5.1 Direct Variation

DO IT NOW!!

You and a friend decide to have a friendly bicycle race on Saturday. How far ahead will you be after one hour, given that you travel at the following speeds?

a) Heading North your speed is 12km/h, your friend's speed is 10 km/h

b) Heading South you travel 5 km in 30 minutes, your friend travels 5 km in 20 minutes

Speed Calculations:

$$\boxed{speed = \frac{distance}{time} = \frac{km}{hour}}$$

a) My speed = 12 km/h

Friend = 10 km/h

∴ after one hour, I will be 2 km ahead of my friend.

b) Me $= \frac{5\text{ km}}{0.5\text{ hrs}} = 10$ km/h

Friend $= \frac{5\text{ km}}{\frac{1}{3}\text{ hour}} = 15$ km/h

∴ my friend will be 5 km ahead of me after 1 hour.

Speed is an example of <u>a rate of change</u>

Part A: Investigate Direct Variation using Tables

<u>**Example 1**</u>**:** Use a table to organize the information in the following problem.

a) How long does it take to fill a 3 000 litre hot tub if a water truck supplies water at a rate of 250 litres per hour and the tub is initially empty?

Time (h)	Water (l)
0	0
1	250
2	500
3	750
4	1000
5	1250
6	1500
7	1750
8	2000
9	2250
10	2500
11	2750
12	3000

b) What if the water truck was able to double the flow rate?

Time (h)	Water (l)
0	0
1	500
2	1000
3	1500
4	2000
5	2500
6	3000

Which variable is dependent? (If a variable is dependent its value cannot be known without knowing the other variable's value). **To determine the dependent variable ask yourself the following key questions:**

Does the amount of water DEPEND on time?

OR

Does time DEPEND on the amount of water ?

∴ Dependent (y) = water
Independent (x) = time

$$\textit{Rate of Change} = \frac{\Delta \textit{dependent variable}}{\Delta \textit{independent variable}} = \frac{\Delta y}{\Delta x} = \frac{\Delta \textit{water}}{\Delta \textit{time}}$$

a) Calculate the rate of change of the amount of water in the hot tub between the following times, using the first table:

i) between 0 and 1 hour:

$$\text{rate} = \frac{250\ L}{1\ h} = 250\ L/h$$

ii) between 0 and 2 hours:

$$\text{rate} = \frac{\Delta\text{water}}{\Delta\text{time}} = \frac{500L - 0L}{2h - 0h} = \frac{500L}{2h} = 250\ L/h$$

iii) between 2 and 5 hours:

$$\text{rate} = \frac{\Delta\text{water}}{\Delta\text{time}} = \frac{1250L - 500L}{5h - 2h} = \frac{750L}{3h} = 250\ L/h$$

What do you notice about these rates?

They are all the same. The rate is constant.

When the rate is ***constant,*** it is often referred to as the ***constant of variation***.

A **direct variation** is a relationship between two variables in which one variable is a constant multiple of the other. The rate of change is constant (constant of variation). A direct variation is a situation in which two quantities, such as hours and pay, increase or decrease at the same rate. That is, as one quantity doubles, the other quantity also doubles. The variables are said to be directly proportional.

For example: *if you get paid hourly, when you work twice as many hours, you will make twice as much money. How much you make is a constant multiple of how many hours you work.*

<u>Example 2</u>: Identify the dependent and independent variables in the following rates of change.

	Dependent variable (y)	Independent variable (x)
The car travelled at 75 km/h.	km	hours
In November the temperature drops 1.2 degrees Celsius per day.	degrees Celsius	days

<u>Example 3</u>:

Determine the constant of variation (rates of change) given the data in the following tables.

Recall: Rate of change (constant of variation) $\overset{m}{}$ $= \dfrac{\Delta\text{dependent } y}{\Delta\text{independent } x}$

a)

Hours worked (h) x	Money made ($) y
0	**0**
1	35
2	70
3	105

Δx Δy

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{35-0}{1-0}$$

$$= \frac{\$35}{1h}$$

$$= \$35 \text{ /hour}$$

b)

Mass of grain (kg) x	Cost ($) y
0	**0**
20	125
40	250
60	375

Δx Δy

$m = \frac{\Delta y}{\Delta x}$

$= \frac{125-0}{20-0}$

$= \frac{\$125}{20\text{ kg}}$

$= \$6.25/\text{kg}$

i) What is the same in the relationships given in parts a) and b)?

Initial value of (0,0)

Both tables represent **direct variations**, the independent variable directly affects the dependent variable.

Direct variation will always have (0 , 0) as the initial value!

ii) Write an equation to model the data in the tables given in Example 3. Remember that 'y' varies directly with 'x'; $\boldsymbol{y = mx}$

a)

hours

$y = 35x$

\$ made

b)

kg

$y = 6.25x$

cost

iii) Which is the independent variable (x)?

a) hours b) kg

Which is the dependent variable (y)?

a) \$ made b) cost

In general, the equation of a **direct variation** is always in the form $y = mx$, where m is the constant of variation

Part B: Investigate Direct Variations using Graphs

<u>Example 4</u>: The cost (y) to do electrical work varies directly with time (x). Electric company charges $25 per hour to do electrical work while AC-DC electrical charges $50 per hour.

a) Write equations to model each relationship.

Electric company:

$m = \$25/h$

$y = 25x$ (hours) (cost)

AC-DC electrical:

$m = \$50/h$

$y = 50x$ (hours) (cost)

b) Use the equations to complete the tables for 0 to 4 hours.

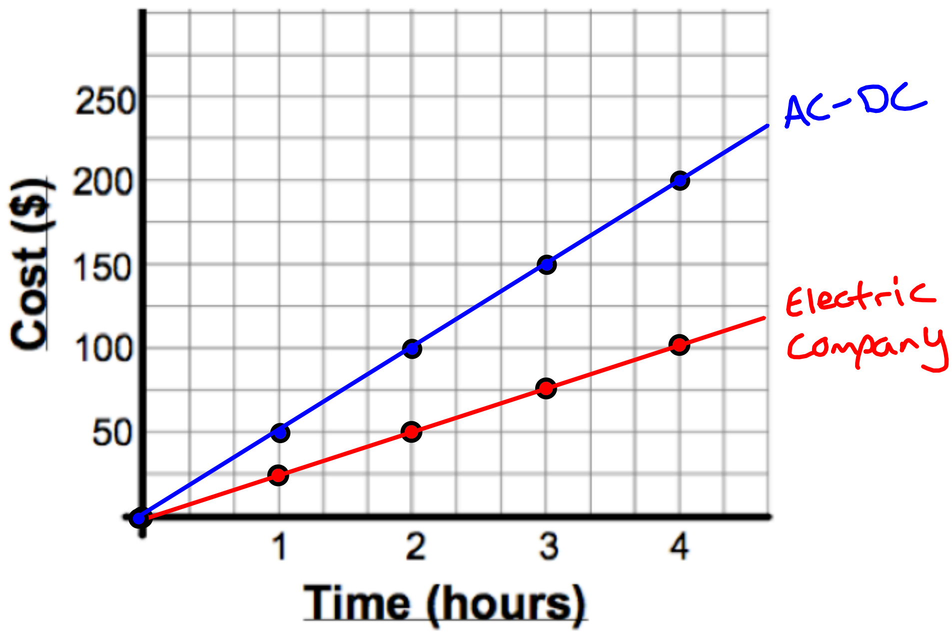
Electric company:

Hours (h)	Cost ($)
0	0
1	25
2	50
3	75
4	100

AC-DC electrical:

Hours (h)	Cost ($)
0	0
1	50
2	100
3	150
4	200

c) Graph the data for both companies on the same Cartesian coordinate grid.



d) Looking at the **graph or the table,** which company should I choose if I have a job that requires 3 hours of electrical work?

Electric Company

e) How can we show the rate of change on the graph? How does the steepness of the line relate to the rate of change?

A steeper line shows a higher rate of change (m).

f) What is the same for both graphs?

- Lines are straight (constant rate of change)
- Start at the origin (0,0)

In general, the graph of a **direct variation** is a straight line which always passes through the origin

Part C: Interpreting Direct Variation Word Problems

Example 5:

The distance travelled by Mr. Jensen when he drives varies directly with time. His car travels 630 km in 3 hours.

y x

i) What is the constant of variation? $\text{constant of variation} = \frac{\Delta dependent}{\Delta independent} = \frac{\Delta y}{\Delta x}$

$$m = \frac{\Delta y}{\Delta x} = \frac{630\text{ km}}{3\text{ hrs}} = 210\text{ km/h}$$

ii) Write an equation to represent the relationship between the variables

$$y = 210x$$

time

distance

iii) Which variable is the dependent/independent variable?

dependent (y) = distance

independent (x) = time

Example 6:

The total cost varies directly with the number of MP3's downloaded. 13 MP3 downloads costs $12.87

x y

i) What is the constant of variation?

$$m = \frac{\Delta y}{\Delta x} = \frac{\$12.87}{13 \text{ MP3's}} = \$0.99 / \text{MP3}$$

ii) Write an equation to represent the relationship between the variables

$$y = 0.99x$$

cost MP3's

iii) Which variable is the dependent/independent variable?

dependent (y) = cost

independent (x) = # of MP3's

<u>Consolidate</u>:

Direct variation occurs when the dependent variable is a constant multiple of the independent variable.

Direct variation can be defined algebraically as $y = mx$ where *m* is the constant of variation.

The graph of a direct variation is a straight line that passes through the origin.