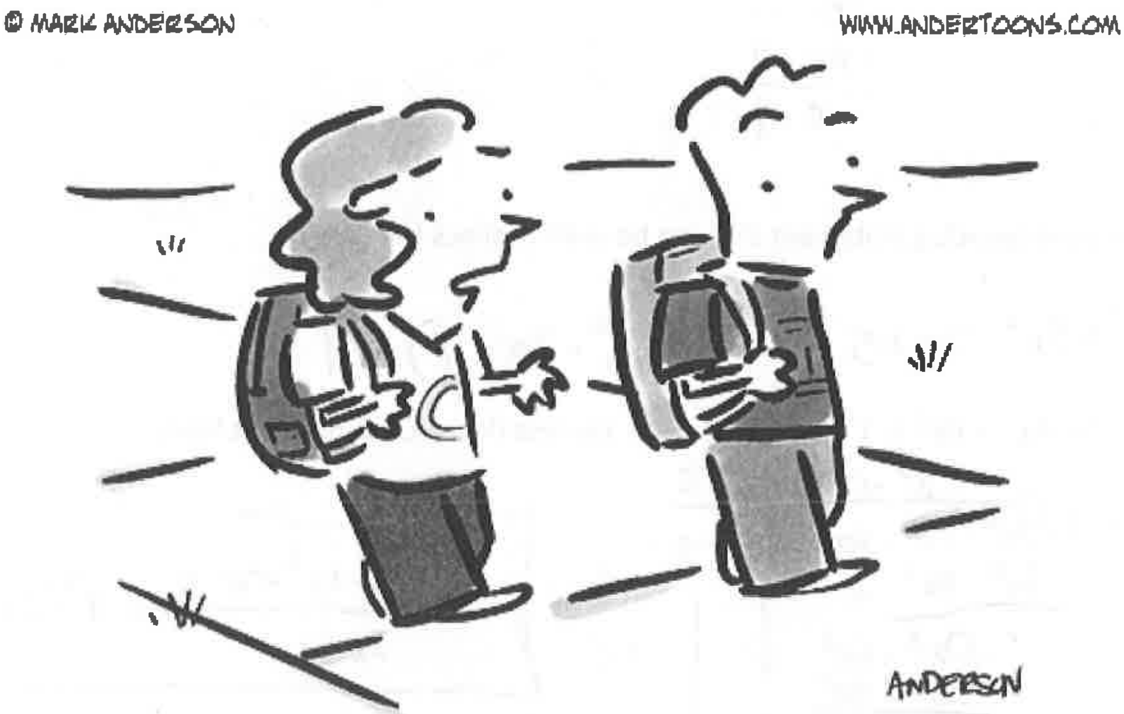
UNIT 2

Chapter 2- Factor Theorem and Inequalities

WORKBOOK

MHF4U



"I don't like long division; I always feel bad for the remainders."

W1 – 2.1 – Long Division of Polynomials and The Remainder Theorem
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1) Use the remainder theorem to determine the remainder when $2x^3 + 7x^2 - 8x + 3$ is divided by each binomial.

a) $x + 1$

$P(-1) = 2(-1)^3 + 7(-1)^2 - 8(-1) + 3$
$= 16$

b) $x - 2$

$P(2) = 2(2)^3 + 7(2)^2 - 8(2) + 3$
$= 31$

c) $x + 3$

$P(-3) = 2(-3)^3 + 7(-3)^2 - 8(-3) + 3$
$= 36$

2)a) Divide $x^3 + 3x^2 - 2x + 5$ by $x + 1$. Express the result in quotient form.

$$\begin{array}{r}
x^2 + 2x - 4 \\
x+1 \overline{\big) x^3 + 3x^2 - 2x + 5} \\
\underline{x^3 + x^2} \qquad\qquad \\
2x^2 - 2x \qquad \\
\underline{2x^2 + 2x} \qquad \\
-4x + 5 \\
\underline{-4x - 4} \\
R = 9
\end{array}$$

$$\boxed{\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}}$$

b) Write the corresponding statement that can be used to check the division.

$$x^3 + 3x^2 - 2x + 5 = (x+1)(x^2 + 2x - 4) + 9$$

3) Divide $3x^4 - 4x^3 - 6x^2 + 17x - 8$ by $3x - 4$. Express the result in quotient form.

$$\begin{array}{r}
x^3 + 0x^2 - 2x + 3 \\
3x-4 \overline{\big) 3x^4 - 4x^3 - 6x^2 + 17x - 8} \\
\underline{3x^4 - 4x^3} \qquad\qquad\qquad\qquad \\
0x^3 - 6x^2 \qquad\qquad \\
\underline{0x^3 - 0x^2} \qquad\qquad \\
-6x^2 + 17x \qquad \\
\underline{-6x^2 + 8x} \qquad \\
9x - 8 \\
\underline{9x - 12} \\
R = 4
\end{array}$$

$$\boxed{\frac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{3x - 4} = x^3 - 2x + 3 + \frac{4}{3x - 4}}$$

b) Write the corresponding statement that can be used to check the division.

$$3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x-4)(x^3 - 2x + 3) + 4$$

4) Perform each division. Express the result in quotient form.

a) $x^3 + 7x^2 - 3x + 4$ divided by $x + 2$

$$x^2+5x-13$$
$$x+2\,\overline{)\,x^3+7x^2-3x+4}$$
$$x^3+2x^2$$
$$5x^2-3x$$
$$5x^2+10x$$
$$-13x+4$$
$$-13x-26$$
$$R=30$$

$$\boxed{\frac{x^3+7x^2-3x+4}{x+2} = x^2+5x-13+\frac{30}{x+2}}$$

b) $6x^3 + x^2 - 14x - 6$ divided by $3x + 2$

$$2x^2-1x-4$$
$$3x+2\,\overline{)\,6x^3+x^2-14x-6}$$
$$6x^3+4x^2$$
$$-3x^2-14x$$
$$-3x^2-2x$$
$$-12x-6$$
$$-12x-8$$
$$R=2$$

$$\boxed{\frac{6x^3+x^2-14x-6}{3x+2} = 2x^2-x-4+\frac{2}{3x+2}}$$

c) $10x^3 + 11 - 9x^2 - 8x$ divided by $5x - 2$

$$2x^2-1x-2$$
$$5x-2\,\overline{)\,10x^3-9x^2-8x+11}$$
$$10x^3-4x^2$$
$$-5x^2-8x$$
$$-5x^2+2x$$
$$-10x+11$$
$$-10x+4$$
$$R=7$$

$$\boxed{\frac{10x^3-9x^2-8x+11}{5x-2} = 2x^2-x-2+\frac{7}{5x-2}}$$

d) $11x - 4x^4 - 7$ divided by $x - 3$

$$-4x^3-12x^2-36x-97$$
$$x-3\,\overline{)\,-4x^4+0x^3+0x^2+11x-7}$$
$$-4x^4+12x^3$$
$$-12x^3+0x^2$$
$$-12x^3+36x^2$$
$$-36x^2+11x$$
$$-36x^2+108x$$
$$-97x-7$$
$$-97x+291$$
$$R=-298$$

$$\boxed{\frac{-4x^4+11x-7}{x-3} = -4x^3-12x^2-36x-97-\frac{298}{x-3}}$$

e) $6x^3 + x^2 + 7x + 3$ divided by $3x + 2$

$$2x^2-1x+3$$
$$3x+2\,\overline{)\,6x^3+x^2+7x+3}$$
$$6x^3+4x^2$$
$$-3x^2+7x$$
$$-3x^2-2x$$
$$9x+3$$
$$9x+6$$
$$R=-3$$

$$\boxed{\frac{6x^3+x^2+7x+3}{3x+2} = 2x^2-1x+3-\frac{3}{3x+2}}$$

f) $8x^3 + 4x^2 - 31$ divided by $2x - 3$

$$4x^2+8x+12$$
$$2x-3\,\overline{)\,8x^3+4x^2+0x-31}$$
$$8x^3-12x^2$$
$$16x^2+0x$$
$$16x^2-24x$$
$$24x-31$$
$$24x-36$$
$$R=5$$

$$\boxed{\frac{8x^3+4x^2-31}{2x-3} = 4x^2+8x+12+\frac{5}{2x-3}}$$

g) $6x^2 - 6 + 8x^3$ divided by $4x - 3$

$$2x^2+3x+\tfrac{9}{4}$$
$$4x-3\,\overline{)\,8x^3+6x^2+0x-6}$$
$$8x^3-6x^2$$
$$12x^2+0x$$
$$12x^2-9x$$
$$9x-6$$
$$9x-\tfrac{27}{4}$$
$$R=\tfrac{3}{4}$$

$$\boxed{\frac{8x^3+6x^2-6}{4x-3} = 2x^2+3x+\frac{9}{4}+\frac{3}{4(4x-3)}}$$

5) The volume, in cubic cm, of a rectangular box can be modelled by the polynomial expression $2x^3 + 17x^2 + 38x + 15$. Determine possible dimensions of the box if the height, in cm, is given by $x + 5$.

$$\begin{array}{r|l} & 2x^2 + 7x + 3 \\ \hline x+5 & 2x^3 + 17x^2 + 38x + 15 \\ & 2x^3 + 10x^2 \\ \hline & 7x^2 + 38x \\ & 7x^2 + 35x \\ \hline & 3x + 15 \\ & 3x + 15 \\ \hline & R = 0 \end{array}$$

$2x^3 + 17x^2 + 38x + 15 = (x+5)(2x^2+7x+3)$

Factor

$= (x+5)(x+3)(2x+1)$

height legth width

P 6 S 7 $\frac{3}{1} = \frac{6}{2}$ $\frac{1}{2}$

6) Determine the value of k such that when $P(x) = kx^3 + 5x^2 - 2x + 3$ is divided by $x + 1$, the remainder is 7.

$P(-1) = k(-1)^3 + 5(-1)^2 - 2(-1) + 3$

$7 = -1k + 5 + 2 + 3$

$7 = -1k + 10$

$-3 = -1k$

$\boxed{k = 3}$

ANSWER KEY

1) a) 16 **b)** 31 **c)** 36

2)a) $\frac{x^3+3x^2-2x+5}{x+1} = x^2 + 2x - 4 + \frac{9}{x+1}$ **b)** $x^3 + 3x^2 - 2x + 5 = (x+1)(x^2+2x-4) + 9$

3)a) $\frac{3x^4-4x^3-6x^2+17x-8}{3x-4} = x^3 - 2x + 3 + \frac{4}{3x-4}$ **b)** $3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x-4)(x^3-2x+3) + 4$

4)a) $\frac{x^3+7x^2-3x+4}{x+2} = x^2 + 5x - 13 + \frac{30}{x+2}$ **b)** $\frac{6x^3+x^2-14x-6}{3x+2} = 2x^2 - x - 4 + \frac{2}{3x+2}$

c) $\frac{10x^3-9x^2-8x+11}{5x-2} = 2x^2 - x - 2 + \frac{7}{5x-2}$ **d)** $\frac{-4x^4+11x-7}{x-3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x-3}$

e) $\frac{6x^3+x^2+7x+3}{3x+2} = 2x^2 - x + 3 - \frac{3}{3x+2}$ **f)** $\frac{8x^3+4x^2-31}{2x-3} = 4x^2 + 8x + 12 + \frac{5}{2x-3}$

g) $\frac{6x^2-6+8x^3}{4x-3} = 2x^2 + 3x + \frac{9}{4} + \frac{3}{4(4x-3)}$

5) $2x^3 + 17x^2 + 38x + 15 = (x+5)(x+3)(2x+1)$

6) $k = 3$

W2 – 2.1 – Synthetic Division
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SOLUTIONS

1) Calculate each of the following using synthetic division. Express your answer using the statement that could be used to check the division.

a) $x^3 - 7x - 6$ divided by $x - 3$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & \downarrow & 3 & 9 & 6 \quad + \\ \hline \times & 1 & 3 & 2 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$\boxed{x^3 - 7x - 6 = (x-3)(x^2+3x+2)}$$

b) $2x^3 - 7x^2 - 7x + 19$ divided by $x - 1$

$$\begin{array}{r|rrrr} 1 & 2 & -7 & -7 & 19 \\ & \downarrow & 2 & -5 & -12 \quad + \\ \hline \times & 2 & -5 & -12 & 7 \\ & x^2 & x & \# & R \end{array}$$

$$\boxed{2x^3 - 7x^2 - 7x + 19 = (x-1)(2x^2-5x-12) + 7}$$

c) $6x^4 + 13x^3 - 34x^2 - 47x + 28$ divided by $x + 3$

$$\begin{array}{r|rrrrr} -3 & 6 & 13 & -34 & -47 & 28 \\ & \downarrow & -18 & 15 & 57 & -30 \quad + \\ \hline & 6 & -5 & -19 & 10 & -2 \\ & x^3 & x^2 & x & \# & R \end{array}$$

$$\boxed{\begin{aligned} &6x^4 + 13x^3 - 34x^2 - 47x + 28 \\ &= (x+3)(6x^3 - 5x^2 - 19x + 10) - 2 \end{aligned}}$$

d) $2x^3 + x^2 - 22x + 20$ divided by $2x - 3$

$= 2(x - \frac{3}{2})$

$$\begin{array}{r|rrrr} 3/2 & 2 & 1 & -22 & 20 \\ & \downarrow & 3 & 6 & -24 \quad + \\ \hline \times & 2 & 4 & -16 & -4 \\ & \multicolumn{3}{c}{\underbrace{\hphantom{2 \quad 4 \quad -16}}_{\div 2}} & R \\ & 1 & 2 & -8 & \\ & x^2 & x & \# & \end{array}$$

$$\boxed{\begin{aligned} &2x^3 + x^2 - 22x + 20 \\ &= (2x-3)(x^2+2x-8) - 4 \end{aligned}}$$

e) $12x^4 - 56x^3 + 59x^2 + 9x - 18$ divided by $2x + 1$

$= 2(x + \frac{1}{2})$

$$\begin{array}{r|rrrrr} -1/2 & 12 & -56 & 59 & 9 & -18 \\ & \downarrow & -6 & 31 & -45 & 18 \\ \hline & 12 & -62 & 90 & -36 & 0 \\ & \multicolumn{4}{c}{\underbrace{\hphantom{12 \quad -62 \quad 90 \quad -36}}_{\div 2}} & R \\ & 6 & -31 & 45 & -18 & \end{array}$$

$$\boxed{12x^4 - 56x^3 + 59x^2 + 9x - 18 = (2x+1)(6x^3 - 31x^2 + 45x - 18)}$$

f) $6x^3 - 15x^2 - 2x + 5$ divided by $2x - 5$ $= 2(x - \frac{5}{2})$

$$\begin{array}{r|rrrr} 5/2 & 6 & -15 & -2 & 5 \\ & \downarrow & 15 & 0 & -5 \quad + \\ \hline \times & 6 & 0 & -2 & 0 \\ & \multicolumn{3}{c}{\underbrace{\hphantom{6 \quad 0 \quad -2}}_{\div 2}} & R \\ & 3 & 0 & -1 & \\ & x^2 & x & \# & \end{array}$$

$$\boxed{6x^3 - 15x^2 - 2x + 5 = (2x-5)(3x^2-1)}$$

g) $x^3 - 2x + 1$ divided by $x - 4$

4	1	0	-2	1	
	↓	4	16	56	+
x	1	4	14	57	
	x^2	x	#	R	

$$\boxed{x^3-2x+1 = (x-4)(x^2+4x+14)+57}$$

h) $x^3 + 2x^2 - 6x + 1$ divided by $x + 2$

-2	1	2	-6	1	
	↓	-2	0	12	+
x	1	0	-6	13	
	x^2	x	#	R	

$$\boxed{x^3+2x^2-6x+1=(x+2)(x^2-6)+13}$$

2) Divide $x^4 - 16x^3 + 4x^2 + 10x - 11$ by each of the following binomials...

a) $x - 2$

2	1	-16	4	10	-11	
	↓	2	-28	-48	-76	+
x	1	-14	-24	-38	-87	
	x^3	x^2	x	#	R	

$$\boxed{\begin{aligned}&x^4-16x^3+4x^2+10x-11\\&=(x-2)(x^3-14x^2-24x-38)-87\end{aligned}}$$

b) $x + 4$

-4	1	-16	4	10	-11	
	↓	-4	80	-336	1304	+
x	1	-20	84	-326	1293	
	x^3	x^2	x	#	R	

$$\boxed{\begin{aligned}&x^4-16x^3+4x^2+10x-11\\&=(x+4)(x^3-20x^2+84x-326)+1293\end{aligned}}$$

3) Are either of the binomials in question #2 factors of $x^4 - 16x^3 + 4x^2 + 10x - 11$? Explain.

No, because there is a non-zero Remainder for each.

ANSWER KEY

1a) $x^3 - 7x - 6 = (x-3)(x^2+3x+2)$ **b)** $2x^3 - 7x^2 - 7x + 19 = (x-1)(2x^2 - 5x - 12) + 7$

c) $6x^4 + 13x^3 - 34x^2 - 47x + 28 = (x+3)(6x^3 - 5x^2 - 19x + 10) - 2$

d) $2x^3 + x^2 - 22x + 20 = (2x-3)(x^2+2x-8) - 4$

e) $12x^4 - 56x^3 + 59x^2 + 9x - 18 = (2x+1)(6x^3 - 31x^2 + 45x - 18)$ **f)** $6x^3 - 15x^2 - 2x + 5 = (2x-5)(3x^2-1)$

g) $x^3 - 2x + 1 = (x-4)(x^2+4x+14) + 57$ **h)** $x^3 + 2x^2 - 6x + 1 = (x+2)(x^2-6) + 13$

2)a) $x^4 - 16x^3 + 4x^2 + 10x - 11 = (x-2)(x^3 - 14x^2 - 24x - 38) - 87$
b) $x^4 - 16x^3 + 4x^2 + 10x - 11 = (x+4)(x^3 - 20x^2 + 84x - 326) + 1293$

3) No, because for each division problem, there is a remainder.

W3 – 2.2 – Factor Theorem
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1) Determine if $x+3$ is a factor of each polynomial:

a) x^3+x^2-x+6

$f(-3)=(-3)^3+(-3)^2-(-3)+6$
$=-27+9+3+6$
$=-9$

$\therefore$ Not a factor

b) $2x^3+9x^2+10x+3$

$f(-3)=2(-3)^3+9(-3)^2+10(-3)+3$
$=-54+81-30+3$
$=0$

$\therefore$ a factor

c) x^3+27

$f(-3)=(-3)^3+27$
$=0$

$\therefore$ a factor

2) Find possible factors of the following polynomials using integral zero theorem. Then, factor the polynomial.

a) x^3+3x^2-6x-8

Possible Factors: $\pm1, \pm2, \pm4, \pm8$
$f(-1)=0$; $\therefore$ $x+1$ is a factor

-1	1	3	-6	-8
	$\downarrow$	-1	-2	8
$\times$	1	2	-8	0
	x^2	x	#	R

$x^3+3x^2-6x-8=(x+1)(x^2+2x-8)$
$\boxed{=(x+1)(x+4)(x-2)}$

b) $x^3+4x^2-15x-18$

Possible factors: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$
$f(-1)=0$; $\therefore$ $x+1$ is a factor

-1	1	4	-15	-18
	$\downarrow$	-1	-3	18
$\times$	1	3	-18	0
	x^2	x	#	R

$x^3+4x^2-15x-18=(x+1)(x^2+3x-18)$
$\boxed{=(x+1)(x+6)(x-3)}$

c) $x^3-3x^2-10x+24$

Possible Factors: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$
$f(2)=0$; $\therefore$ $x-2$ is a factor

2	1	-3	-10	24
	$\downarrow$	2	-2	-24
$\times$	1	-1	-12	0
	x^2	x	#	R

$x^3-3x^2-10x+24=(x-2)(x^2-x-12)$
$\boxed{=(x-2)(x-4)(x+3)}$

3) Factor by grouping:

a) x^3+x^2-9x-9

$=(x^3+x^2)+(-9x-9)$
$=x^2(x+1)-9(x+1)$
$=(x+1)(x^2-9)$
$\boxed{=(x+1)(x-3)(x+3)}$

b) $2x^3-x^2-72x+36$

$=(2x^3-x^2)+(-72x+36)$
$=x^2(2x-1)-36(2x-1)$
$=(2x-1)(x^2-36)$
$\boxed{=(2x-1)(x-6)(x+6)}$

4) Determine a value of k so that x+2 is a factor of $x^3 - 2kx^2 + 6x - 4$.

$f(-2) = (-2)^3 - 2(k)(-2)^2 + 6(-2) - 4$

$0 = -8 - 8k - 12 - 4$

$0 = -24 - 8k$

$24 = -8k$

$\boxed{k = -3}$

5) Find possible factors of the following polynomials using integral zero theorem. Then, factor the polynomial.

a) $3x^3 + x^2 - 22x - 24$

Possible factors: $\pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}, \pm 3, \pm 4, \pm\frac{4}{3}, \pm 6, \pm 8, \pm\frac{8}{3}, \pm 12, \pm 24$

$f(-2) = 0$; ∴ $x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 3 & 1 & -22 & -24 \\ & \downarrow & -6 & 10 & 24 \\ \hline \times & 3 & -5 & -12 & 0 \\ & x^2 & x & \# & R \end{array}$$

P: -36, S: -5, $\frac{-3}{1} = \frac{-9}{3}$, $\frac{4}{3}$

$3x^3 + x^2 - 22x - 24 = (x+2)(3x^2 - 5x - 12)$

$\boxed{= (x+2)(x-3)(3x+4)}$

b) $2x^3 - 9x^2 + 10x - 3$

Possible factors: $\pm 1, \pm\frac{1}{2}, \pm 3, \pm\frac{3}{2}$

$f(1) = 0$; ∴ $x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 10 & -3 \\ & \downarrow & 2 & -7 & 3 \\ \hline \times & 2 & -7 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

P: 6, S: -7, $\frac{-3}{1} = \frac{-6}{2}$, $\frac{-1}{2}$

$2x^3 - 9x^2 + 10x - 3 = (x-1)(2x^2 - 7x + 3)$

$\boxed{= (x-1)(x-3)(2x-1)}$

c) $6x^3 - 11x^2 - 26x + 15$

Possible factors: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 3, \pm\frac{3}{2}, \pm 5, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \ldots$

$f(3) = 0$; ∴ $x-3$ is a factor

$$\begin{array}{r|rrrr} 3 & 6 & -11 & -26 & 15 \\ & \downarrow & 18 & 21 & -15 \\ \hline \times & 6 & 7 & -5 & 0 \\ & x^2 & x & \# & R \end{array}$$

P: -30, S: 7, $\frac{5}{3} = \frac{10}{6}$, $\frac{-3}{6} = \frac{-1}{2}$

$6x^3 - 11x^2 - 26x + 15 = (x-3)(6x^2 + 7x - 5)$

$\boxed{= (x-3)(3x+5)(2x-1)}$

d) $4x^3 + 3x^2 - 4x - 3$

Possible factors: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}$

$f(1) = 0$, ∴ $x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 4 & 3 & -4 & -3 \\ & \downarrow & 4 & 7 & 3 \\ \hline \times & 4 & 7 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

P: 12, S: 7, $\frac{1}{1} = \frac{4}{4}$, $\frac{3}{4}$

$4x^3 + 3x^2 - 4x - 3 = (x-1)(4x^2 + 7x + 3)$

$\boxed{= (x-1)(x+1)(4x+3)}$

6) Factor each polynomial

a) $2x^3 + 5x^2 - x - 6$

Possible Factors: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

$f(1) = 0$; $\therefore$ $x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -1 & -6 \\ & \downarrow & 2 & 7 & 6 \\ \hline \times & 2 & 7 & 6 & 0 \\ & x^2 & x & \# & R \end{array}$$

P: 12, S: 7

$\frac{2}{1} = \frac{4}{2}$, $\frac{3}{2}$

$2x^3 + 5x^2 - x - 6 = (x-1)(2x^2 + 7x + 6)$

$\boxed{= (x-1)(x+2)(2x+3)}$

b) $4x^3 - 7x - 3$

Possible Factors: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$

$f(-1) = 0$; $\therefore$ $x+1$ is a factor

$$\begin{array}{r|rrrr} -1 & 4 & 0 & -7 & -3 \\ & \downarrow & -4 & 4 & 3 \\ \hline \times & 4 & -4 & -3 & 0 \\ & x^2 & x & \# & R \end{array}$$

P: -12, S: -4

$\frac{-3}{2} = \frac{-6}{4}$, $\frac{2}{4} = \frac{1}{2}$

$4x^3 - 7x - 3 = (x+1)(4x^2 - 4x - 3)$

$\boxed{= (x+1)(2x-3)(2x+1)}$

c) $x^4 - 15x^2 - 10x + 24$

Possible factors: $\pm 1, \pm 2, \pm 4$

$f(1) = 0$; $\therefore$ $x-1$ is a Factor

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -15 & -10 & 24 \\ & \downarrow & 1 & 1 & -14 & -24 \\ \hline \times & 1 & 1 & -14 & -24 & 0 \\ & x^3 & x^2 & x & \# & R \end{array}$$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$f(-2) = 0$; $\therefore$ $x+2$ is a Factor

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & \downarrow & -2 & 2 & 24 \\ \hline \times & 1 & -1 & -12 & 0 \\ & x^2 & x & \# & R \end{array}$$

$x^4 - 15x^2 - 10x + 4 = (x-1)(x^3 + x^2 - 14x - 24)$

$= (x-1)(x+2)(x^2 - x - 12)$

$= (x-1)(x+2)(x-4)(x+3)$

ANSWER KEY

1)a) No **b)** Yes **c)** Yes

2)a) $(x-2)(x+1)(x+4)$ **b)** $(x-3)(x+1)(x+6)$ **c)** $(x-4)(x-2)(x+3)$

3)a) $(x-3)(x+1)(x+3)$ **b)** $(x-6)(x+6)(2x-1)$

4) $k=-3$

5)a) $(x-3)(x+2)(3x+4)$ **b)** $(x-3)(x-1)(2x-1)$ **c)** $(x-3)(2x-1)(3x+5)$ **d)** $(x-1)(x+1)(4x+3)$

6)a) $(x-1)(x+2)(2x+3)$ **b)** $(x+1)(2x-3)(2x+1)$ **c)** $(x-4)(x-1)(x+2)(x+3)$

1) Determine the solutions of the following polynomials.

a) $(3x+2)(x+9)(x-2)=0$

$3x+2=0$ $x+9=0$ $x-2=0$

$x_1=-\frac{2}{3}$ $x_2=-9$ $x_3=2$

$(-\frac{2}{3},0)$, $(-9,0)$, $(2,0)$

b) $(x^2+1)(x-4)=0$

$x^2+1=0$ $x-4=0$

$x^2=-1$ $x=4$

$\therefore$ No solutions

$(4,0)$

2) Determine the solutions of the following polynomials by factoring. Use the tools you have learned this unit to help you. (remainder theorem, integral zero theorem, division etc.)

a) $x^3-4x^2-3x+18=0$

Possible Factors: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

$f(-2)=0$; $\therefore x+2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -3 & 18 \\ & \downarrow & -2 & 12 & -18 \\ \hline \times & 1 & -6 & 9 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x+2)(x^2-6x+9)=0$

$(x+2)(x-3)^2=0$

$x+2=0$ $x-3=0$

$\boxed{x_1=-2}$ $\boxed{x_2=3}$

$\boxed{\text{Solutions: } (-2,0) \text{ and } (3,0)}$

b) $x^3-3x^2-4x+12=0$

Possible Factors: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$f(2)=0$; $\therefore x-2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & \downarrow & 2 & -2 & -12 \\ \hline \times & 1 & -1 & -6 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x-2)(x^2-x-6)=0$

$(x-2)(x-3)(x+2)=0$

$x-2=0$ $x-3=0$ $x+2=0$

$\boxed{x_1=2}$ $\boxed{x_2=3}$ $\boxed{x_3=-2}$

$\boxed{\text{Solutions: } (2,0), (3,0), \text{ and } (-2,0)}$

c) $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

Possible Factors: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

$f(-1) = 0$; ∴ $x+1$ is a factor

-1	1	-1	-11	9	18	
	↓	-1	2	9	-18	+
x	1	-2	-9	18	0	
	x^3	x^2	x	#	R	

$(x+1)(x^3 - 2x^2 - 9x + 18) = 0$

$(x+1)[x^2(x-2) - 9(x-2)] = 0$

$(x+1)(x-2)(x^2-9) = 0$

$(x+1)(x-2)(x-3)(x+3) = 0$

$\boxed{x_1 = -1 \quad x_2 = 2 \quad x_3 = 3 \quad x_4 = -3}$

Solutions:
(-1,0), (2,0), (3,0), and (-3,0)

d) $x^3 - 64 = 0$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(x-4)(x^2 + 4x + 16) = 0$

↓ $\boxed{x = 4}$

↓ check $b^2 - 4ac = (4)^2 - 4(1)(16)$
$= -48$
No solutions

Solution: (4,0)

e) $2x^3 - 7x^2 + 10x - 5 = 0$

Possible Factors: $\pm1, \pm\frac{1}{2}, \pm5, \pm\frac{5}{2}$

$f(1) = 0$; ∴ $x-1$ is a factor

1	2	-7	10	-5	
	↓	2	-5	5	+
x	2	-5	5	0	
	x^2	x	#	R	

$(x-1)(2x^2 - 5x + 5) = 0$

↓ $x - 1 = 0$

$\boxed{x = 1}$

↓ check $b^2 - 4ac = (-5)^2 - 4(2)(5)$
$= -15$
∴ No solutions

Solution: (1,0)

3) Solve each equation by first factoring the sum or difference of cubes.

a) $x^3 - 8 = 0$

$(x-2)(x^2+2x+4)=0$

$x-2=0$

$\boxed{x=2}$

Check $b^2-4ac=(2)^2-4(1)(4)$
$= -12$
$\therefore$ No solution

$\boxed{\text{Solution: } (2,0)}$

b) $x^3 + 27 = 0$

$(x+3)(x^2-3x+9)=0$

$x+3=0$

$\boxed{x=-3}$

Check $b^2-4ac=(-3)^2-4(1)(9)$
$= -27$
$\therefore$ No solution

$\boxed{\text{Solution: } (-3,0)}$

4) Solve by factoring

$x^3 - 4x^2 - 7x + 10 = 0$

Possible Factors: $\pm1, \pm2, \pm5, \pm10$

$f(1)=0$; $\therefore$ $x-1$ is a factor

$$\begin{array}{c|cccc} 1 & 1 & -4 & -7 & 10 \\ & \downarrow & 1 & -3 & -10 \quad + \\ \hline \times & 1 & -3 & -10 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x-1)(x^2-3x-10)=0$

$(x-1)(x-5)(x+2)=0$

$x-1=0$ $\quad$ $x-5=0$ $\quad$ $x+2=0$

$\boxed{x_1=1}$ $\quad$ $\boxed{x_2=5}$ $\quad$ $\boxed{x_3=-2}$

$\boxed{\text{Solutions: } (1,0), (5,0), \text{and } (-2,0)}$

b) $2x^3 - 11x^2 + 12x + 9 = 0$

Possible Factors: $\pm1, \pm\frac{1}{2}, \pm3, \pm\frac{3}{2}, \pm9, \pm\frac{9}{2}$

$f(3)=0$; $\therefore$ $x-3$ is a factor

$$\begin{array}{c|cccc} 3 & 2 & -11 & 12 & 9 \\ & \downarrow & 6 & -15 & -9 \quad + \\ \hline \times & 2 & -5 & -3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x-3)(2x^2-5x-3)=0$

$(x-3)(x-3)(2x+1)=0$

$(x-3)^2(2x+1)=0$

$x-3=0$ $\quad$ $2x+1=0$

$\boxed{x_1=3}$ $\quad$ $\boxed{x_2=-\frac{1}{2}}$

$\boxed{\text{Solutions: } (3,0) \text{ and } (-\tfrac{1}{2},0)}$

c) $x^4 - x^3 - 2x - 4 = 0$

Possible factors: $\pm 1, \pm 2, \pm 4$

$f(-1) = 0$; $x+1$ is a factor

$$\begin{array}{r|rrrrrl}
-1 & 1 & -1 & 0 & -2 & -4 & \\
 & \downarrow & -1 & 2 & -2 & 4 & + \\
\hline
x & 1 & -2 & 2 & -4 & 0 & \\
 & x^3 & x^2 & x & \# & R &
\end{array}$$

$(x+1)(x^3 - 2x^2 + 2x - 4) = 0$

$(x+1)[x^2(x-2) + 2(x-2)] = 0$

$(x+1)(x-2)(x^2+2) = 0$

$x+1=0$ $\quad$ $x-2=0$ $\quad$ $x^2+2=0$

$\boxed{x_1 = -1}$ $\quad$ $\boxed{x_2 = 2}$ $\quad$ No solution

$\boxed{\text{Solutions: } (-1,0) \text{ and } (2,0)}$

ANSWER KEY

1a) $\left(-\frac{2}{3}, 0\right), (-9,0), (2,0)$ **b)** (4,0)

2a) (-2,0) and (3,0) **b)** (3,0), (-2,0), (2,0) **c)** (-1,0), (2,0), (-3,0), (3,0) **d)** (4,0) **e)** (1,0)

3)a) (2, 0) **b)** (-3, 0)

4)a) (5, 0), (-2, 0), (1, 0) **b)** (-0.5, 0) and (3, 0) **c)** (-1, 0) and (2, 0)

W5 – 2.4 – Families of Polynomial Functions
MHF4U
Jensen

The zeros of a quadratic function are -7 and -3.

a) Determine an equation for the family of quadratic functions with these zeros.

$y = k(x+7)(x+3)$

b) Write equations for two functions that belong to this family.

$y = 87(x+7)(x+3)$
$y = 71(x+7)(x+3)$

c) Determine an equation for the member of the family that passes through the point (2, 18).

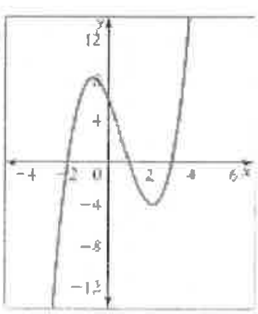
$18 = k(2+7)(2+3)$
$18 = 45k$
$k = \frac{2}{5}$

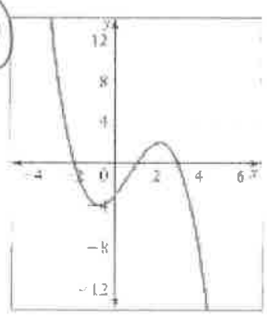
$y = \frac{2}{5}(x+7)(x+3)$

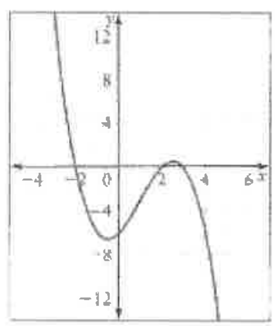
2) Examine the following functions. Which function does not belong to the same family?

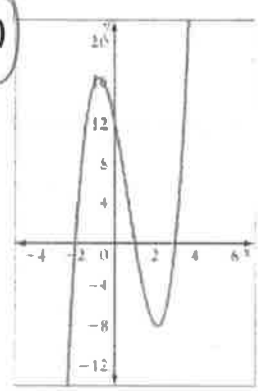
a) $y = 1.5(x+4)(x-5)(x-2)$
b) $y = -1.5(x-2)(x-5)(x+4)$
c) $y = 1.5(x-2)(x+4)(x-2)$
d) $y = 3(x-5)(x-2)(x+4)$

3) The graphs of four polynomial functions are given. Which graphs represent functions that belong to the same family?

A)


B)


C)


D)


4)a) Determine an equation for the family of cubic functions with zeros -2, -1, and $\frac{1}{2}$

$y = k(x+2)(x+1)(2x-1)$

b) Write equations for two functions that belong to this family.

$y = 66(x+2)(x+1)(2x-1)$

$y = 68(x+2)(x+1)(2x-1)$

c) Determine an equation for the member of the family whose graph has a y-intercept of 6.

$6 = k(0+2)(0+1)[2(0)-1]$

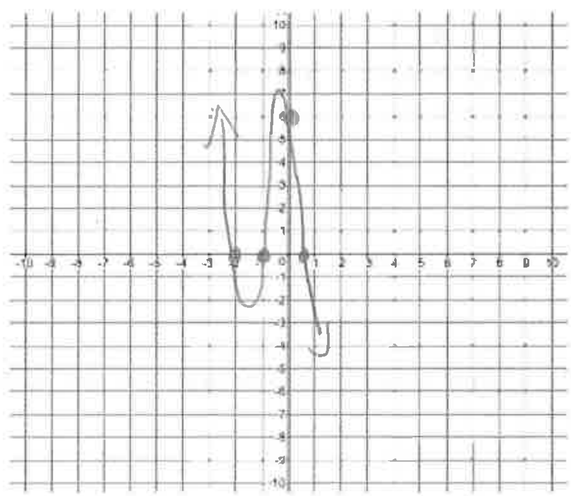
$6 = k(2)(1)(-1)$

$6 = -2k$

$k = -3$

$y = -3(x+2)(x+1)(2x-1)$

d) Sketch a graph of the function from part c).



5)a) Determine an equation for the family of cubic functions with zeros $1 \pm \sqrt{2}$ and $-\frac{1}{2}$

factors: $x = 1 \pm \sqrt{2}$

$x - 1 = \pm\sqrt{2}$

$(x-1)^2 = 2$

$x^2 - 2x + 1 = 2$

$x^2 - 2x - 1 = 0$

$(x^2 - 2x - 1)$

$x = \frac{-1}{2}$

$2x = -1$

$2x + 1 = 0$

$(2x+1)$

$\boxed{y = k(x^2 - 2x - 1)(2x+1)}$

b) Determine an equation for the member of the family whose graph passes through the point (3, 35).

$35 = k[(3)^2 - 2(3) - 1][2(3)+1]$

$35 = k(2)(7)$

$35 = 14k$

$\frac{35}{14} = k$

$k = \frac{5}{2}$

$\boxed{y = \frac{5}{2}(x^2 - 2x - 1)(2x+1)}$

6)a) Determine an equation for the family of quartic functions with zeros 3 (order 2) and $-4 \pm \sqrt{3}$.

Factors: $x = 3$

$x - 3 = 0$

$(x-3)^2$

$x = -4 \pm \sqrt{3}$

$x + 4 = \pm\sqrt{3}$

$(x+4)^2 = 3$

$x^2 + 8x + 16 = 3$

$x^2 + 8x + 13 = 0$

$(x^2 + 8x + 13)$

$\boxed{y = k(x-3)^2(x^2+8x+13)}$

b) Determine an equation for the member of the family whose graph passes through the point (1, -22).

$-22 = k(1-3)^2[(1)^2 + 8(1) + 13]$

$-22 = k(4)(22)$

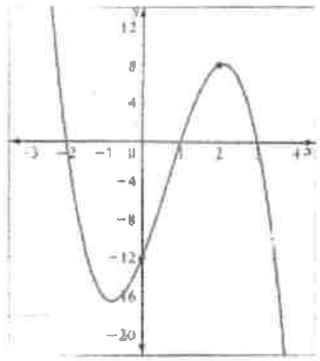
$-1 = 4k$

$k = \frac{-1}{4}$

$\boxed{y = \frac{-1}{4}(x-3)^2(x^2 + 8x + 13)}$

7) Determine an equation for each of the following functions

a)



$y = k(x+2)(x-1)(x-3)$

$-12 = k(0+2)(0-1)(0-3)$

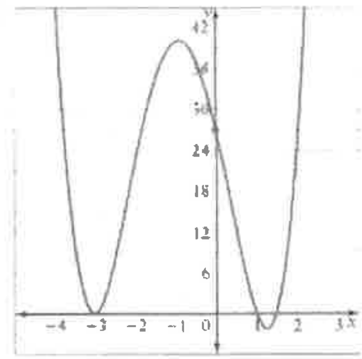
$-12 = k(2)(-1)(-3)$

$-12 = 6k$

$k = -2$

$\boxed{y = -2(x+2)(x-1)(x-3)}$

b)



$y = k(x+3)^2(x-1)(2x-3)$

$27 = k(0+3)^2(0-1)[2(0)-3]$

$27 = k(9)(-1)(-3)$

$27 = 27k$

$k = 1$

$\boxed{y = (x+3)^2(x-1)(2x-3)}$

ANSWER KEY

1)a) $y = k(x+7)(x+3)$ **b)** answer will vary **c)** $y = \frac{2}{5}(x+7)(x+3)$ **2)** C **3)** A, B, D

4)a) $y = k(x+2)(x+1)(2x-1)$ **b)** answer wil vary **c)** $y = -3(x+2)(x+1)(2x-1)$ **d)** see posted

5)a) $y = k(x^2-2x-1)(2x+1)$ **b)** $y = \frac{5}{2}(x^2-2x-1)(2x+1)$

6)a) $y = k(x-3)^2(x^2+8x+13)$ **b)** $y = -\frac{1}{4}(x-3)^2(x^2+8x+13)$

7)a) $y = -2(x+2)(x-1)(x-3)$ **b)** $y = (x+3)^2(x-1)(2x-3)$

W6 – 2.5 – Solving Inequalities
MHF4U
Jensen

1) Solve each linear inequality

a) $x + 3 \leq 5$

$x \leq 2$

b) $7x < 4 + 3x$

$4x < 4$

$x < 1$

2) Solve each inequality by graphing

a) $(x + 3)(x - 2) > 0$

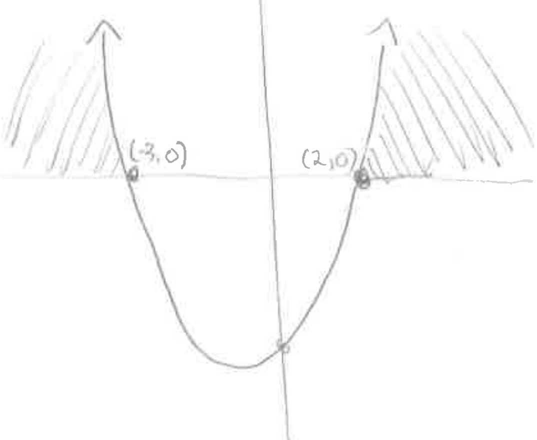
Factor Table

	$-\infty$ to -3	-3 to 2	2 to ∞
	-4	0	3
$x+3$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
overall	$+$	$-$	$+$

Solution: $x < -3$ or $x > 2$

$x \in (-\infty, -3) \cup (2, \infty)$

Graph: Degree 2; Positive L.C.



b) $(x + 2)(3 - x)(x + 1) < 0$

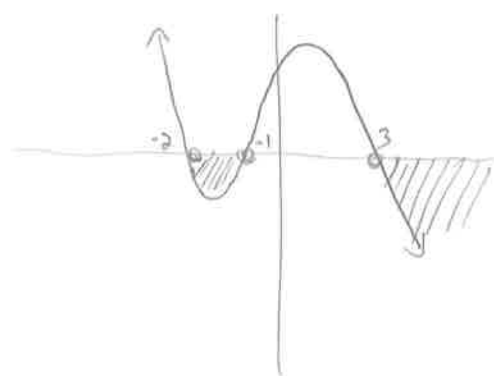
Factor Table:

	$-\infty$ to -2	-2 to -1	-1 to 3	3 to ∞
	-3	-1.5	0	4
$x+2$	$-$	$+$	$+$	$+$
$3-x$	$+$	$+$	$+$	$-$
$x+1$	$-$	$-$	$+$	$+$
overall	$+$	$-$	$+$	$-$

Solution: $-2 < x < -1$ or $x > 3$

$x \in (-2, -1) \cup (3, \infty)$

Graph: Degree 3; Negative L.C.



3) Solve each of the following polynomial inequalities

a) $x^2 - 7x + 10 \geq 0$

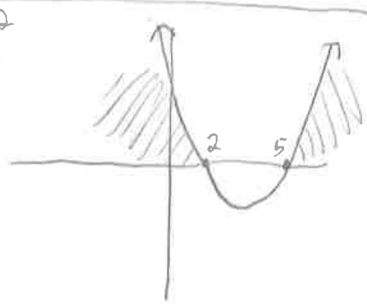
$(x-2)(x-5) \geq 0$

	$-\infty$ 0	2 3	5 6 ∞
$x-2$	$-$	$+$	$+$
$x-5$	$-$	$-$	$+$
overall	$+$	$-$	$+$

Solution: $x \leq 2$ or $x \geq 5$

$x \in (-\infty, 2] \cup [5, \infty)$

Degree 2
+ L.C.



b) $x^3 + 6x^2 - 16x > 0$

$x(x^2 + 6x - 16) > 0$

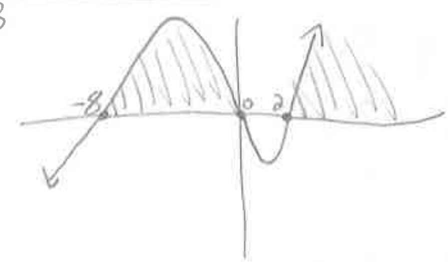
$x(x+8)(x-2) > 0$

	$-\infty$ -10	-8 -1	0 1	2 3 ∞
x	$-$	$-$	$+$	$+$
$x+8$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
overall	$-$	$+$	$-$	$+$

Solution: $-8 < x < 0$ or $x > 2$

$x \in (-8, 0) \cup (2, \infty)$

Degree 3
+ L.C.



c) $-x^2 + 36 \geq 0$

$-1(x^2 - 36) \geq 0$

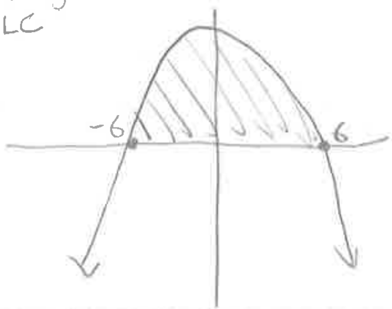
$-1(x-6)(x+6) \geq 0$

	$-\infty$ -7	-6 0	6 7 ∞
-1	$-$	$-$	$-$
$x-6$	$-$	$-$	$+$
$x+6$	$-$	$+$	$+$
overall	$-$	$+$	$-$

Solution: $-6 \leq x \leq 6$

$x \in [-6, 6]$

Even deg.
- LC



d) $x^4 - 26x^2 + 25 > 0$

$(x^2 - 25)(x^2 - 1) > 0$

$(x-5)(x+5)(x-1)(x+1) > 0$

	$-\infty$ -6	-5 -2	-1 0	1 2	5 6 ∞
$x-5$	$-$	$-$	$-$	$-$	$+$
$x+5$	$-$	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$	$+$
overall	$+$	$-$	$+$	$-$	$+$

Solution: $x < -5$ or $-1 < x < 1$ or $x > 5$

$x \in (-\infty, -5) \cup (-1, 1) \cup (5, \infty)$

e) $x^3 - 3x^2 \geq 25x - 75$

$x^3 - 3x^2 - 25x + 75 \geq 0$

$[x^2(x-3) - 25(x-3)] \geq 0$

$(x-3)(x^2-25) \geq 0$

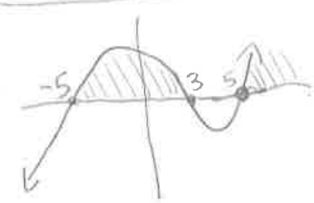
$(x-3)(x-5)(x+5) \geq 0$

	$-\infty$ to -5: -6	-5 to 3: 0	3 to 5: 4	5 to ∞: 6
$x-3$	−	−	+	+
$x-5$	−	−	−	+
$x+5$	−	+	+	+
overall	−	+	−	+

Solution: $-5 \leq x \leq 3$ or $x \geq 5$

$x \in [-5, 3] \cup [5, \infty)$

odd deg.
+ LC



f) $-x^3 + 28x + 48 \geq 0$

$f(-2) = 0$; ∴ $x+2$ is a factor

-2	-1	0	28	48
	↓	2	-4	-48
x	-1	2	24	0
	x^2	x	#	R

$(x+2)(-x^2+2x+24) \geq 0$

$-1(x+2)(x^2-2x-24) \geq 0$

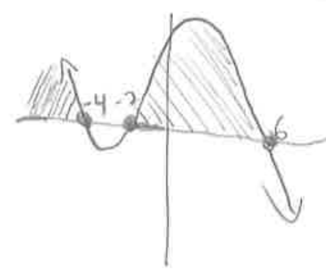
$-1(x+2)(x-6)(x+4) \geq 0$

	$-\infty$ to -4: -5	-4 to -2: -3	-2 to 6: 0	6 to ∞: 7
-1	−	−	−	−
$x+2$	−	−	+	+
$x-6$	−	−	−	+
$x+4$	−	+	+	+
overall	+	−	+	−

Solution: $x \leq -4$ or $-2 \leq x \leq 6$

$x \in (-\infty, -4] \cup [-2, 6]$

odd Deg
− LC



g) $x^3 - 2x^2 - 5x + 6 < 0$

$f(1) = 0$; ∴ $x-1$ is a factor

1	1	-2	-5	6
	↓	1	-1	-6
x	1	-1	-6	0
	x^2	x	#	R

$(x-1)(x^2-x-6) < 0$

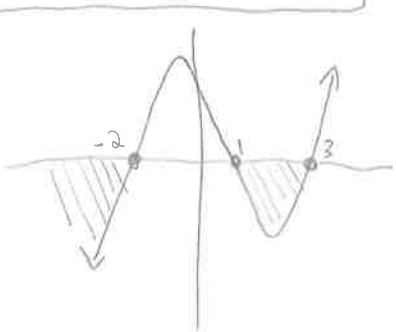
$(x-1)(x-3)(x+2) < 0$

	$-\infty$ to -2: -3	-2 to 1: 0	1 to 3: 2	3 to ∞: 4
$x-1$	−	−	+	+
$x-3$	−	−	−	+
$x+2$	−	+	+	+
overall	−	+	−	+

Solution: $x < -2$ or $1 < x < 3$

$x \in (-\infty, -2) \cup (1, 3)$

odd deg
+ LC



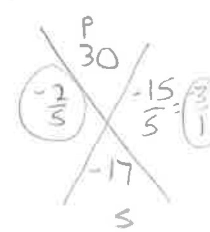
h) $5x^3 - 12x^2 - 11x + 6 \leq 0$

$f(-1) = 0$; $x+1$ is a factor

-1	5	-12	-11	6
	↓	-5	17	-6
x	5	-17	6	0
	x^2	x	#	R

$(x+1)(5x^2 - 17x + 6) \leq 0$

$(x+1)(5x-2)(x-3) \leq 0$

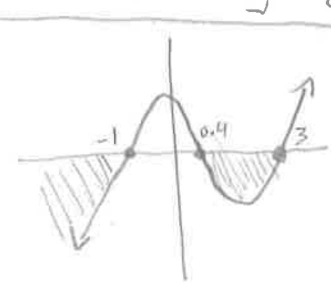


	$-\infty$ to -1: -2	-1 to $2/5$: 0	$2/5$ to 3: 1	3 to ∞: 4
$x+1$	−	+	+	+
$5x-2$	−	−	+	+
$x-3$	−	−	−	+
overall	−	+	−	+

Solution: $x \leq -1$ or $2/5 \leq x \leq 3$

$x \in (-\infty, -1] \cup [2/5, 3]$

Odd deg.
+ LC

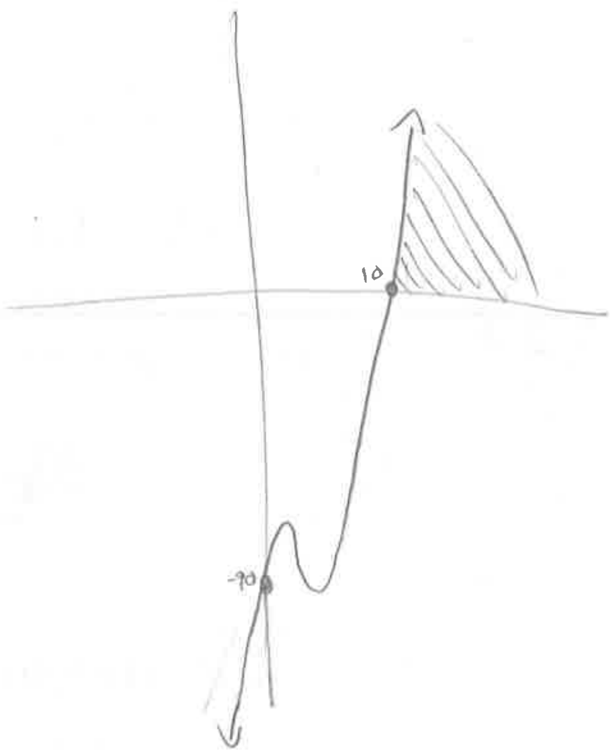


4) The price, p, in dollars, of a stock t years after 1999 can be modelled by the function $p(t) = 0.5t^3 - 5.5t^2 + 14t$. When will the stock be more than \$90? You may use technology to help you determine the solution.

$0.5t^3 - 5.5t^2 + 14t > 90$

$0.5t^3 - 5.5t^2 + 14t - 90 > 0$

using Desmos:



Solution: $t > 10$

$t \in (10, \infty)$

$\therefore$ The stock will be more than \$90 after 10 years.

ANSWER KEY

1)a) $x \leq 2$ **b)** $x < 1$
2)a) $x < -3$ or $x > 2$ **b)** $-2 < x < -1$ or $x > 3$
3)a) $x \leq 2$ or $x \geq 5$ **b)** $-8 < x < 0$ or $x > 2$ **c)** $-6 \leq x \leq 6$ **d)** $x < -5$ or $-1 < x < 1$ or $x > 5$
e) $-5 \leq x \leq 3$ or $x \geq 5$ **f)** $x \leq -4$ or $-2 \leq x \leq 6$ **g)** $x < -2$ or $1 < x < 3$
h) $x \leq -1$ or $\frac{2}{5} < x < 3$
4) after 10 years (2009)