### 5.5 First Differences

## DO IT NOW

If a tennis ball falls out of the third story window of a building will its motion be linear? The height of the ball over time is recorded in the following table.

Graph the relation and determine if it represents linear motion.

| Time (seconds) | Height (meters) |
| :---: | :---: |
| 0 | 30 |
| 1 | 29 |
| 2 | 26 |
| 3 | 21 |
| 4 | 14 |
| 5 | 5 |
| 6 | 0 |



Time (seconds)
Rate of change is not costant.

## Part 1:Recall

We know from graphing lines that if the slope (rise and the run) is constant then the relation will form a straight line.

$$
\text { slope }=m=\frac{\text { rise }}{\text { run }}=\frac{\Delta x}{\Delta y}
$$

Therefore, we need to determine if the changes in $x$ and $y$ are constant in a table to determine if a relation is linear.

## Part 2: What are First differences

First differences are the differences between consecutive $y$-values in tables of values with evenly spaced $x$-values.

If the first differences of a relation are constant, the relation is LNEAR

If the first differences of a relation are not constant, the relation is NON-LINEAR
Notice that the $x-$
values change by a
constant amount. This
is a requirement to
work with first
differences!

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |  <br> First Differences <br> are constant! <br> linear relation |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | $3-0=3$ |  |
| 1 | 3 | $6-3=3$ |  |
| 2 | 6 | $9-6=3$ |  |
| 3 | 9 | $12-9=3$ |  |
| 4 | 12 |  |  |

## Part 3: Calculating First Differences

Complete a table of values for each equation given. Then determine if the first differences are constant and state whether the relation is linear or non linear.

Example 1:
$y=-2 x+7$


Conclusion:
the first differences are Constant
therefore the relationship is Linear

## Example 2:

$y=x^{2}$


Conclusion:
the first differences are Not Constant therefore the relationship is Non-Linear

## Example 3:

$$
y=2^{x}
$$

$\left.\begin{array}{|c|c|}\hline x & y \\ \hline & \\ \hline 0 & 1 \\ \text { First Differences } \\ \hline 1 & 2\end{array}\right)$

## Conclusion:

the first differences are Not Constant therefore the relationship is Non Linear

## Part 4: Check Your Understanding

Use first differences to determine which of these relations are linear and which are non linear.

## Example 4:

| $\boldsymbol{x}$ | $\boldsymbol{y}$ | First Differences |
| :--- | :--- | :--- |
| 0 | 7 |  |
| 1 | 3 | $-1-3=-4$ |
| 2 | -1 | $-5-(-1)=-4$ |
| 3 | -5 | $-9-(-5)=-4$ |
| 4 | -9 |  |

Type of relation: Linear

Example 5:

| $\boldsymbol{x}$ | $\boldsymbol{y}$ | First Differences |
| :--- | :--- | :--- |
| 2 | -5 |  |
| 3 | 10 | $25-10=15$ |
| 4 | 25 | $40-25=15$ |
| 5 | 40 | $55-40=15$ |
| 6 | 55 |  |

Type of relation:Linear

## Example 6:

| $\boldsymbol{x}$ | $\boldsymbol{y}$ | First Differences |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -2 | -10 |  |  |  |  |  |
| -1 | -2 | $0-(-2)=2$ |  |  |  |  |
| 0 | 0 | $2-0=2$ |  |  |  |  |
| 1 | 2 | $10-2=8$ |  |  |  |  |
| 2 | 10 |  |  |  |  |  |

Type of relation:Non-linear

