5.6 - Connecting Slope, Rate of Change, and First Differences

Part 1: Do It Now

a) Calculate the first differences

x	y	First Differences
0	-3	
3	1	$1-(-3)=4$
6	5	$5-1=4$
9	9	$9-5=4$
12	13	$13-9=4$

Type of relation:

LINEAR

b) Using the table of values, what is the constant of variation (slope)?

Point 1: $(\overset{x_1}{0},\overset{y_1}{-3})$ Point 2: $(\overset{x_2}{3},\overset{y_2}{1})$

Remember: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{3-0} = \frac{4}{3}$$

c) What is the initial value (y-intercept)?

when $x=0$, $y=-3$. $\therefore$ $b=-3$

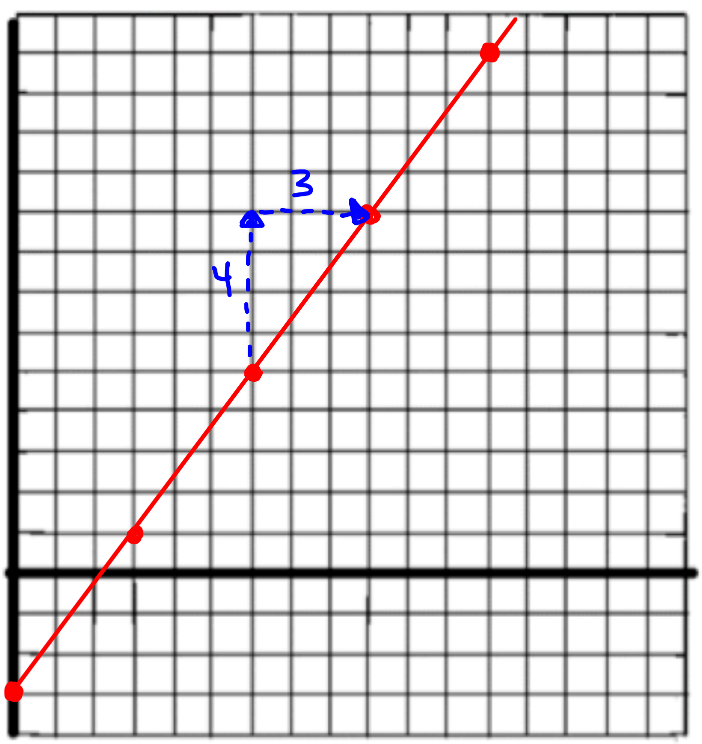
d) Is this a direct variation or partial variation?

Partial because $b \neq 0$

e) Write an equation for the relation in the form $y = mx + b$ using the constant of variation (m) and the initial value (b)

$$y = \frac{4}{3}x - 3$$

f) Graph the relation



g) Find the slope of the line from the graph. How does this relate to the constant of variation?

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{4}{3}$$

This is the same as the constant of variation.

h) What is the y-intercept? How does this relate to the initial value?

$$\text{y-intercept} = b = -3$$

This is the same as the initial value.

i) Write the equation of the line in the form $y = mx + b$ using the slope and y-intercept

$$y = \frac{4}{3}x - 3$$

Part 2: The Rule of Four

A relation can be represented in a variety of ways so that it can be looked at from different points of view. A mathematical relation can be described in four ways:

1. Using words
2. Using a graph
3. Using a table of values
4. Using an equation

Part 3: Write an equation when the relation is represented in words

Remember that the equation of a line is $y = mx + b$

Considering that a line is really just a set of ordered pairs, (x, y), it makes sense that the equation of a line needs to contain the variables x and y. These variables will define the coordinates that make up the line.

This means that the only 2 values that need to be determined in order to write the equation of a linear relation are *m* and *b*.

When a linear relation is represented in words m is the rate of change and b is the initial value.

Linear relation represented in words: ***m*** **= rate of change (slope)**
b **= initial value (y-intercept)**

Example 1: Write an equation for the following relationship by first identifying the value of m and b.

The Copy Centre charges $75 to design a poster plus 25 cents **for each copy**.

$m = 0.25$ $b = 75$

And the equation of this linear relation is:

cost → $y = 0.25x + 75$

\# of copies

If The Copy Centre changed their cost per flyer to 35 cents for each copy the equation would become:

$y = 0.35x + 75$

If The Copy Centre changed their design cost to $125 the equation will become: $y = 0.25x + 125$

Example 2: y varies directly with x. When $x = 2, y = 8$.

a) What is the initial value? (y-intercept)

All direct variations have an initial value of 0.

$\therefore b = 0$

b) What is the slope of the line? Point 1: $(\overset{x_1}{0}, \overset{y_1}{0})$ Point 2: $(\overset{x_2}{2}, \overset{y_2}{8})$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{2-0} = \frac{8}{2} = 4$$

c) Write an Equation for this relationship

$$y = 4x$$

Example 3: y varies partially with x. When $x = 0, y = 3$, and when $x = 2, y = 8$.

a) What is the initial value? (y-int)

when $x=0, y=3$. $\therefore b = 3$

b) What is the slope of the line? Point 1: $(\overset{x_1}{0}, \overset{y_1}{3})$ Point 2: $(\overset{x_2}{2}, \overset{y_2}{8})$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-3}{2-0} = \frac{5}{2}$$

c) Write an Equation for this relationship

$$y = \frac{5}{2}x + 3$$

Part 4: Write an equation when the relation is represented in a table of values

Remember:

$$slope = rate\ of\ change = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b = initial\ value = y - intercept = value\ of\ y\ when\ x\ is\ 0$$

Example 4: Determine the equation of the following linear relations using the tables provided:

a)

x	y
x_1 0	y_1 6
x_2 1	y_2 9
2	12
3	15
4	18

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9-6}{1-0}$

$= \frac{3}{1}$

$= 3$

$b = 6$

Equation: $y = 3x + 6$

b)

x	y
x_1 -2	y_1 3
x_2 0	y_2 5
2	7
4	9
6	11

$m = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{5-3}{0-(-2)}$

$= \dfrac{2}{2}$

$= 1$

$b = 5$

Equation:

$y = x + 5$

What should we do if the initial value isn't in the table?

c)

x	y
0	-17
1	-14
2	-11
x_1 3	y_1 -8
x_2 4	y_2 -5

$m = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{-5-(-8)}{4-3}$

$= \dfrac{3}{1}$

$= 3$

$b = -17$

Equation: $y = 3x - 17$

d)

x	y
-8	-5
x_1 -6	y_1 -10
x_2 -4	y_2 -15
-2	-20
0	-25

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-15-(-10)}{-4-(-6)}$

$= \frac{-5}{2}$

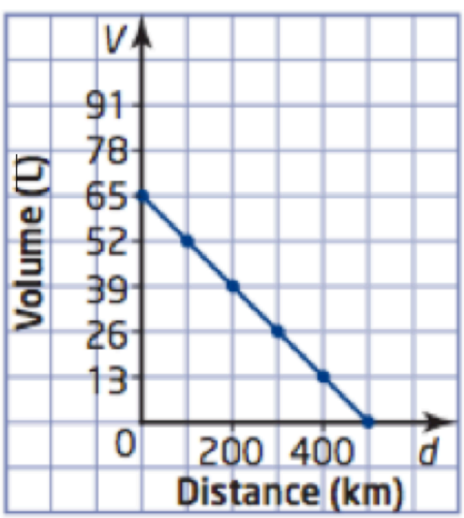
$b = -25$

Equation: $y = -\frac{5}{2}x - 25$

Part 5: Write an equation when the relation is represented as a graph

Example 5: The graph shows the relationship between the volume of gasoline remaining in a car's fuel tank and the distance driven.

> **Remember:** $m = slope = \frac{rise}{run}$
>
> $b = initial\ value = y - intercept$



Slope: $m = \frac{rise}{run}$

$= \frac{-65}{500}$

$= \frac{-13}{100}$

y-intercept: $b = 65$

Equation: $y = -\frac{13}{100}x + 65$