

5.3 Binomial Probability Distributions

MDM4U

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Part 1: Properties of a Binomial Experiment

In a Binomial Experiment, the number of **successes** in n trials is a discrete random variable - X . X is termed a **binomial random variable** and its probability distribution is called a **binomial distribution**.

Properties of Binomial Experiments (Bernoulli Trials)

1. There are n identical trials
2. There are only two possible outcomes. Success or failure.
3. The probability of success is the same in every trial (trials are independent of one another)

Part 2: Investigating Binomial Experiments

Consider an experiment where you roll a single die 4 times.

a) What is the probability that your first two rolls are 6's and your next two rolls will be something other than a 6?

$P[(\text{roll } 1 = 6) \text{ and } (\text{roll } 2 = 6) \text{ and } (\text{roll } 3 \neq 6) \text{ and } (\text{roll } 4 \neq 6)]$

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}$$

$$= \frac{25}{1296}$$

Note: This is a Bernoulli Trial because there are only two possible outcomes - success is a roll of 6, failure is a roll that is not 6. The probability of success is the same for every roll. The trials are independent of each other.

b) Find the probability of the event that the roll of 6 will appear exactly twice in any of the four available positions in the table.

Note: The number of ways 6 can be placed in two of the four entries is the same as counting the number of ways two objects can be selected from four available objects. $C(4, 2)$

$$P(\text{two } 6\text{'s}) = \frac{25}{1296} \times C(4, 2) = \frac{25}{1296} \times 6 = \frac{150}{1296} = \frac{25}{216}$$

c) Complete a theoretical probability distribution for the number of 6's showing in four rolls

Probability of zero 6's

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

Probability of one 6

$$= \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \binom{4}{1} = \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^3 \binom{4}{1} = \frac{500}{1296} = \frac{125}{324}$$

Probability of two 6's

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \binom{4}{2} = \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \binom{4}{2} = \frac{150}{1296} = \frac{25}{216}$$

Probability of three 6's

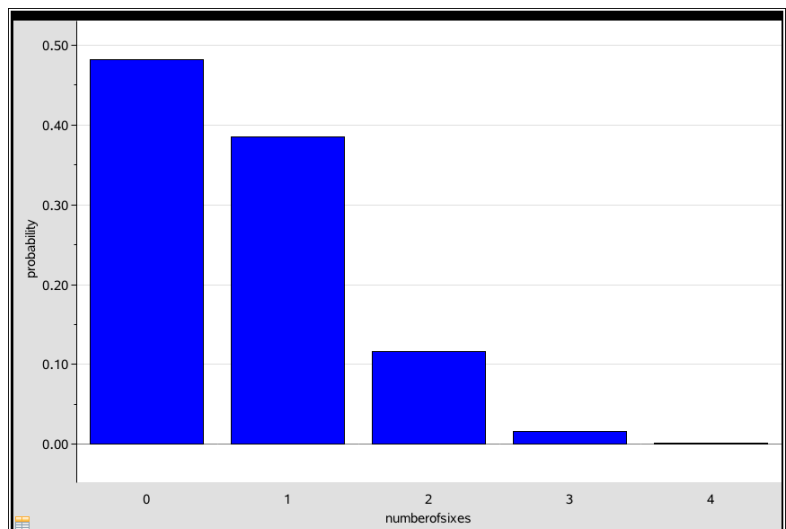
$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \binom{4}{3} = \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1 \binom{4}{3} = \frac{20}{1296} = \frac{5}{324}$$

Probability of four 6's

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$$

Use a chart to display the probability distribution...

# of 6's, x	$P(x)$
0	$\binom{4}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$
1	$\binom{4}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3 = \frac{500}{1296}$
2	$\binom{4}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{150}{1296}$
3	$\binom{4}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1 = \frac{20}{1296}$
4	$\binom{4}{4} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = \frac{1}{1296}$



Part 2: Binomial Probability Formula

In a binomial experiment with n Bernoulli trials, each with a probability of success p , the probability of k successes in the n trials is given by:

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n-k}$$

Where X is the discrete random variable corresponding to the number of successes.

Example 1: Each child of a particular set of parents has probability 0.25 of having type O blood. Suppose the parents have 5 children.

a) Find the probability that exactly 3 of the children have type O blood

$$n = 5$$

$$P(X = k) = \binom{n}{k} (p)^k (1 - p)^{n-k}$$

$$p = 0.25$$

$$P(X = 3) = \binom{5}{3} (0.25)^3 (0.75)^2$$

$$k = 3$$

$$P(X = 3) = 0.08789$$

There is an 8.79% chance that 3 of their children will have type O blood.

b) Should the parents be surprised if fewer than 2 of the children have type O blood?

$$P(X < 2) = P(X = 0) + P(X = 1)$$

$$P(X < 2) = \binom{5}{0} (0.25)^0 (0.75)^5 + \binom{5}{1} (0.25)^1 (0.75)^4$$

$$P(X < 2) = 0.6328125$$

There is a 63.28% chance that fewer than 2 of their children have type O blood so they should not be surprised.

Example 2: During the 2010 NHL season when Crosby led the NHL in Goals and Points, he had a 17% shooting percentage. Determine the probability that in a game where he takes four shots, he gets three goals.

$$n = 4$$

$$P(X = 3) = \binom{4}{3} (0.17)^3 (0.83)^1 = 0.0163$$

$$p = 0.17$$

$$k = 3$$

There is approximately a 1.63% chance of Crosby getting a hat-trick when he takes 4 shots.

Example 3: A basketball player has 3 shots in a free-throw competition. Historically, the player has a probability of 65% of scoring on a free throw. Assuming the probability is constant...

a) Determine the probability distribution for the number of free throws made

# of Free Throws Made, x	Probability, $P(x)$
0	$\binom{3}{0} (0.65)^0 (0.35)^3 = 0.042875$
1	$\binom{3}{1} (0.65)^1 (0.35)^2 = 0.238875$
2	$\binom{3}{2} (0.65)^2 (0.35)^1 = 0.443625$
3	$\binom{3}{3} (0.65)^3 (0.35)^0 = 0.274625$

b) Calculate the players expected number of free throws made in the competition.

Note: to calculate the expected value for a binomial probability distribution, you can use either of the following two formulas:

i) $E(X) = \sum x \cdot P(x)$ (this formula works for ALL types of probability distributions)

$$= 0(0.042875) + 1(0.238875) + 2(0.443625) + 3(0.274625)$$

$$= 1.95$$

ii) $E(X) = np$ (this formula only works for binomial probability distributions)

$$= 3(0.65)$$

$$= 1.95$$

Example 4: A candy company makes candy-coated chocolates, 40% of which are red. The production line mixes the candies randomly and packages ten per box.

a) What is the probability that 3 of the candies in a given box are red?

$$P(X = 3) = \binom{10}{3} (0.4)^3 (0.6)^7 = 0.21499$$

There is about a 21.50% chance that exactly three of the candies are red.

b) What is the probability that a given box has at least 2 red candies?

$$P(X \geq 2) = 1 - [P(0) + P(1)]$$

$$P(X \geq 2) = 1 - \left[\binom{10}{0} (0.4)^0 (0.6)^{10} + \binom{10}{1} (0.4)^1 (0.6)^9 \right]$$

$$P(X \geq 2) = 1 - 0.0463574016$$

$$P(X \geq 2) = 0.9536$$

There is about a 95.36% chance that a given box has at least 2 red candies.

Part 3: Using the Ti-84

`binompdf(n, p, k)` computes $P(X = k)$

`binomcdf(n, p, k)` computes $P(X \leq k)$

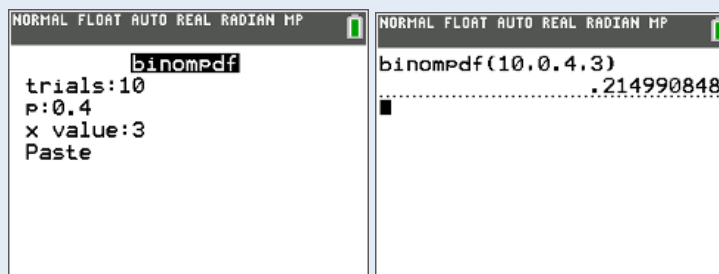
Note: the `binomcdf` command only computes the probability of getting k or FEWER successes

Example 4: A candy company makes candy-coated chocolates, 40% of which are red. The production line mixes the candies randomly and packages ten per box.

a) What is the probability that 3 of the candies in a given box are red?

How to use the calculator:

- 2nd → VARS (DISTR) → `binompdf`(→ trials: 10 → p: 0.4 → x value: 3 → PASTE



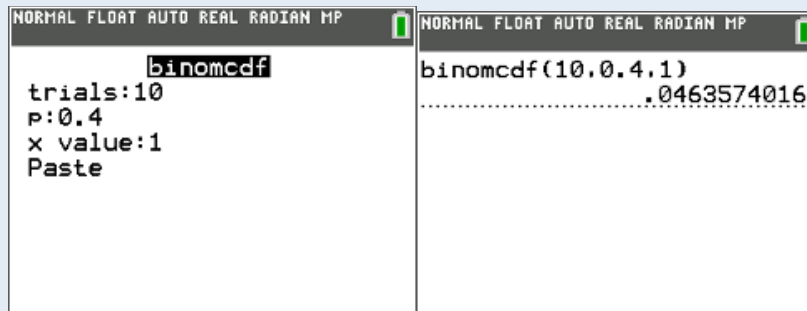
What you need to write:

$$P(X = 3) = \text{binompdf}(n=10, p=0.4, k=3) = 0.21499$$

There is about a 21.50% chance that exactly three of the candies are red.

b) What is the probability that a given box has at least 2 red candies?

- 2nd → VARS (DISTR) → binomcdf(→ trials: 10 → p: 0.4 → x value: 1 → PASTE



```
NORMAL FLOAT AUTO REAL RADIAN MP
binomcdf
trials:10
p:0.4
x value:1
Paste

NORMAL FLOAT AUTO REAL RADIAN MP
binomcdf(10,0.4,1)
.....0463574016
```

What you need to write:

$$P(X \geq 2) = 1 - P(X \leq 1)$$

$$= 1 - \text{binomcdf}(n=10, p=0.4, k=1) = 0.0463574016$$

$$= 0.9536$$

There is about a 95.36% chance that a given box has at least 2 red candies.