 3n + 2 - 20 = 0$$

$$n^2 - 3n - 18 = 0$$

$$(n-6)(n+3) = 0$$

$$n = 6 \qquad \cancel{n = -3}$$