

6.6 Find an Equation for a Line Given Two Points

MPM1D

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1. Find the equation for the line passing through each pair of points

a) P(2,3) and Q(5,6)

$$\begin{aligned}m &= \frac{6-3}{5-2} & y &= mx+b \\ &= \frac{3}{3} & 3 &= (1)(2)+b \\ &= 1 & 3 &= 2+b \\ & & 3-2 &= b \\ & & b &= 1\end{aligned}$$

$$y = x + 1$$

b) A(4,-1) and B(0,5)

$$\begin{aligned}m &= \frac{5-(-1)}{0-4} & y &= mx+b \\ &= \frac{6}{-4} & 5 &= (-\frac{3}{2})(0)+b \\ &= -\frac{3}{2} & 5 &= 0+b \\ & & b &= 5\end{aligned}$$

$$y = -\frac{3}{2}x + 5$$

c) U(-3,4) and V(-2,-6)

$$\begin{aligned}m &= \frac{-6-4}{-2-(-3)} & y &= mx+b \\ &= \frac{-10}{1} & 4 &= (-10)(-3)+b \\ &= -10 & 4 &= 30+b \\ & & 4-30 &= b \\ & & b &= -26\end{aligned}$$

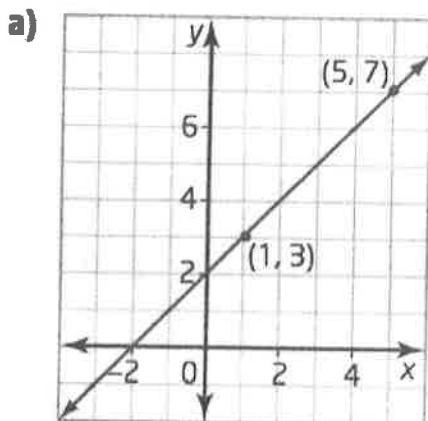
$$y = -10x - 26$$

d) L($\frac{1}{2}, 0$) and M($\frac{7}{2}, -5$)

$$\begin{aligned}m &= \frac{-5-0}{\frac{7}{2}-\frac{1}{2}} & y &= mx+b \\ &= \frac{-5}{\frac{6}{2}} & 0 &= (-\frac{5}{3})(\frac{1}{2})+b \\ &= -\frac{5}{3} & 0 &= -\frac{5}{6}+b \\ & & b &= \frac{5}{6}\end{aligned}$$

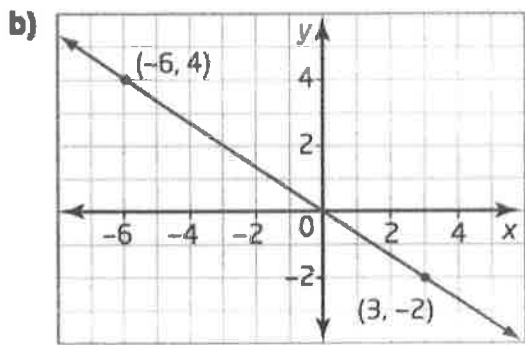
$$y = -\frac{5}{3}x + \frac{5}{6}$$

2. Find the equation for each line



$$\begin{aligned}m &= \frac{7-3}{5-1} & y &= mx+b \\ &= \frac{4}{4} & 3 &= (1)(1)+b \\ &= 1 & 3 &= 1+b \\ & & 3-1 &= b \\ & & b &= 2\end{aligned}$$

$$y = x + 2$$



$$m = \frac{4 - (-2)}{-6 - 3}$$

$$= \frac{6}{-9}$$

$$= -\frac{2}{3}$$

$$y = mx + b$$

$$4 = \left(-\frac{2}{3}\right)(-6) + b$$

$$4 = 4 + b$$

$$4 - 4 = b$$

$$b = 0$$

$$y = -\frac{2}{3}x$$

3. a) Find an equation for the line with an x -intercept of 4 and a y -intercept of -2.

$$\begin{array}{l} x_1, y_1 \\ (4, 0) \\ x_2, y_2 \\ (0, -2) \end{array}$$

$$m = \frac{-2 - 0}{0 - 4}$$

$$= \frac{-2}{-4}$$

$$= \frac{1}{2}$$

$$y = mx + b$$

$$-2 = \left(\frac{1}{2}\right)(0) + b$$

$$-2 = b$$

$$y = \frac{1}{2}x - 2$$

b) Find an equation for the line whose x - and y -intercepts are both -5

$$\begin{array}{l} x_1, y_1 \\ (-5, 0) \\ x_2, y_2 \\ (0, -5) \end{array}$$

$$m = \frac{-5 - 0}{0 - (-5)}$$

$$= \frac{-5}{5}$$

$$= -1$$

$$y = mx + b$$

$$-5 = (-1)(0) + b$$

$$-5 = b$$

$$y = -x - 5$$

4. Find the equation of a line passing through each pair of points.

a) M(0,3) and N(5,3)

$$m = \frac{3 - 3}{5 - 0}$$

$$= \frac{0}{5}$$

$$= 0$$

$$y = mx + b$$

$$3 = (0)(0) + b$$

$$3 = b$$

$$y = 3$$

b) K(-2,6) and L(-2,-4)

$$\begin{aligned} m &= \frac{-4-6}{-2-(-2)} \\ &= \frac{-10}{0} \\ &= \text{undefined} \end{aligned}$$

∴ it is a vertical line with an x-intercept of -2.

$$\boxed{x = -2}$$

5. A bowling alley has a fixed base cost and charges a variable per game rate. It costs \$20.50 for five games and \$28.50 for nine games

a) What is the variable cost (slope)

$$(5, 20.5)$$

$$(9, 28.5)$$

$$\begin{aligned} m &= \frac{28.5 - 20.5}{9 - 5} = 2 \\ &= \frac{8}{4} \end{aligned}$$

$$\boxed{m = 2}$$

b) Find an equation for the line relating, C, in dollars, and number of games, g, in the form $C = mg + b$

$$C = mg + b$$

$$20.5 = 2(5) + b$$

$$20.5 = 10 + b$$

$$10.5 = b$$

$$C = 2g + 10.5$$

d) What is the C-intercept? What does it mean?

$$(0, 10.5)$$

The base cost is \$10.50.

f) Use the equation to find the cost of 20 games

$$C = 2(20) + 10.5$$

$$= 40 + 10.5$$

$$= 50.5$$

6. Fiona is walking at a constant speed in front of a motion sensor. After 2 s, she is 1.5 m from the sensor. 2 s later, she is 4.5 m from the sensor.

a) Is Fiona moving toward or away from the sensor? How do you know?

away; distance from the sensor is increasing

b) How fast is Fiona walking? (find the slope)

(2, 1.5)

$$m = \frac{4.5 - 1.5}{4 - 2}$$

She is walking at a speed of 1.5 m/s.

(4, 4.5)

$$= \frac{3}{2}$$

$$= 1.5$$

c) Find the equation that describes Fiona's motion in the form $d = mt + b$

$$d = mt + b$$

$$d = 1.5t - 1.5$$

$$1.5 = (1.5)(2) + b$$

$$1.5 = 3 + b$$

$$-1.5 = b$$

d) What is the d-intercept and what does it represent?

-1.5; this means her initial position was 1.5 m behind the sensor.

7. Workers at a laboratory get the same raise each year. Colette, who has been working at the lab for 5 years, earns \$17.25/h. Lee, who has been working at the lab for 1 year, earns \$14.25/h. The equation relating wage and number of years worked is of the form $w = mn + b$, where w is the hourly wage and n is the number of years worked.

a) (5, 17.25) and (1, 14.25) are two points on the line. Explain why.

x-coordinate represents years of experience

y-coordinate represents wage.

b) Find the slope and the w-intercept of this line, and explain what they mean.

$$m = \frac{17.25 - 14.25}{5 - 1}$$

$$= \frac{3}{4}$$

$$= 0.75$$

Rate of increase in pay per year.

$$y = mx + b$$

$$14.25 = (0.75)(1) + b$$

$$14.25 - 0.75 = b$$

$$13.5 = b$$

Initial pay with no experience

c) Write the equation of the line

$$y = 0.75x + 13.5$$

d) Maria has been working at the lab for 7 years. Determine her hourly wage.

$$\begin{aligned} y &= 0.75(7) + 13.5 \\ &= 5.25 + 13.5 \\ &= 18.75 \end{aligned}$$

$$\boxed{\$18.75}$$

e) What wage does the linear model predict for a worker who has been with the lab for 25 years? Does this seem reasonable? Explain.

$$\begin{aligned} y &= 0.75(25) + 13.5 \\ &= 18.75 + 13.5 \\ &= 32.25 \end{aligned}$$

$$\boxed{\$32.25, \text{ yes this is reasonable.}}$$

8. Anil's family is driving home to Toronto. Anil hopes that they will make it back in time to see the hockey game on television. While travelling at a fairly constant speed, he observes two signs along the trip.

a) How fast Anil's family travelling?

$$(0, 240)$$

$$(2.5, 40)$$

$$m = \frac{240 - 40}{0 - 2.5}$$

$$= \frac{200}{-2.5}$$

$$= -80$$

They are travelling 80 km/h towards home.

At 4:30 P.M.:

Toronto 240 km

At 7:00 P.M.:

Toronto 40 km

b) Find a linear equation that relates distance from home, in kilometers, to time travelled, in hours.

$$y = mx + b$$

$$240 = (-80)(0) + b$$

$$240 = b$$

$$y = -80x + 240$$

c) The game starts at 7:45 P.M. Will they make it back to Toronto in time? If yes, how much spare time will Anil have to make it to the TV? If not, how late will he be? What assumptions must you make

$$y = -80x + 240$$

The game starts 3.25 hours from when they started driving. How long will it take them to drive the 240 km?

$$0 = -80x + 240$$

$$-240 = -80x$$

$$x = 3$$

It would take 3 hours to get home. They will have 15 minutes to spare