5.1 Probability Distributions

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Part 1: Introducing Probability Distribution

What is a *probability distribution*?

A table, formula or graph that provides the probabilities of a discrete random variable assuming any of all of its possible values.

Discrete Random Variable: A variable that has a unique value for each outcome.

A ***probability distribution*** must satisfy the following criteria:

1. The probability of each value of the discrete random variable is between 0 and 1, inclusive.

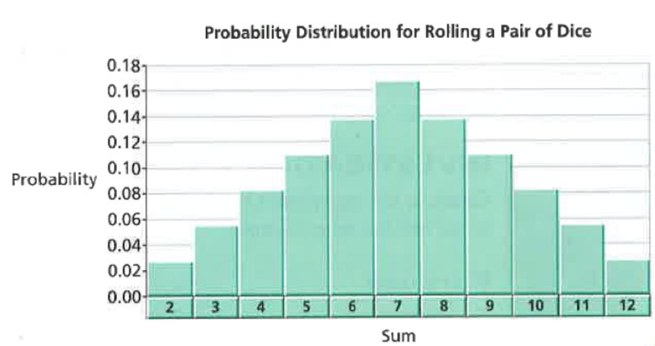
2. The sums of all the probabilities is 1.

Consider the experiment in which two six-sided dice are rolled. Suppose one die is red and the other is blue. Create a **table** to represent the probability distribution for the possible sums of the dice:

Sum	Theoretical Probability
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

Blue Die \ Red Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The **graph** below is another way to represent the probability distribution. This graph also provides the probability of each sum occurring when a pair of dice is rolled.



Part 2: Expected Value

The mean of a probability distribution is often called the expected value of the distribution. The mean is an average value and need not be a point of the sample space.

The formula used is:

$$E(x) = \mu = \sum x \cdot P(x)$$

The expected value of the distribution of dice sums is the result of adding each possible sum value multiplied by its probability.

$$E(sum) = 2 \cdot P(2) + 3 \cdot P(3) + \ \dots + 12 \cdot P(12)$$

$$E(sum) = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right)$$

$$= 7$$

Part 3: Making Probability Distributions

Example 1: Consider a simple game in which you roll a single die. If you roll an even number, you gain that number of points, and, if you roll an odd number, you lose that number of points. Show the probability distribution of points in this game and calculate the expected number of points per roll.

Number Rolled	Points, x	Probability, $P(x)$
1	-1	$\frac{1}{6}$
2	2	$\frac{1}{6}$
3	-3	$\frac{1}{6}$
4	4	$\frac{1}{6}$
5	-5	$\frac{1}{6}$
6	6	$\frac{1}{6}$

$$E(points) = -1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) - 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) - 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$= \frac{3}{6}$$

$$= 0.5$$

You would expect that the score in this game would average out to 0.5 points per roll.

Example 2: A summer camp has seven 4.6 meter canoes, ten 5.0 meter canoes, four 5,2 meter canoes, and four 6.1 meter canoes. Canoes are assigned randomly for campers going on a canoe trip. Show the probability distribution for the length of an assigned canoe. Then calculated the expected length of an assigned canoe.

Length of Canoe (m), x	**Probability, $P(x)$**
4.6	$\frac{7}{25}$
5.0	$\frac{10}{25}$
5.2	$\frac{4}{25}$
6.1	$\frac{4}{25}$

$$E(length) = 4.6\left(\frac{7}{25}\right) + 5.0\left(\frac{10}{25}\right) + 5.2\left(\frac{4}{25}\right) + 6.1\left(\frac{4}{25}\right)$$

$$= 5.1\ m$$

The expected length of the canoe is 5.1 meters.

Example 3: A school raffle sold 1500 tickets at $2 each. There are four prizes of $500, $250, $150, and $75.

a) Create a probability distribution for the amount of money you could win.

Winnings, x	**Probability, $P(x)$**
500	$\frac{1}{1500}$
250	$\frac{1}{1500}$
150	$\frac{1}{1500}$
75	$\frac{1}{1500}$
0	$\frac{1496}{1500}$

b) Calculate your expected gain if you buy a ticket

Expected gain = $E(winnings) - cost\ of\ ticket$

$$= \left[500\left(\frac{1}{1500}\right) + 250\left(\frac{1}{1500}\right) + 150\left(\frac{1}{1500}\right) + 76\left(\frac{1}{1500}\right) + 0\left(\frac{1496}{1500}\right)\right] - 2$$

$$= 0.65 - 2$$

$$= -1.35$$

You can expect to lost $1.35 if you buy a ticket