UNIT 3

Chapter 6/7- Logarithmic and Exponential Functions

WORKBOOK

SOLUTION
MANUAL

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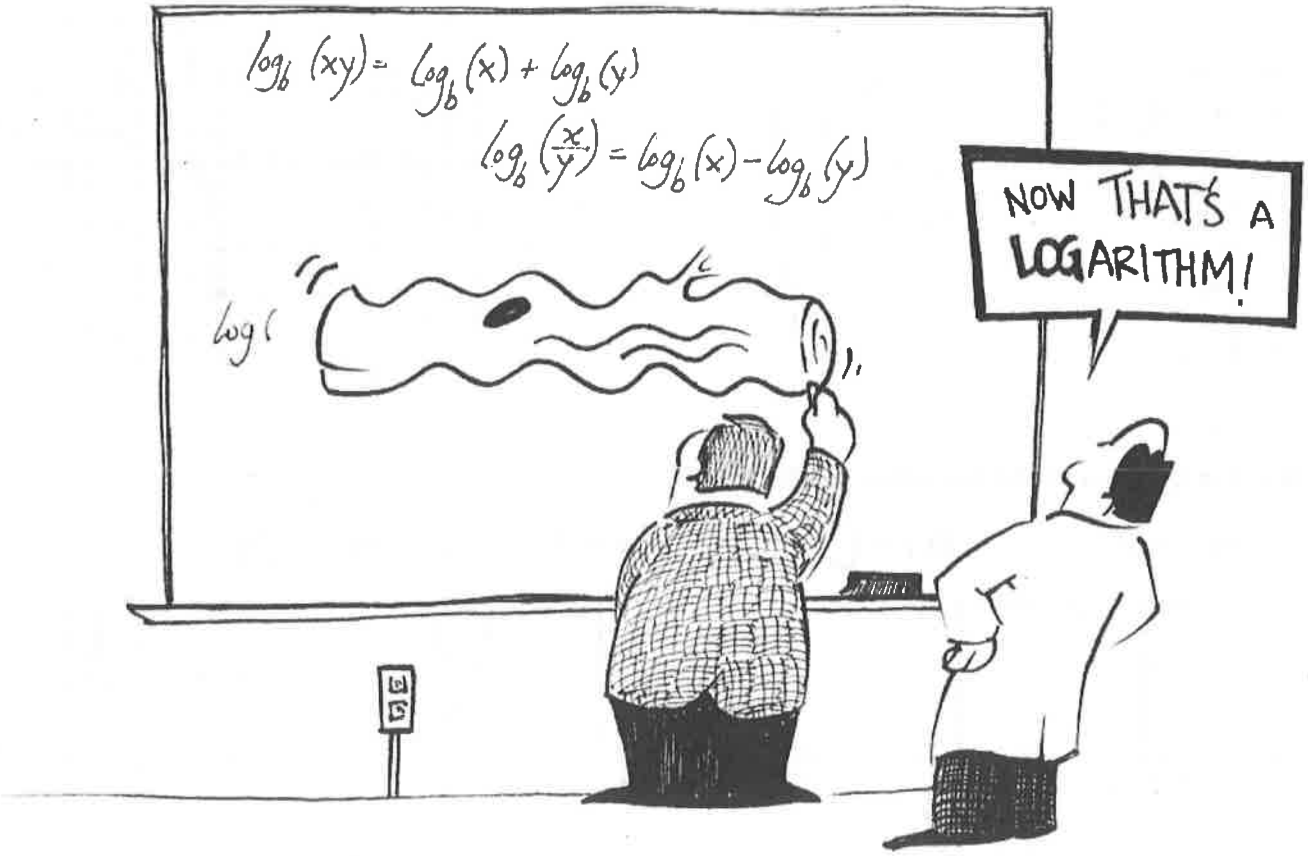

W1 – 6.1/6.2 – Intro to Logarithms and Review of Exponentials

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SOLUTIONS

1) Sketch a graph of each function. Then, sketch a graph of the inverse of each function. Label each graph with its equation.

a) $y = 2^x$

$f(x) = 2^x$

x	y
-2	0.25
-1	0.5
0	1
1	2
2	4

Find eq^n of $f^{-1}(x)$

$x = 2^y$

$\log x = \log 2^y$

$\log x = y \log 2$

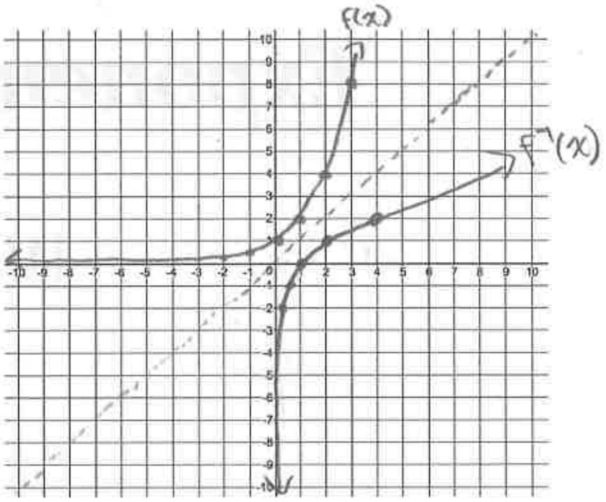
$\frac{\log x}{\log 2} = y$

$y = \log_2 x$

$f^{-1}(x) = \log_2 x$

$f^{-1}(x) = \log_2 x$

x	y
0.25	-2
0.5	-1
1	0
2	1
4	2



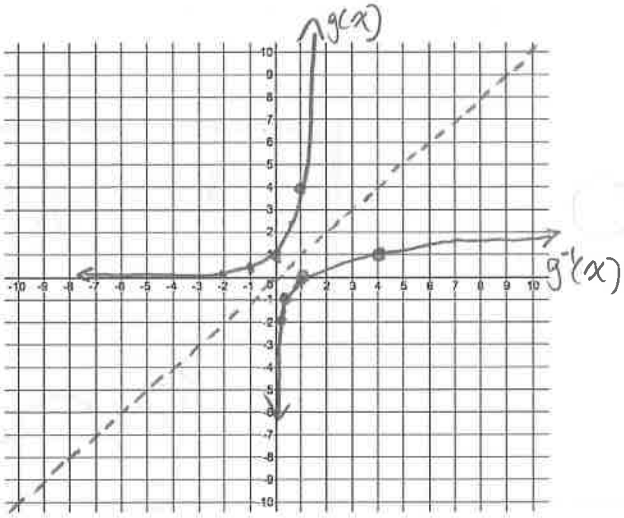
b) $y = 4^x$

$g(x) = 4^x$

x	y
-2	0.0625
-1	0.25
0	1
1	4
2	16

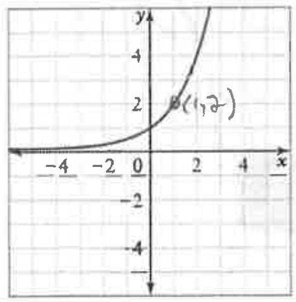
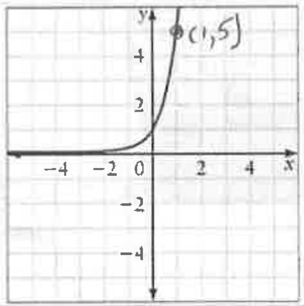
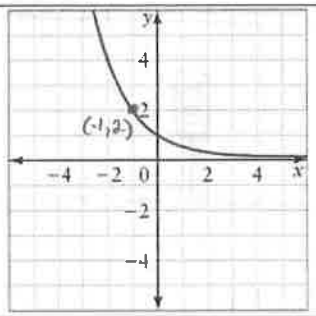
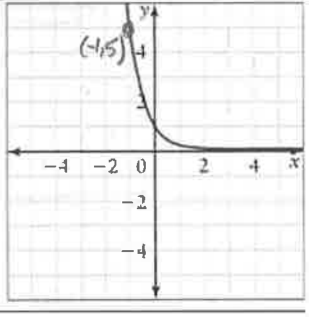
$g^{-1}(x) = \log_4 x$

x	y
0.0625	-2
0.25	-1
1	0
4	1
16	2



2) Match each equation to its corresponding graph.

A) $y = 5^x$ **B)** $y = \left(\frac{1}{2}\right)^x$ **C)** $y = 2^x$ **D)** $y = \left(\frac{1}{5}\right)^x$

			
C	A	B	D

3) An influenza virus is spreading according to the function $N = 10(2)^t$, where N is the number of people infected and t is the time, in days.

How many people have the virus at each time?

i) initially, when $t = 0$

$N = 10$

ii) after 1 day

$N = 10(2)^1$

$= 20$

iii) after 2 days

$N = 10(2)^2$

$= 40$

iv) after 3 days

$N = 10(2)^3$

$= 80$

b) After how many days will 40960 people be infected?

$40960 = 10(2)^t$

$4096 = 2^t$

$\log 4096 = \log 2^t$

$\log 4096 = t \log(2)$

$t = \frac{\log 4096}{\log 2}$

$t = 12$ days

4) Rewrite each equation in logarithmic form

$4^3 = 64$

$\log_4 64 = 3$

b) $128 = 2^7$

$\log_2 128 = 7$

c) $5^{-2} = \frac{1}{25}$

$\log_5\left(\frac{1}{25}\right) = -2$

d) $\left(\frac{1}{2}\right)^2 = 0.25$

$\log_{\frac{1}{2}} 0.25 = 2$

e) $6^x = y$

$\log_6 y = x$

f) $10^5 = 100\ 000$

$\log_{10} 100\ 000 = 5$

g) $\frac{1}{27} = 3^{-3}$

$\log_3\left(\frac{1}{27}\right) = -3$

5) Evaluate each logarithm

a) $\log_2 64$

$= \log_2(2^6)$

$= 6$

b) $\log_3 27$

$= \log_3(3^3)$

$= 3$

c) $\log_2\left(\frac{1}{4}\right)$

$= \log_2(2^{-2})$

$= -2$

d) $\log_4\left(\frac{1}{64}\right)$

$= \log_4(4^{-3})$

$= -3$

e) $\log_5 125$

$= \log_5(5^3)$

$= 3$

f) $\log_2 1024$

$= \log_2(2^{10})$

$= 10$

6) Evaluate each common logarithm

a) $\log 1000$

$= \log(10^3)$

$= 3$

b) $\log\left(\frac{1}{10}\right)$

$= \log(10^{-1})$

$= -1$

c) $\log 1$

$= \log(10^0)$

$= 0$

d) $\log 0.001$

$= \log(10^{-3})$

$= -3$

e) $\log 10^{-4}$

$= -4$

f) $\log 1\,000\,000$

$= \log(10^6)$

$= 6$

7) Rewrite in exponential form

a) $\log_7 49 = 2$

$7^2 = 49$

b) $5 = \log_2 32$

$2^5 = 32$

c) $\log 10\,000 = 4$

$10^4 = 10000$

d) $w = \log_b z$

$b^w = z$

e) $\log_2 8 = 3$

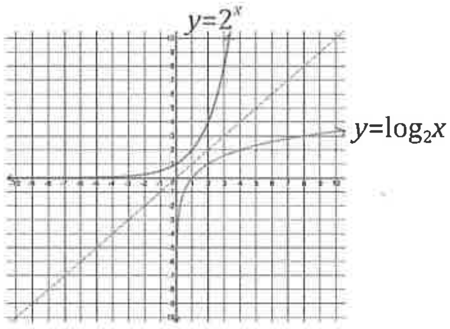
$2^3 = 8$

f) $-2 = \log\left(\frac{1}{100}\right)$

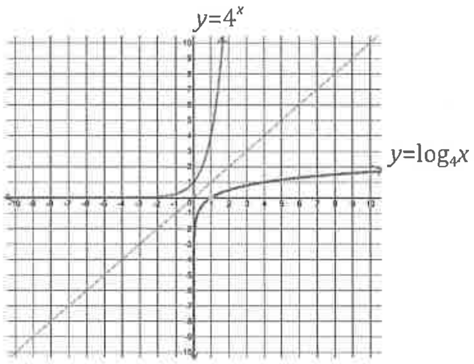
$10^{-2} = \frac{1}{100}$

ANSWER KEY

a)



b)



2) C A B D

3)a)i) 10 **ii)** 20 **iii)** 40 **iv)** 80 **b)** 12 days

4)a) $\log_4 64 = 3$ **b)** $\log_2 128 = 7$ **c)** $\log_5 \left(\frac{1}{25}\right) = -2$ **d)** $\log_{\frac{1}{2}} 0.25 = 2$ **e)** $\log_6 y = x$
f) $\log_{10} 100\,000 = 5$ **g)** $\log_3 \left(\frac{1}{27}\right) = -3$

5)a) 6 **b)** 3 **c)** -2 **d)** -3 **e)** 3 **f)** 10

6)a) 3 **b)** -1 **c)** 0 **d)** -3 **e)** -4 **f)** 6

a) $7^2 = 49$ **b)** $2^5 = 32$ **c)** $10^4 = 10\,000$ **d)** $b^w = z$ **e)** $2^3 = 8$ **f)** $10^{-2} = \frac{1}{100}$

W2 – 6.4 – Power Law of Logarithms

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SOLUTIONS

1) Evaluate.

a) $\log_2 16^3$

$= 3\log_2(2^4)$

$= 3(4)$

$= 12$

b) $\log_4 8^2$

$= \log_4 64$

$= \log_4(4^3)$

$= 3$

c) $\log 100^{-4}$

$= -4\log(10^2)$

$= -4(2)$

$= -8$

d) $\log 0.1^{\frac{1}{2}}$

$= \frac{1}{2}\log(10^{-1})$

$= \frac{1}{2}(-1)$

$= -\frac{1}{2}$

e) $\log_2 \sqrt{8}$

$= \frac{1}{2}\log(2^3)$

$= \frac{1}{2}(3)$

$= \frac{3}{2}$

f) $\log_3\left(\sqrt[3]{81}\right)^6$

$= \log_3(81^{1/3})^6$

$= \log_3(81)^2$

$= 2\log_3(3^4)$

$= 2(4)$

$= 8$

2) Solve for t to two decimal places.

a) $10 = 4^t$

$\log_4 10 = t$

$\frac{\log 10}{\log 4} = t$

$t = 1.66$

b) $5^t = 250$

$\log_5 250 = t$

$\frac{\log 250}{\log 5} = t$

$t = 3.43$

c) $2 = 1.08^t$

$\log_{1.08} 2 = t$

$\frac{\log 2}{\log 1.08} = t$

$t = 9.01$

d) $500 = 100(1.06)^t$

$5 = 1.06^t$

$\log_{1.06} 5 = t$

$\frac{\log 5}{\log 1.06} = t$

$t = 27.62$

3) An investment earns 7% interest, compounded annually. The amount, A, that the investment is worth as a function of time, t, in years, is given by $A(t) = 500(1.07)^t$.

a) Use the equation to determine the value of the investment after 4 years.

$A(4) = 500(1.07)^4$

$= \$655.40$

b) How long will it take for the investment to double in value?

$1000 = 500(1.07)^t$

$2 = 1.07^t$

$\log_{1.07} 2 = t$

$\frac{\log 2}{\log 1.07} = t$

$t = 10.2$ years

4) Use the change of base formula to evaluate each of the following. Round to 3 decimal places.

a) $\log_3 23$

$= \frac{\log 23}{\log 3}$

$= 2.854$

b) $\log_6 20$

$= \frac{\log 20}{\log 6}$

$= 1.672$

c) $-\log_{12} 4$

$= -\frac{\log 4}{\log 12}$

$= -0.558$

d) $\log_{\frac{1}{2}} 30$

$= \frac{\log 30}{\log\left(\frac{1}{2}\right)}$

$= -4.907$

5) Write each as a single logarithm

a) $\frac{\log 8}{\log 5}$

$= \log_5 8$

b) $\frac{\log 17}{\log 9}$

$= \log_9 17$

c) $\frac{\log\left(\frac{1}{2}\right)}{\log\left(\frac{2}{3}\right)}$

$= \log_{\frac{2}{3}}\left(\frac{1}{2}\right)$

d) $\frac{\log(x+1)}{\log(x-1)}$

$= \log_{(x-1)}(x+1)$

6)a) Evaluate $\log_2 8^5$ without using the power law of logarithms.

$= \log_2 (2^3)^5$

$= \log_2 (2^{15})$

$= 15$

b) Evaluate the same expression by applying the power law of logarithms.

$= 5 \cdot \log_2 (2^3)$

$= 5(3)$

$= 15$

c) Which method do you prefer?

answers will vary

7) Solve for x, correct to 3 decimal places.

a) $2 = \log 3^x$

$2 = x \cdot \log 3$

$\frac{2}{\log 3} = x$

$x = 4.192$

b) $100 = 10 \log 1000^x$

$10 = \log 1000^x$

$10 = x \log 1000$

$\frac{10}{\log 1000} = x$

$x = 3.333$

c) $4 = \log_3 15^x$

$3^4 = 15^x$

$81 = 15^x$

$\log 81 = \log 15^x$

$\log 81 = x \cdot \log 15$

$x = \frac{\log 81}{\log 15}$

$x = 1.623$

ANSWER KEY

) 12 **b)** 3 **c)** -8 **d)** $-\frac{1}{2}$ **e)** $\frac{3}{2}$ **f)** 8

2)a) 1.66 **b)** 3.43 **c)** 9.01 **d)** 27.62

3)a) $655.40 **b)** 10.2 years

4)a) 2.854 **b)** 1.672 **c)** -0-.558 **d)** -4.907

5)a) $\log_5 8$ **b)** $\log_9 17$ **c)** $\log_{\frac{2}{3}}\left(\frac{1}{2}\right)$ **d)** $\log_{(x-1)}(x+1)$

6)a) 15 **b)** 15 **c)** answers will vary

7)a) 4.192 **b)** 3.333 **c)** 1.623

W3 – 7.3 – Product and Quotient Laws of Logarithms

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SOLUTIONS

1) Simplify using laws of logarithms and then evaluate.

a) $\log 9 + \log 6$

$= \log(9 \times 6)$

$= \log(54)$

$= 1.732$

b) $\log 48 - \log 6$

$= \log\left(\frac{48}{6}\right)$

$= \log(8)$

$= 0.903$

c) $\log_3 7 + \log_3 3$

$= \log_3(7 \times 3)$

$= \log_3(21)$

$= \frac{\log(21)}{\log(3)}$

$= 2.771$

2) Simplify each algebraic expression.

a) $\log x + \log y + \log(2z)$

$= \log(2xyz)$

b) $\log_2 a + \log_2(3b) - \log_2(2c)$

$= \log_2\left(\frac{3ab}{2c}\right)$

c) $2\log m + 3\log n - 4\log y$

$= \log m^2 + \log n^3 - \log y^4$

$= \log\left(\frac{m^2 n^3}{y^4}\right)$

3) Evaluate using the product law of logarithms.

a) $\log_6 18 + \log_6 2$

$= \log_6(18 \times 2)$

$= \log_6(36)$

$= \frac{\log 36}{\log 6}$

$= 2$

b) $\log 40 + \log 2.5$

$= \log(40 \times 2.5)$

$= \log(100)$

$= 2$

c) $\log_{12} 8 + \log_{12} 2 + \log_{12} 9$

$= \log_{12}(8 \times 2 \times 9)$

$= \log_{12}(144)$

$= \frac{\log 144}{\log 12}$

$= 2$

4) Evaluate using the quotient law of logarithms.

a) $\log_3 54 - \log_3 2$

$= \log_3\left(\frac{54}{2}\right)$

$= \log_3(27)$

$= 3$

b) $\log 50\,000 - \log 5$

$= \log\left(\frac{50\,000}{5}\right)$

$= \log(10\,000)$

$= 4$

c) $\log_4 320 - \log_4 5$

$= \log_4\left(\frac{320}{5}\right)$

$= \log_4(64)$

$= 3$

5) Evaluate, using the laws of logarithms

a) $3\log_{16} 2 + 2\log_{16} 8 - \log_{16} 2$

$= \log_{16}(2^3) + \log_{16}(8^2) - \log_{16}(2)$

$= \log_{16}(8) + \log_{16}(64) - \log_{16}(2)$

$= \log_{16}(256)$

$= 2$

b) $\log 20 + \log 2 + \frac{1}{3}\log 125$

$= \log(20 \times 2) + \log(125^{1/3})$

$= \log(40) + \log(5)$

$= \log(40 \times 5)$

$= \log(200)$

$\doteq 2.301$

6) Write as a sum or difference of logarithms. Simplify, if possible.

a) $\log_7(cd)$

$= \log_7(c) + \log_7(d)$

b) $\log_3\left(\frac{m}{n}\right)$

$= \log_3(m) - \log_3(n)$

c) $\log(uv^3)$

$= \log(u) + \log(v^3)$

$= \log(u) + 3\log(v)$

d) $\log\left(\frac{a\sqrt{b}}{c^2}\right)$

$= \log a + \log(b^{1/2}) - \log(c^2)$

$= \log a + \frac{1}{2}\log b - 2\log c$

e) $\log_2 10$

$= \log_2(2 \times 5)$

$= \log_2(2) + \log_2(5)$

$= 1 + \log_2(5)$

7) Simplify

$\log\left(\frac{x^2}{\sqrt{x}}\right)$

$= \log x^2 - \log x^{1/2}$

$= 2\log x - \frac{1}{2}\log x$

$= \frac{3}{2}\log x$

b) $\log\sqrt{k} + \log(\sqrt{k})^3 + \log\sqrt[3]{k^2}$

$= \log(k^{1/2}) + \log(k^{3/2}) + \log(k^{2/3})$

$= \frac{1}{2}\log k + \frac{3}{2}\log k + \frac{2}{3}\log k$

$= \frac{3}{6}\log k + \frac{9}{6}\log k + \frac{4}{6}\log k$

$= \frac{8}{3}\log k$

c) $\log(x^2 - 4) - \log(x - 2)$

$= \log\left(\frac{x^2-4}{x-2}\right)$

$= \log\left[\frac{(x-2)(x+2)}{x-2}\right]$

$= \log(x+2)$

d) $\log(x^2 - x - 6) - \log(2x - 6)$

$= \log\left(\frac{x^2-x-6}{2x-6}\right)$

$= \log\left[\frac{(x-3)(x+2)}{2(x-3)}\right]$

$= \log\left(\frac{x+2}{2}\right)$

ANSWER KEY

1) $\log 54 = 1.732$ **b)** $\log 8 = 0.903$ **c)** $\log_3 21 = 2.771$ **2)a)** $\log(2xyz)$ **b)** $\log_2\left(\frac{3ab}{2c}\right)$ **c)** $\log\left(\frac{m^2n^3}{y^4}\right)$

3)a) 2 **b)** 2 **c)** 2 **4)a)** 3 **b)** 4 **c)** 3 **5)a)** 2 **b)** 2.301

6)a) $\log_7 c + \log_7 d$ **b)** $\log_3 m - \log_3 n$ **c)** $\log u + 3\log v$ **d)** $\log a + \frac{1}{2}\log b - 2\log c$ **e)** $1 + \log_2 5$

7)a) $\frac{3}{}\log x$ **b)** $\frac{8}{}\log k$ **c)** $\log(x+2)$ **d)** $\log\left(\frac{x+2}{}\right)$

W4 – 7.1/7.2 – Solving Exponential Equations
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SOLUTIONS

1) Write each expression with base 2.

a) 4^6

$= (2^2)^6$

$= 2^{12}$

b) 8^3

$= (2^3)^3$

$= 2^9$

c) $\left(\frac{1}{8}\right)^2$

$= (2^{-3})^2$

$= 2^{-6}$

d) 14

$2^x = 14$

$x = \log_2 14$

$x = \frac{\log 14}{\log 2}$

$\therefore 2^{\frac{\log 14}{\log 2}} = 14$

2) Write each expression as a power of 4.

a) $(\sqrt{16})^3$

$= 4^3$

b) $\sqrt[3]{16}$

$= 16^{1/3}$

$= (4^2)^{1/3}$

$= 4^{2/3}$

$4^x = 128$

$x = \log_4 128$

$x = \frac{\log 128}{\log 4}$

$x = 3.5$

$x = \frac{7}{2}$

c) $\sqrt{64}\times\left(\sqrt[4]{128}\right)^3$

$= 64^{1/2} \times 128^{3/4}$

$= (4^3)^{1/2} \times (4^{7/2})^{3/4}$

$= 4^{3/2} \times 4^{21/8}$

$= 4^{33/8}$

3) Solve each equation

a) $2^{4x} = 4^{x+3}$

$2^{4x} = (2^2)^{x+3}$

$2^{4x} = 2^{2x+6}$

$4x = 2x+6$

$2x = 6$

$\boxed{x = 3}$

b) $3^{w+1} = 9^{w-1}$

$3^{w+1} = (3^2)^{w-1}$

$3^{w+1} = 3^{2w-2}$

$w+1 = 2w-2$

$\boxed{3 = w}$

c) $4^{3x} = 8^{x-3}$

$(2^2)^{3x} = (2^3)^{x-3}$

$2^{6x} = 2^{3x-9}$

$6x = 3x-9$

$3x = -9$

$\boxed{x = -3}$

d) $125^{2y-1} = 25^{y+4}$

$(5^3)^{2y-1} = (5^2)^{y+4}$

$6y-3 = 2y+8$

$4y = 11$

$\boxed{y = \frac{11}{4}}$

4) Consider the equation $10^{2x} = 100^{2x-5}$

a) Solve this equation by expressing both sides as powers of a common base.

$10^{2x} = (10^2)^{2x-5}$

$10^{2x} = 10^{4x-10}$

$2x = 4x-10$

$10 = 2x$

$\boxed{5 = x}$

b) Solve the same equation by taking the common logarithm of both sides.

$\log 10^{2x} = \log 100^{2x-5}$

$2x \log 10 = (2x-5)\log 100$

$2x(1) = (2x-5)(2)$

$2x = 4x-10$

$10 = 2x$

$\boxed{x = 5}$

5) Solve $2^{3x} > 4^{x+1}$

$2^{3x} > (2^2)^{x+1}$

$2^{3x} > 2^{2x+2}$

$3x > 2x+2$

$\boxed{x > 2}$

6) Solve for t. Round answers to 2 decimal places.

a) $2 = 1.07^t$

$\log_{1.07} 2 = t$

$\frac{\log 2}{\log 1.07} = t$

$\boxed{t = 10.24}$

b) $\frac{100}{10} = \frac{10(1.04)^t}{10}$

$10 = 1.04^t$

$\log 10 = \log 1.04^t$

$\log 10 = t \cdot \log 1.04$

$t = \frac{\log 10}{\log 1.04}$

$\boxed{t = 58.71}$

c) $15 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

$\log_{1/2}(15) = \frac{t}{4}$

$\frac{\log(15)}{\log(\frac{1}{2})} = \frac{t}{4}$

$4\left[\frac{\log(15)}{\log(0.5)}\right] = t$

$\boxed{t = -15.63}$

7) Solve each equation. Round answers to 3 decimal places.

$2^x = 3^{x-1}$

$\log 2^x = \log 3^{x-1}$

$x \log 2 = (x-1) \log 3$

$x \log 2 = x \log 3 - \log 3$

$\log 3 = x \log 3 - x \log 2$

$\log 3 = x(\log 3 - \log 2)$

$x = \frac{\log 3}{\log 3 - \log 2}$

$\boxed{x = 2.71}$

b) $5^{x-2} = 4^x$

$\log 5^{x-2} = \log 4^x$

$(x-2)\log 5 = x \log 4$

$x \log 5 - 2\log 5 = x \log 4$

$x \log 5 - x \log 4 = 2 \log 5$

$x(\log 5 - \log 4) = 2 \log 5$

$x = \frac{2 \log 5}{\log 5 - \log 4}$

$\boxed{x = 14.425}$

c) $7^{2x+1} = 4^{x-2}$

$\log 7^{2x+1} = \log 4^{x-2}$

$(2x+1)\log 7 = (x-2)\log 4$

$2x \log 7 + \log 7 = x \log 4 - 2\log 4$

$2x \log 7 - x \log 4 = -2\log 4 - \log 7$

$x(2\log 7 - \log 4) = -2\log 4 - \log 7$

$x = \frac{-2\log 4 - \log 7}{2\log 7 - \log 4}$

$\boxed{x = -1.883}$

8) Solve $2^{2x} + 2^x - 6 = 0$ using the quadratic formula. Clearly identify any extraneous roots.

$(2^x)^2 + (2^x) - 6 = 0$

Let $k = 2^x$

$k^2 + k - 6 = 0$

$(k+3)(k-2) = 0$

$k = -3, 2$

Case 1

$-3 = 2^x$

$\log_2(-3) = x$

$\therefore$ NO SOLUTIONS

CASE 2

$2 = 2^x$

$\boxed{x = 1}$

9) Solve $8^{2x} - 2(8^x) - 5 = 0$ using the quadratic formula. Clearly identify any extraneous roots.

$(8^x)^2 - 2(8^x) - 5 = 0$

Let $k = 8^x$

$k^2 - 2k - 5 = 0$

$k = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$

$k = \frac{2 \pm \sqrt{24}}{2}$

$k = \frac{2 \pm 2\sqrt{6}}{2}$

$k = \frac{\cancel{2}(1 \pm \sqrt{6})}{\cancel{2}}$

Case 1

$8^x = 1 + \sqrt{6}$

$\log_8(1+\sqrt{6}) = x$

$\frac{\log(1+\sqrt{6})}{\log 8} = x$

$\boxed{x = 0.595}$

Case 2

$8^x = 1 - \sqrt{6}$

↑

NO SOLUTIONS

10) Use the decay equation for polonium-218, $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{3.1}}$, A is the amount remaining after t minutes and A_0 is the initial amount.

a) How much will remain after 90 seconds from an initial sample of 50 mg?

$A(1.5) = 50\left(\frac{1}{2}\right)^{1.5/3.1}$

$A(1.5) = 35.75$ mg.

b) How long will it take for this sample to decay to 10% of its initial amount of 50 mg?

$5 = 50\left(\frac{1}{2}\right)^{t/3.1}$

$0.1 = \left(\frac{1}{2}\right)^{t/3.1}$

$\log_{1/2}(0.1) = \frac{t}{3.1}$

$\frac{\log 0.1}{\log(\frac{1}{2})} = \frac{t}{3.1}$

$3.321928095 = \frac{t}{3.1}$

$t = 10.298$ minutes.

11) A 20-mg sample of thorium-233 decays to 17 mg after 5 minutes.

a) What is the half-life of thorium-233?

$$17 = 20\left(\tfrac{1}{2}\right)^{5/h}$$

$$0.85 = \left(\tfrac{1}{2}\right)^{5/h}$$

$$\log_{1/2}(0.85) = \frac{5}{h}$$

$$\frac{\log(0.85)}{\log(0.5)} = \frac{5}{h}$$

$$h = \frac{5\log(0.5)}{\log(0.85)}$$

$$\boxed{h = 21.325 \text{ minutes}}$$

b) How long will it take this sample to decay to 1 mg?

$$1 = 20\left(\tfrac{1}{2}\right)^{t/21.3}$$

$$0.05 = \left(\tfrac{1}{2}\right)^{t/21.3}$$

$$\log_{1/2}(0.05) = \frac{t}{21.3}$$

$$\frac{\log(0.05)}{\log(0.5)} = \frac{t}{21.3}$$

$$4.321928095 = \frac{t}{21.3}$$

$$\boxed{t = 92.06 \text{ min}}$$

ANSWER KEY

1)a) 2^{12} **b)** 2^9 **c)** 2^{-6} **d)** $2^{\frac{\log 14}{\log 2}}$ **2)a)** 4^3 **b)** $4^{\frac{2}{3}}$ **c)** $4^{\frac{33}{8}}$ **3)a)** 3 **b)** 3 **c)** -3 **d)** $\frac{11}{4}$

4)a) 5 **b)** 5 **5)** $x > 2$ **6)a)** 10.24 **b)** 58.71 **c)** -15.63 **7)a)** 2.710 **b)** 14.425 **c)** -1.883

8) $x = 1$ is the only solution; $2^x = -3$ or $x = \frac{\log(-3)}{\log 2}$ is an extraneous root

, $x = \frac{\log(1+\sqrt{6})}{\log 8} \cong 0.6$ is the only solution; $8^x = 1 - \sqrt{6}$ or $x = \frac{\log(1-\sqrt{6})}{\log 8}$ is an extraneous root

10)a) 35.75 mg **b)** 10.3 min **11)a)** 21.3 min **b)** 92.06 min

W5 – 7.4 – Solving Logarithmic Equations
MHF4U
Jensen

1) Find the roots of each equation

a) $2 = \log(x + 25)$

$10^2 = x + 25$

$100 = x + 25$

$\boxed{x = 75}$

b) $1 - \log(w - 7) = 0$

$1 = \log(w - 7)$

$10^1 = w - 7$

$10 = w - 7$

$\boxed{w = 17}$

c) $6 - 3\log(2n) = 0$

$6 = 3\log(2n)$

$2 = \log(2n)$

$10^2 = 2n$

$100 = 2n$

$\boxed{n = 50}$

2) Solve

a) $5 = \log_2(2x - 10)$

$2^5 = 2x - 10$

$32 = 2x - 10$

$42 = 2x$

$\boxed{x = 21}$

b) $9 = \log_5(x + 100) + 6$

$3 = \log_5(x + 100)$

$125 = x + 100$

$\boxed{x = 25}$

c) $\log_3(n^2 - 3n + 5) = 2$

$3^2 = n^2 - 3n + 5$

$9 = n^2 - 3n + 5$

$0 = n^2 - 3n - 4$

$0 = (n - 4)(n + 1)$

$\boxed{n = 4}$ $\boxed{n = -1}$

3) Solve. Make sure to reject any extraneous roots.

a) $\log x + \log(x - 4) = 1$

$\log[x(x - 4)] = 1$

$10^1 = x(x - 4)$

$10 = x^2 - 4x$

$0 = x^2 - 4x - 10$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}$

$x = \frac{4 \pm \sqrt{56}}{2}$

$x = \underline{4 \pm 2\sqrt{14}}$

$x = \frac{\cancel{2}(2 \pm \sqrt{14})}{\cancel{2}}$

$x = 2 \pm \sqrt{14}$

Reject $2 - \sqrt{14}$

$\boxed{x = 2 + \sqrt{14}}$

b) $\log x^3 - \log 2 = \log(2x^2)$

$\log x^3 - \log(2x^2) = \log 2$

$\log\left(\frac{x^3}{2x^2}\right) = \log 2$

$\log\left(\frac{x}{2}\right) = \log 2$

$\frac{x}{2} = 2$

$\boxed{x = 4}$

c) $\log(v-1) = 2 + \log(v-16)$

$$\log(v-1) - \log(v-16) = 2$$
$$\log\left(\frac{v-1}{v-16}\right) = 2$$
$$10^2 = \frac{v-1}{v-16}$$
$$100(v-16) = v-1$$
$$100v - 1600 = v - 1$$
$$99v = 1599$$
$$\boxed{v = \frac{533}{33}}$$

d) $\log(k+2) + \log(k-1) = 1$

$$\log[(k+2)(k-1)] = 1$$
$$10^1 = (k+2)(k-1)$$
$$10 = k^2 - 1k + 2k - 2$$
$$0 = k^2 + k - 12$$
$$0 = (k+4)(k-3)$$

↓ ↓

$k = -4$
Reject

$$\boxed{k=3}$$

4) Solve. Check for extraneous roots.

a) $\log\sqrt{x^2-3x} = \frac{1}{2}$

$$\log(x^2-3x)^{1/2} = \frac{1}{2}$$
$$\frac{1}{2}\log(x^2-3x) = \frac{1}{2}$$
$$\log(x^2-3x) = 1$$
$$10^1 = x^2-3x$$
$$0 = x^2-3x-10$$
$$x = \frac{3 \pm \sqrt{(-3)^2-4(1)(-10)}}{2(1)}$$

b) $\log\sqrt{x^2+48x} = 1$

$$\log(x^2+48x)^{1/2} = 1$$
$$\frac{1}{2}\log(x^2+48x) = 1$$
$$\log(x^2+48x) = 2$$
$$10^2 = x^2+48x$$
$$0 = x^2+48x-100$$
$$0 = (x+50)(x-2)$$

↓ ↓

$$\boxed{x=-50} \quad \boxed{x=2}$$

5) Solve. Check for extraneous roots.

a) $\log_2(x+5) - \log_2(2x) = 8$

$$\log_2\left(\frac{x+5}{2x}\right) = 8$$
$$2^8 = \frac{x+5}{2x}$$
$$256 = \frac{x+5}{2x}$$
$$512x = x+5$$
$$511x = 5$$
$$\boxed{x = \frac{5}{511}}$$

b) $\log(2k+4) = 1 + \log k$

$$\log(2k+4) - \log(k) = 1$$
$$\log\left(\frac{2k+4}{k}\right) = 1$$
$$10^1 = \frac{2k+4}{k}$$
$$10k = 2k+4$$
$$8k = 4$$
$$\boxed{k = \frac{1}{2}}$$

ANSWER KEY

1)a) 75 **b)** 17 **c)** 50 **2)a)** 21 **b)** 25 **c)** 4, -1 **3)a)** $2+\sqrt{14}$ **b)** 4 **c)** $\frac{533}{33}$ **d)** 3
4)a) 5, -2 **b)** -50, 2 **5)a)** $\frac{5}{511}$ **b)** $\frac{1}{2}$

L6 – 6.5 – Applications of Logarithms AND Exponentials in Physical Sciences
MHF4U
Jensen

Exponential Formulas

$A(t)=A_0(1+i)^t$	$A(t)=A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$	$A(t)=A_0(2)^{\frac{t}{D}}$
general, where i is percent growth(+) or decay(-)	half-life, H is the half-life period	doubling, D is the doubling period

Logarithmic Formulas

$pH=-\log[H^+]$	$\beta_2-\beta_1=10\log\left(\frac{I_2}{I_1}\right)$	$M=\log(\frac{I}{I_0})$
Where pH is acidity and [H+] is concentration of hydronium ions mol/L	Where β is loudness in dB and I is intensity of sound in W/m^2	Where M is magnitude measure by richters, I is intensity

1) The half-life of a radioactive form of tritium is about 2 years. How much of a 5-kg sample of this material would remain after …

a) 8 years

$A(8) = 5\left(\frac{1}{2}\right)^{8/2}$

$A(8) = 5\left(\frac{1}{2}\right)^{4}$

$A(8) = \frac{5}{16}$

$\boxed{A(8) = 0.3125\text{ kg}}$

b) 12 months = 1 year

$A(1) = 5\left(\frac{1}{2}\right)^{1/2}$

$\boxed{\approx 3.536\text{ Kg}}$

2) The population of Littleton is currently (2014) 23000, and is increasing exponentially with a growth rate of 2% per year. Estimate when Littleton will have a population of 30000.

$30\,000 = 23\,000(1.02)^t$

$\left(\frac{30}{23}\right) = (1.02)^t$

$\log_{1.02}\left(\frac{30}{23}\right) = t$

$\frac{\log\left(\frac{30}{23}\right)}{\log 1.02} = t$

$\boxed{t = 13.42\text{ years}}$

3) The population of purple martins in Algonquin park was estimated to be 35000 in 1992. Ten years later, in 2002, the population had risen to 44400.

a) What is the annual growth rate for the purple martin population?

$$44\,400 = 35000(b)^{10} \quad\to\quad 1.024 = b$$

$$\frac{222}{175} = (b)^{10}$$

$$\sqrt[10]{\frac{222}{175}} = b$$

∴ growth rate is 2.4%
per year.

b) Estimate the population for 2010 to the nearest hundred.

$$A(8) = 44\,400(1+0.024)^8$$

$$A(8) = 53\,676.3$$

$$\boxed{A(8) \simeq 53\,700.}$$

4) After an accident at a nuclear plant, the radiation level in the plant was 950 R (roentgens). Five hours later the level was 800 R. How long will it take before safe levels of radiation are reached, which is less than 0.01 R?

$$800 = 950(b)^5$$

$$\sqrt[5]{\frac{800}{950}} = b$$

$$b = 0.9662$$

$$0.01 = 950(0.9662)^t$$

$$\frac{0.01}{950} = 0.9662^t$$

$$\frac{\log\left(\frac{0.01}{950}\right)}{\log(0.9662)} = t$$

$$\boxed{t = 333.3 \text{ hours}}$$

5) The value of a new minivan drops 40% after the first year, and then decreases exponentially at a rate of 12% per year after that. When will a minivan that cost $35000 new be worth less than $10000?

Value after 1st year = 35 000 (0.6) = 21000

$$A(t) = A_0(1-i)^t$$

$$10\,000 = 21\,000(1-0.12)^t$$

$$\frac{10}{21} = (0.88)^t$$

$$\log_{0.88}\left(\frac{10}{21}\right) = t$$

$$\frac{\log\left(\frac{10}{21}\right)}{\log(0.88)} = t$$

$$t \simeq 5.8 \text{ years}$$

1 year at 40% and then
5.8 years at 12%; This means
it would take about 6.8
years to be worth less
than $10000

6) A crab fossil contains 38.6% of its original Carbon14 isotope, which has a half life of 5370 years. Approximately how old is the crab fossil?

$$A(t) = A_0\left(\frac{1}{2}\right)^{t/H}$$

$$\frac{0.386(A_0)}{A_0} = \frac{A_0\left(\frac{1}{2}\right)^{t/5370}}{A_0}$$

$$0.386 = \left(\frac{1}{2}\right)^{t/5370}$$

$$\log(0.386) = \log\left(\frac{1}{2}\right)^{t/5370}$$

$$\log(0.386) = \frac{t}{5370}\log\left(\frac{1}{2}\right)$$

$$\frac{\log(0.386)}{\log(0.5)} = \frac{t}{5370}$$

$$\frac{5370\log(0.386)}{\log(0.5)} = t$$

$$\boxed{t = 7374.8 \text{ years old}}$$

7) A Trimark mutual fund has track record of 4.2% growth per year. What is the doubling period for this investment?

$$2 = 1(1.042)^t$$

$$\log 2 = \log(1.042)^t$$

$$\log 2 = t\log(1.042)$$

$$t = \frac{\log(2)}{\log(1.042)}$$

$$\boxed{t = 16.85 \text{ years}}$$

8) A treatment to help a patient stop smoking involves chewing nicotine gum. Each gum introduces 1.5 mg of nicotine into the patient's system. Nicotine has a half life of 3 hours. The patient will feel the urge to smoke when the level of nicotine drops below 0.45 mg in her system. If she chewed a gum at 8:00 a.m., and another at 10:00 a.m., at what time will she next feel the urge to smoke?

Concentration at 10:00 am

$$A(2) = 1.5\left(\frac{1}{2}\right)^{2/3}$$

$$A(2) = 0.94494$$

then +1.5 mg for the new piece

= 2.44494 mg.

when will 2.44494 mg reduce to 0.45 mg?

$$0.45 = 2.44494\left(\frac{1}{2}\right)^{t/3}$$

$$0.1840535964 = \left(\frac{1}{2}\right)^{t/3}$$

$$\frac{\log 0.1840535964}{\log\left(\frac{1}{2}\right)} = \frac{t}{3}$$

$$\frac{3\log(0.1840535964)}{\log(0.5)} = t$$

$$\boxed{t = 7.3 \text{ hours or about 5:18 pm}}$$

9) Determine the pH of a solution with hydronium ion concentration:

a) 0.01

b) 1.5×10^{-10}

$pH = -\log[H^+]$

$pH = -\log(0.01)$

$pH = -(-2)$

$\boxed{pH = 2}$

$pH = -\log(1.5 \times 10^{-10})$

$\boxed{pH \simeq 9.8}$

10) Determine the hydronium concentration, in moles per litre, of a solution with pH:

a) 8.5

b) 3

$8.5 = -\log[H^+]$

$-8.5 = \log[H^+]$

$[H^+] = 10^{-8.5}$

$\boxed{[H^+] \simeq 3.16 \times 10^{-9} \text{ mol/L}}$

$3 = -\log[H^+]$

$-3 = \log[H^+]$

$[H^+] = 10^{-3}$

$\boxed{[H^+] = 0.001 \text{ mol/L}}$

11) How many times as intense is the sound of a shout as the sound of a whisper?

$B_2 - B_1 = 10 \log\left(\frac{I_2}{I_1}\right)$

$80 - 30 = 10 \log\left(\frac{I_2}{I_1}\right)$

$5 = \log\left(\frac{I_2}{I_1}\right)$

$\frac{I_2}{I_1} = 10^5$

$\frac{I_2}{I_1} = 100\ 000$

∴ A shout is 100 000 as intense as a whisper

12) A loud car stereo has a decibel level of 110 dB. How many times as intense as the sound of a loud car stereo is the sound of a rock concert speaker?

$150 - 110 = 10 \log\left(\frac{I_2}{I_1}\right)$

$4 = \log\left(\frac{I_2}{I_1}\right)$

$10^4 = \frac{I_2}{I_1}$

$10\ 000 = \frac{I_2}{I_1}$

∴ Rock concert is 10 000 times as intense as a car stereo.

β_2 $\quad \frac{I_2}{I_1}$ $\quad = 60dB$

13) The sound intensity of a pin drop is about 1/30 000 of the sound intensity of a normal conversation. What is the decibel level of a pin drop?

β_1

$$\beta_2 - 60 = 10 \log\left(\frac{1}{30000}\right)$$

$$\beta_2 = 10 \log\left(\frac{1}{30\,000}\right) + 60$$

$$\beta_2 \simeq 15.23 \text{ dB}$$

$$\boxed{\text{Decibel level of a pin drop is about } 15.23 \text{ dB}}$$

14) On September 26, 2001, an earthquake in North Bay, Ontario, occurred that was 10 000 times as intense as I_0. What was the measure of this earthquake on the Richter scale?

$$M = \log\left(\frac{I}{I_0}\right)$$

$$M = \log(10\,000)$$

$$\boxed{M = 4}$$

15) On February 10, 2000, an earthquake happen in Welland, Ontario, that measured 2.3 on the Richter scale.

a) How many times as intense was this as a standard low-level earthquake?

$$2.3 = \log\left(\frac{I}{I_0}\right)$$

$$10^{2.3} = \left(\frac{I}{I_0}\right)$$

$$\frac{I}{I_0} \simeq 199.53$$

$$\boxed{\text{About 200 times as intense}}$$

b) On July 22, 2001, an earthquake in St. Catharines measured 1.1 on the Richter scale. How many times as intense as the St.Catharines earthquake was the Welland earthquake?

$$2.3 - 1.1 = 1.2$$

$$1.2 = \log\left(\frac{I}{I_0}\right)$$

$$10^{1.2} = \frac{I}{I_0}$$

$$\frac{I}{I_0} \simeq 15.85$$

$$\boxed{\text{About 16 times as intense}}$$

16) The stellar magnitude scale compares the brightness of stars using the equation $m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$, where m_2 *and* m_1 are the apparent magnitude of the two stars being compared (how bright they appear in the sky) and b_1 and b_2 are their brightness (how much light they actually emit). This relationship does not factor in how far om Earth the stars are.

m_2

a) Sirius is the brightest-appearing star in our sky, with an apparent magnitude of -1.5. How much brighter does Sirius appear than Betelgeuse, whose apparent magnitude in 0.12?

m_1

$$0.12 - (-1.5) = \log\left(\frac{b_1}{b_2}\right)$$

$$1.62 = \log\left(\frac{b_1}{b_2}\right)$$

$$10^{1.62} = \frac{b_1}{b_2}$$

$$\frac{b_1}{b_2} \approx 41.7$$

$\boxed{\text{About 41.7 times brighter}}$

m_2

b) The Sun appears about 1.3 x 10^{10} times as brightly in our sky as does Sirius. What is the apparent magnitude of the Sun?

m_1

$$-1.5 - m_1 = \log(1.3 \times 10^{10})$$

$$-1.5 - \log(1.3 \times 10^{10}) = m_1$$

$$\boxed{m_1 \simeq -11.61}$$

ANSWER KEY – Exponentials

1. a) 8 years **b)** 12 months
0.3125 kg 3.536 kg

2. In approx. 13.418 years

3. a) Growth rate of 2.4% per year
3. b) Approx. 53 700 birds

4. Approx. 333 hours

5. Approx. 7 years

6. Approx. 7375 years

7. Approx. 16.85 years

8. $t \sim 7.3$ hours, so about 5:20 p.m

ANSWER KEY: Logarithms

9. a) 2 **b)** 9.8

10. a) 3.2×10^{-9} mol/L
b) 0.001 mol/L

11. A shout is 100,000 times more intense than a whisper

12. A rock concert speaker is 10,000 times more intense than a loud car speaker

13. A pin drop is 15dB

14. level 4 on the Richter scale

15. a) about 200 times more intense
b) 15.85 times more intense

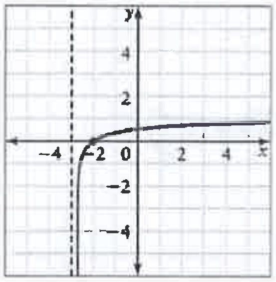
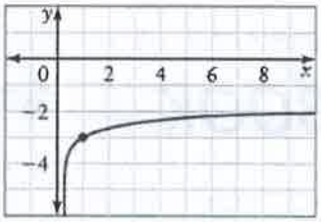
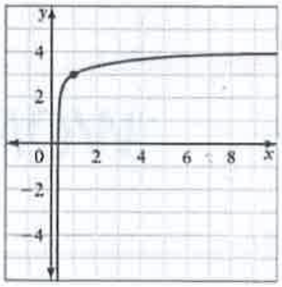
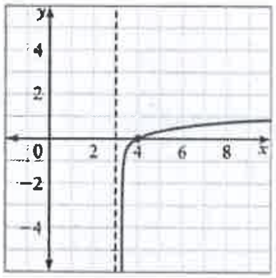
16. a) 41.69 times brighter
b) apparent magnitude is -11.61

W1 – 6.3 Transformations of Exponential and Logarithmic Functions
MHF4U
Jensen

SOLUTIONS

1) Write the letter of the equation under the corresponding graph

A) $y = \log(x-3)$ **B)** $y = \log x - 3$ **C)** $y = \log(x+3)$ **D)** $y = \log x + 3$

			
C	B	D	A

2) Sketch a graph of each of the following logarithmic functions by applying transformations to the parent function. Make sure to identify key points such as asymptotes and x-intercepts.

a) $f(x) = -2\log_2 x - 1$

$y = \log_2 x$

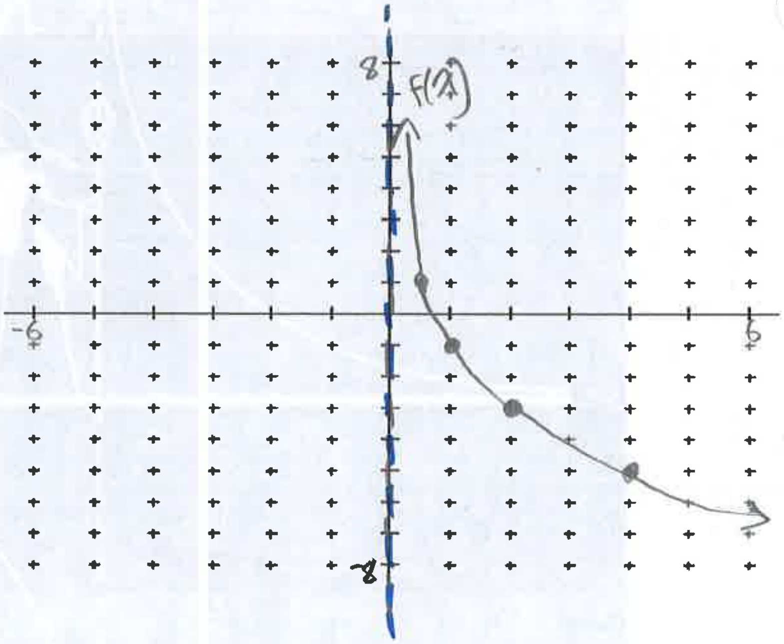
x	y
$\frac{1}{2} = 0.5$	-1
1	0
2	1

VA: $x = 0$

$f(x) = -2\log_2(x) - 1$

x	$-2y-1$
0.5	1
1	-1
2	-3

VA: $x = 0$



b) $g(x) = \log_4(x-1)+4$

$y = \log_4 x$

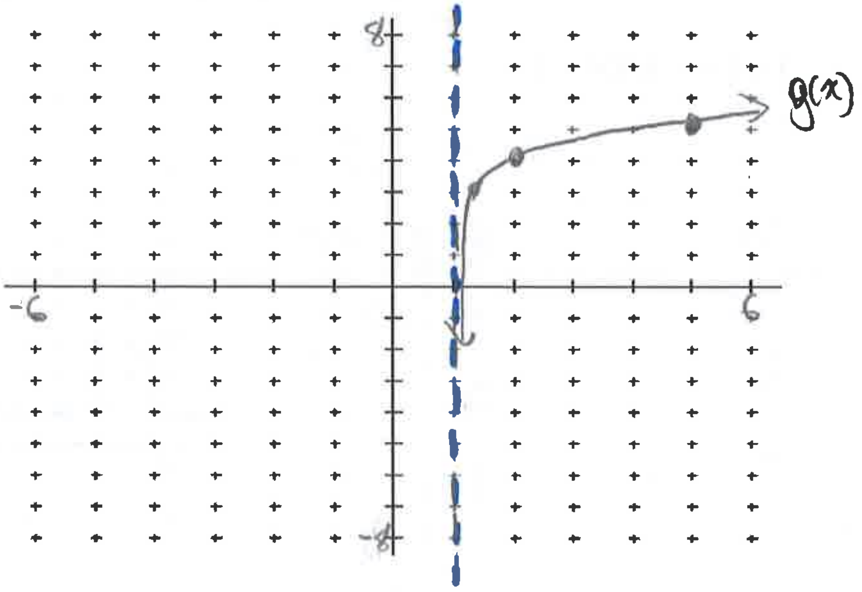
x	y
$1/4 = 0.25$	-1
1	0
4	1

VA: $x = 0$

$g(x) = \log_4(x-1)+4$

$x+1$	$y+4$
1.25	3
2	4
5	5

VA: $x = 1$



c) $h(x) = 4\log_3\left[\frac{1}{2}(x+2)\right]-3$

$y = \log_3 x$

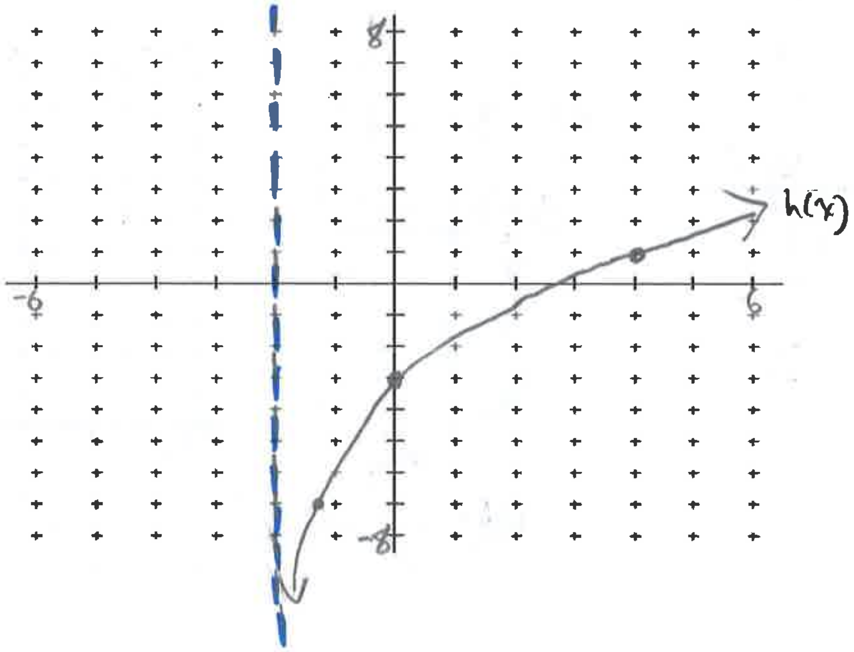
x	y
$1/3 = 0.33$	-1
1	0
3	1

VA: $x = 0$

$h(x) = 4\log_3\left[\frac{1}{2}(x+2)\right]-3$

$2x-2$	$4y-3$
-1.33	-7
0	-3
4	1

VA: $x = -2$



3) Sketch a graph of each of the following exponential functions by applying transformations to the parent function. Make sure to identify key points such as asymptotes and y-intercepts.

a) $f(x) = -3(2)^x + 1$

$y = 2^x$

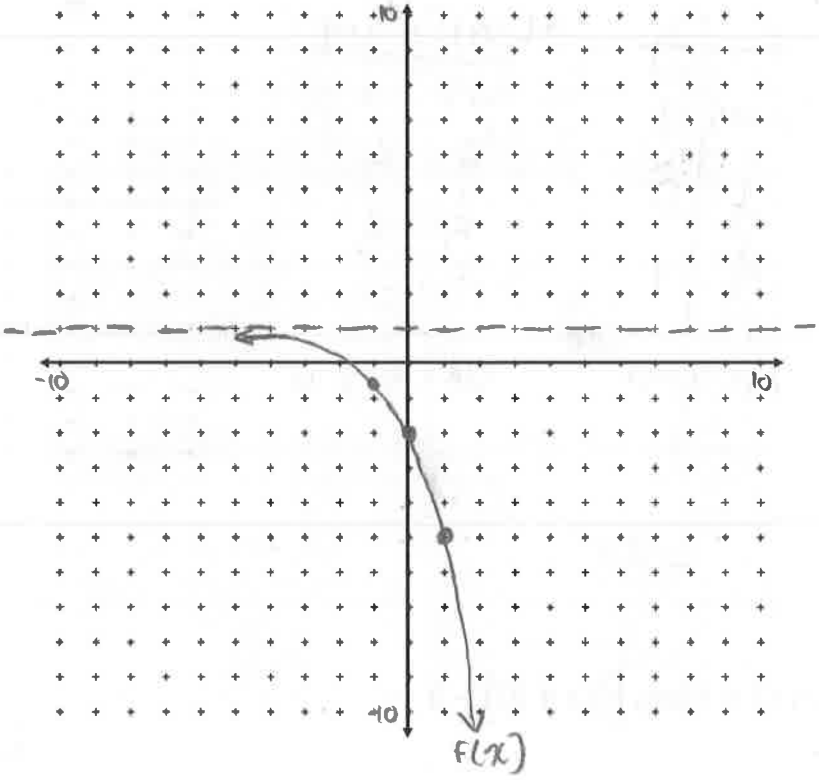
x	y
-1	$\frac{1}{2} = 0.5$
0	1
1	2

HA: $y = 0$

$f(x) = -3(2)^x + 1$

x	$-3y+1$
-1	-0.5
0	-2
1	-5

HA: $y = 1$



b) $g(x) = 3^{x-3} - 4$

$y = 3^x$

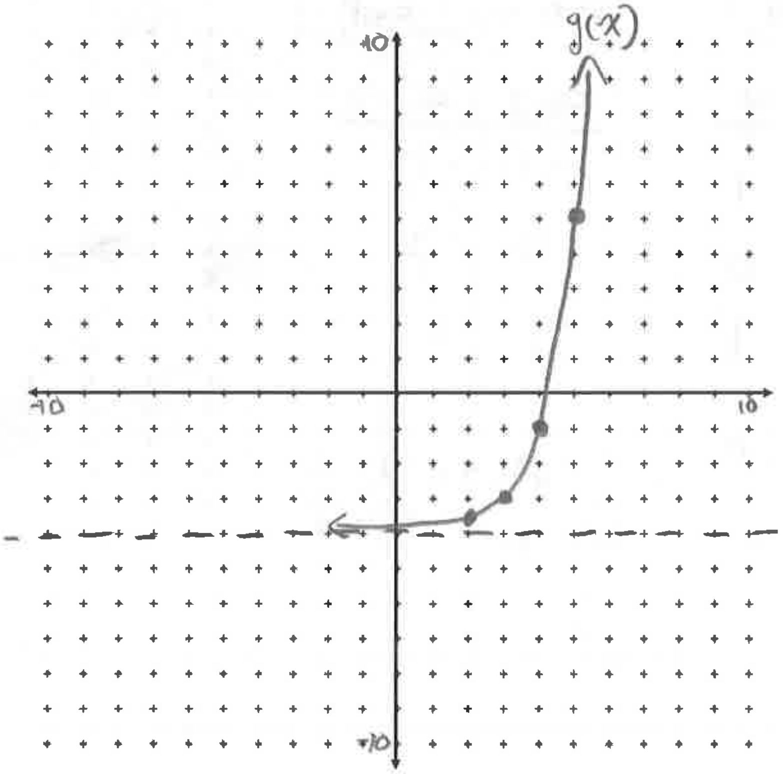
x	y
-1	$\frac{1}{3} = 0.33$
0	1
1	3

HA: $y = 0$

$g(x) = 3^{x-3} - 4$

$x+3$	$y-4$
2	-3.67
3	-3
4	-1

HA: $y = -4$



c) $h(x) = 2(4)^{\frac{1}{2}(x+1)} - 3$

$y = 4^x$

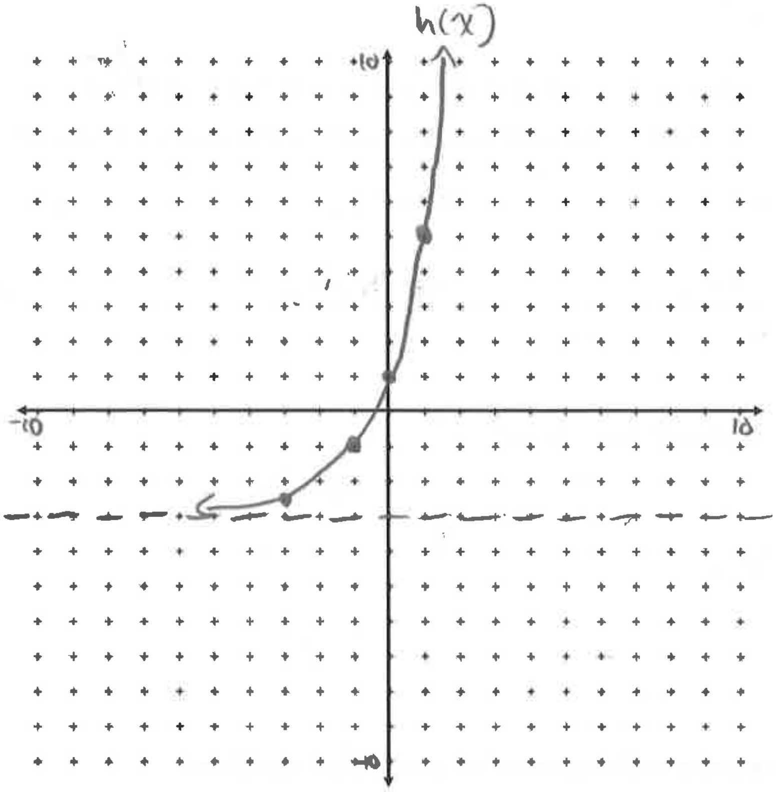
x	y
-1	1/4 = 0.25
0	1
1	4

HA: $y = 0$

$h(x) = 2(4)^{\frac{1}{2}(x+1)} - 3$

$2x-1$	$2y-3$
-3	-2.5
-1	-1
1	5

HA: $y = -3$



W8 – The Natural Logarithm

MHF4U

Jensen

SOLUTIONS

1) Use a calculator to approximate each to the nearest thousandth

a) $\ln 6.2$

$\simeq 1.825$

b) $\ln 2.1$

$\simeq 0.742$

c) $\ln e$

$= 1$

d) e^5

$\simeq 148.413$

2) Expand each logarithm

a) $\ln x^2$

$= 2\ln x$

b) $\ln \sqrt[3]{x}$

$= \ln(x)^{1/3}$

$= \frac{1}{3}\ln x$

c) $\ln \frac{u^3}{wv^4}$

$= \ln u^3 - (\ln w + \ln v^4)$

$= 3\ln u - \ln w - 4\ln v$

3) Condense each expression to a single logarithm

a) $4\ln 2$

$= \ln(2)^4$

$= \ln 16$

b) $\ln 10 - 5\ln 7$

$= \ln 10 - \ln(7)^5$

$= \ln 10 - \ln 16807$

$= \ln\left(\frac{10}{16807}\right)$

c) $3\ln x + 3\ln y$

$= \ln(x)^3 + \ln(y)^3$

$= \ln(x^3y^3)$

4) Solve each equation. Round your answer to 4 decimal places if necessary.

a) $e^x = 2$

$\ln(e)^x = \ln 2$

$x\ln e = \ln 2$

$x(1) = \ln 2$

$x \simeq 0.6931$

b) $e^{-3n} = 83$

$\ln(e)^{-3n} = \ln 83$

$-3n\ln e = \ln 83$

$-3n(1) = \ln 83$

$n = \frac{\ln 83}{-3}$

$n \simeq -1.4729$

c) $e^{k+7} = 26$

$\ln(e)^{k+7} = \ln 26$

$(k+7)\ln(e) = \ln 26$

$(k+7)(1) = \ln 26$

$k+7 = \ln 26$

$k = \ln(26) - 7$

$k \simeq -3.7419$

d) $9e^{1.4p-10} - 10 = 17$

$9e^{1.4p-10} = 27$

$e^{1.4p-10} = 3$

$\ln(e)^{1.4p-10} = \ln 3$

$(1.4p-10)\ln(e) = \ln 3$

$(1.4p-10)(1) = \ln 3$

$\rightarrow 1.4p-10 = \ln(3)$

$p = \frac{\ln(3)+10}{1.4}$

$p \simeq 7.9276$

e) $\ln x = -5$

$$e^{-5} = x$$

$$x \simeq 0.0067$$

g) $\ln(-m) = \ln(m + 10)$

$$-m = m + 10$$

$$-10 = 2m$$

$$m = -5$$

i) $\ln(1 - 8x) - 10 = -7$

$$\ln(1-8x) = 3$$

$$e^{3} = 1-8x$$

$$\frac{e^{3}-1}{-8} = x$$

$$x \simeq -2.3857$$

f) $7.316 = e^{\ln(2x)}$

$$\ln(7.316) = \ln(e)^{\ln(2x)}$$

$$\ln(7.316) = \ln(2x)\ln(e)$$

$$e^{\ln(7.316)} = 2x$$

$$x = \frac{e^{\ln(7.316)}}{2}$$

$$x = 3.658$$

h) $\ln(9x + 1) = \ln(x^2 + 9)$

$$9x+1 = x^{2}+9$$

$$0 = x^{2}-9x+8$$

$$0 = (x-8)(x-1)$$

$$x_2 = 8 \qquad x_2 = 1$$

j) $\ln(5 - 2x^2) + \ln 9 = \ln 43$

$$\ln\left[(5-2x^{2})(9)\right] = \ln(43)$$

$$9(5-2x^{2}) = 43$$

$$45 - 18x^{2} = 43$$

$$-18x^{2} = -2$$

$$x^{2} = \frac{1}{9}$$

$$x = \pm\frac{1}{3}$$