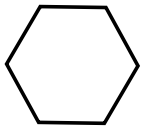
7.3 Angle Relationships in Polygons

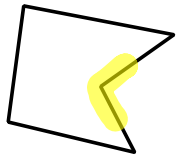
Types of Polygons

Convex Polygon: All interior angles measure less than 180 degrees.



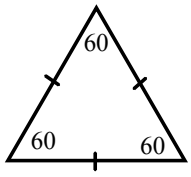
- no part of any line segment joining two points on the polygon goes outside the polygon.

Concave Polygon: Can have interior angles greater than 180 degrees.



- parts of some line segments joining two points on the polygon go outside the polygon.

Regular Polygon: All sides are equal and all interior angles are equal.



Angle Properties in Polygons

The sum of the exterior angles of a convex polygon is 360 degrees.

For a polygon with n sides, the sum of the interior angles, in degrees, is $180(n - 2)$

For a regular polygon with n sides, the measure of each interior angle is equal to: $\frac{180(n-2)}{n}$

For a regular polygon with n sides, the measure of each exterior angle is equal to: $\frac{360}{n}$

Example 1

$\boxed{180(n-2)}$

Calculate the sum of the interior angles of an octagon

8 sides

Sum of interior angles $= 180(n-2)$

$= 180(8-2)$

$= 180(6)$

$= 1080°$

Example 2

$$\frac{180(n-2)}{n}$$

Calculate the measure of each of the interior angles of a *regular* octagon.

interior angle $= \frac{180(n-2)}{n}$

$= \frac{180(8-2)}{8}$

$= \frac{1080}{8}$

$= 135°$

Example 3

$$\frac{360}{n}$$

Calculate the measure of each of the exterior angles of a *regular* octagon.

exterior angle $= \frac{360}{n}$

$= \frac{360}{8}$

$= 45°$

Example 4

How many sides does a polygon have if each of its interior angles measure 140 degrees?

interior angle $= \frac{180(n-2)}{n}$

$140 = \frac{180(n-2)}{n}$

$140n = 180(n-2)$

$140n = 180n - 360$

$360 = 180n - 140n$

$\frac{360}{40} = \frac{40n}{40}$

$9 = n$

The regular polygon has 9 sides.

Example 5

The measure of one of the exterior angles of a regular polygon is 30 degrees. How many sides does it have?

exterior angle $= \frac{360}{n}$

$30 = \frac{360}{n}$

$30n = 360$

$n = \frac{360}{30}$

$n = 12$

The regular polygon has 12 sides.

Example 6

6 sides

Five angles of a hexagon have measures 100°, 110°, 120°, 130°, and 140°. What is the measure of the sixth angle?

sum of interior angles $= 180(n-2)$

$= 180(6-2)$

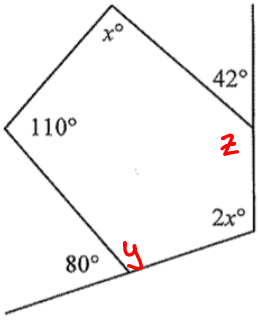
$= 720°$

6th angle $= 720 - 100 - 110 - 120 - 130 - 140$

$= 120°$

Example 7

Solve for x



$y = 180 - 80$ (supplementary)

$= 100°$

$z = 180 - 42$ (supplementary)

$= 138°$

sum of interior angles $= 180(n-2)$

$= 180(5-2)$

$= 540°$

$x + 110 + 100 + 2x + 138 = 540$

$3x = 540 - 110 - 100 - 138$

$3x = 192$

$x = \frac{192}{3}$

$x = 64°$

Complete the following chart and then complete the worksheet

Polygon	Number of Sides	Sum of Interior Angles	Sum of Exterior Angles
Triangle	3	$180(3-2) = 180^\circ$	360°
Quadrilateral	4	$180(4-2) = 360^\circ$	360°
Pentagon	5	$180(5-2) = 540^\circ$	360°
Hexagon	6	$180(6-2) = 720^\circ$	360°
Heptagon	7	$180(7-2) = 900^\circ$	360°
Octagon	8	$180(8-2) = 1080^\circ$	360°
Enneagon	9	$180(9-2) = 1260^\circ$	360°
Decagon	10	$180(10-2) = 1440^\circ$	360°