5.4 Geometric Probability Distributions

MDM4U
Jensen

Part 1: What is a Geometric Probability?

In some board games, you cannot move forward until you roll a specific number, which could take several tries. Manufacturers of products such as switches, relays, and hard drives need to know how many operations their products can perform before failing. In some sports competitions, the winner is the player who scores the most points before missing a shot. In each of these situations, the critical quantity is the waiting time, which is the number of trials it takes for a specific outcome to occur.

Like the binomial distribution, trials in a geometric distribution have only two possible outcomes, success or failure, whose probabilities do not change from one trial to the next. However, the random variable for a geometric distribution is the waiting time (not the number of successes) and this causes significant differences between the binomial and geometric distributions.

The waiting time is the number of trials you have to wait before the event of interest (success) happens. The number of trials isn't fixed – you simply count the number of trials until you get the first success.

Part 2: Getting out of Jail Activity

In the board game Monopoly, one way to get out of jail is to roll doubles. Suppose that this was the only a player could get out of jail. With a pair of dice, keep rolling them until you get out of jail (roll doubles).

a) Record the number of trials it takes you to get out of jail

Waiting time: ________

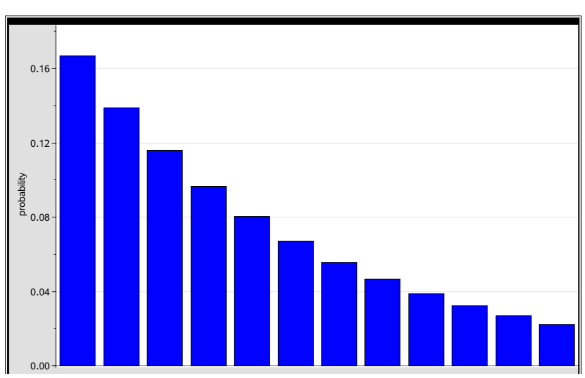
b) Record the following class statistics:

Mean waiting time: ______

Sketch of graph of waiting times:

Should be close to 6

Should have same shape as corresponding probability distribution



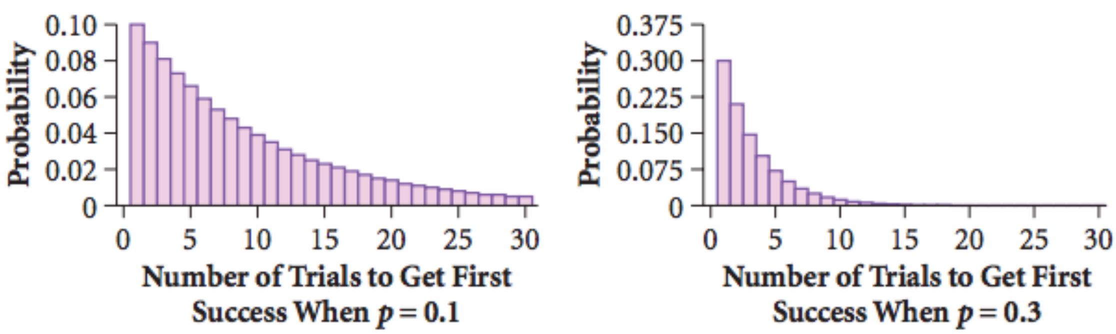
c) What do you think the probability of getting out of jail on your second attempt would be?

$$P(Y = 2) = P(fail\ on\ first\ try) \cdot P(succeed\ on\ second\ try) = \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) = \frac{5}{36} = 0.139$$

d) How many times would you expect to have to roll the dice before getting out of jail?

$$E(Y) = \frac{1}{p} = \frac{1}{\left(\frac{1}{6}\right)} = 6$$

The probability distribution we constructed in this activity is called a geometric distribution. Below are examples of geometric probability distributions for different p values.



Part 3: Geometric Probability Formula

A geometric setting arises when we perform independent trials of the same chance process and record the number of trials it takes to get one success Y. On each trial, the probability p of success must be the same. The possible values of Y are 1, 2, 3, ….. If k is any of these values,

$$P(Y = k) = (1 - p)^{k-1}p$$

k - number of trials until a success occurs (waiting time)

p - probability of success

Part 4: Expected Value of a Geometric Random Variable Formula

If Y is a geometric random variable with probability of success p on each trial, then its mean (expected value) is:

$$E(Y) = \frac{1}{p}$$

Part 5: Using the Geometric Probability Formula

Example 1: As a special promotion for its 20-ounce bottles of soda, a soft drink company printed a message on the inside of each cap. Some of the caps said, "Please try again," while others said, "You're a winner!" The company advertised the promotion with the slogan "1 in 6 wins a prize." Kramer decides to keep buying 20-ounce bottles of soda until he gets a winner.

a) Find the probability that he will have to buy exactly 5 bottles before getting a winner.

$P(Y = 5) = \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right) = 0.0804$ There is about a 8.04% chance he will have to buy 5 bottles to get a winner.

b) What is the probability that he will have to buy 3 or fewer bottles to get a winner.

$P(Y \leq 3) = P(1) + P(2) + P(3)$

$$= \left(\frac{5}{6}\right)^0 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^1 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)$$

$$= 0.4213$$

There is about a 42.13% chance that we will have to buy 3 or fewer bottles to get a winner.

c) Find the expected number of bottles he will have to buy before getting a winner.

$$E(Y) = \frac{1}{p} = \frac{1}{\left(\frac{1}{6}\right)} = 6$$

He should expect to have to buy 6 bottles before getting a winner.

Part 6: Using the Ti-84 For Geometric Distributions

geometpdf(p, k) computes $P(Y = k)$
geometcdf(p, k) computes $P(Y \leq k)$

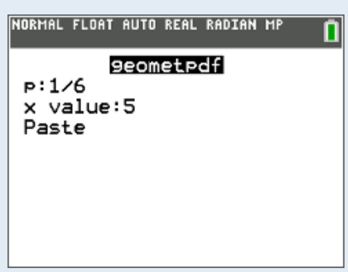
Example 1:

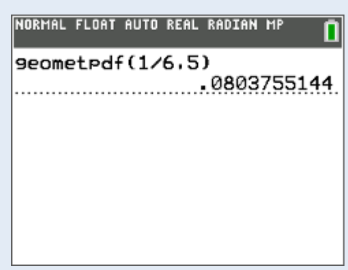
a) Find the probability that he will have to buy exactly 5 bottles before getting a winner.

- 2^{nd} → VARS (DISTR) → geometpdf → p: 1/6 → x value: 5 → Paste

$P(Y = 5) = geometpdf\left(p = \frac{1}{6}, x = 5\right)$

$= 0.0804$



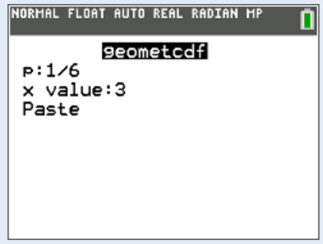


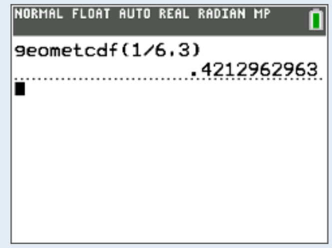
b) What is the probability that he will have to buy 3 or fewer bottles to get a winner.

- 2^{nd} → VARS (DISTR) → geometcdf → p: 1/6 → x value: 3 → Paste

$$P(Y \leq 3) = geometcdf(p = \frac{1}{6}, x = 3)$$

$$= 0.4213$$





Example 2: Your teacher is planning to give you 10 problems for homework. As an alternative, you can agree to play the Lucky Day Game. Here's how it works. A student will be selected at random from your class and asked to pick a day of the week. Then your teacher will use technology to randomly choose a day of the week as the 'lucky day'. If the student picks the correct day, the class will have only one homework problem. If the student picks the wrong day, your teacher will select another student form the class at random and the process is repeated. If the second student gets it right, the class will have two homework problems. If he/she gets it wrong, the process is repeated until a student guesses the right day. Your teacher will assign a number of homework problems that is equal to the total number of guesses made by members of your class.

Do you think the class should play the lucky day game or just accept the 10 homework problems?

In this problem, the random variable of interest is Y = the number of picks it takes to correctly match the lucky day. On each trial, the probability of a correct pick p is 1/7.

a) Find the probability that the class receives exactly 10 homework problems as a result of playing the game.

$$P(Y = 10) = geometpdf\left(p = \frac{1}{7}, x = 10\right) = 0.0357$$

There is about a 3.57% chance the class will get exactly 10 homework problems.

b) Find the probability that the class will get fewer than 10 homework problems.

$$P(Y < 10) = P(Y \leq 9) = geometcdf\left(p = \frac{1}{7}, x = 9\right) = 0.7503$$

There is about a 75.03% chance that the class will get less homework by playing the game.

c) How many homework problems should they expect if they play the game?

$E(Y) = \frac{1}{p} = \frac{1}{\left(\frac{1}{7}\right)} = 7$ They should expect to get 7 problems if they play the game.