Chapter 4/5 Part 2- Trig Identities and Equations

WORKBOOK

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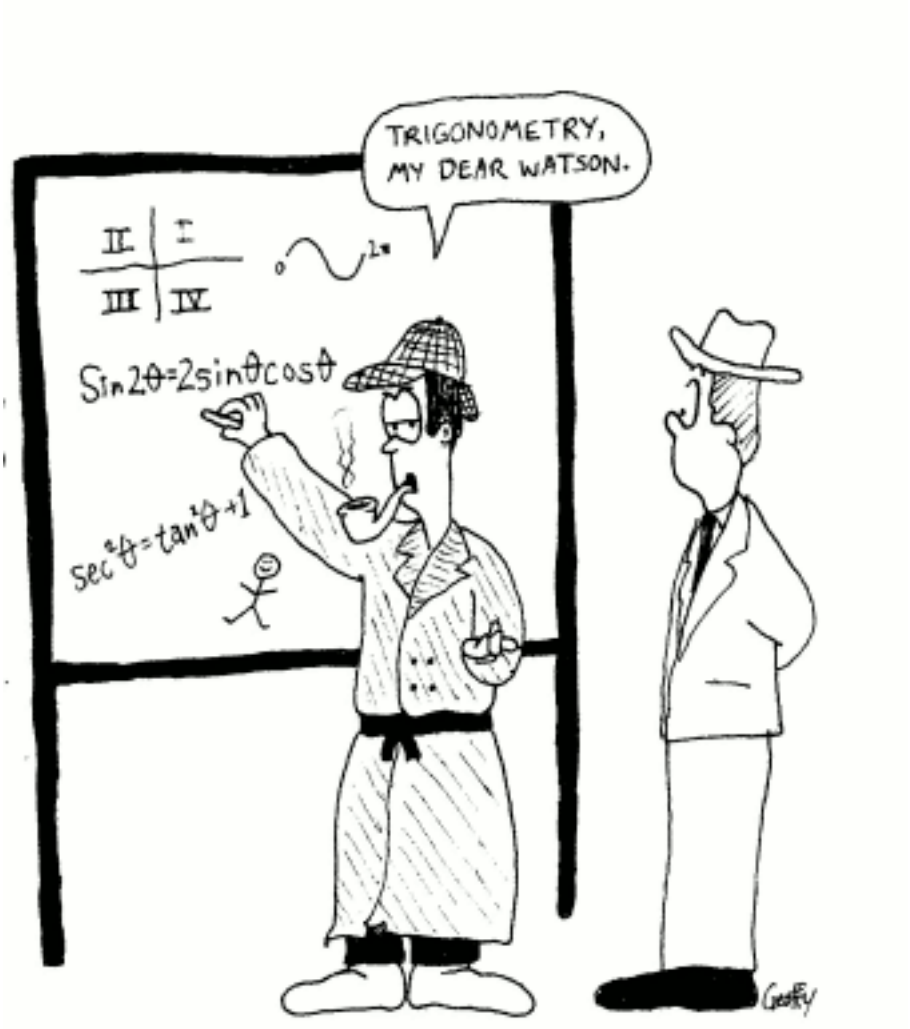

W1 – 4.3 Co-function Identities

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1) Simplify.

a) $\sin x \left(\frac{1}{\cos x}\right)$ **b)** $(\cos x)(\sec x)$ **c)** $1 - \cos^2 x$ **d)** $1 - \sin^2 x$

e) $\frac{\tan x}{\sin x}$ **f)** $(1 - \sin x)(1 + \sin x)$ **g)** $\left(\frac{1}{\tan x}\right)\sin x$ **h)** $\frac{1+\tan^2 x}{\tan^2 x}$

i) $\frac{\sin x \cos x}{1-\sin^2 x}$ **j)** $\frac{1-\cos^2 x}{\sin x \cos x}$

2) Prove the following identities.

a) $\sin^2 x\,(1 + \cot^2 x) = 1$ **b)** $1 - \cos^2 x = \tan x \cos x \sin x$

c) $\cos x \tan^3 x = \sin x \tan^2 x$

d) $1 - 2\cos^2 \theta = \sin^4 \theta - \cos^4 \theta$

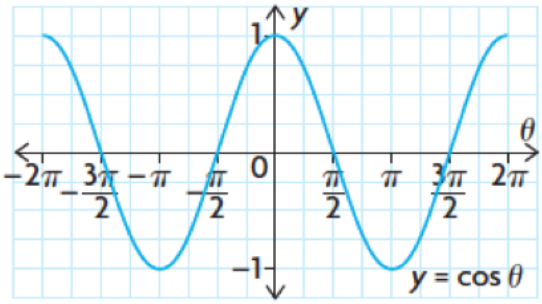
e) $\cot x + \frac{\sin x}{1+\cos x} = \csc x$

f) $\frac{\sec x}{\sin x} + \frac{\csc x}{\cos x} = \frac{2}{\sin x \cos x}$

g) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

h) $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

3)a) Use transformations and the cosine function to write three equivalent expressions for the following graph:



b) Transform your 3 equations from part a) to write the equation of 3 sine functions that represent the graph.

4) Use the co-function identities to write an expression that is equivalent to each of the following expressions.

a) $\sin\frac{\pi}{6}$ **b)** $\cos\frac{5\pi}{12}$ **c)** $\cos\frac{5\pi}{16}$

5) Write an expression that is equivalent to each of the following expressions, using the related acute angle.

a) $\sin\frac{7\pi}{8}$ **b)** $\cos\frac{13\pi}{12}$ **c)** $\cos\frac{11\pi}{6}$

6) Given that $\sin\frac{\pi}{6}=\frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{\pi}{3}=\frac{1}{2}$

7) Given that $\sin\frac{\pi}{6}=\frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{2\pi}{3}=-\frac{1}{2}$

8) Given that $\csc\frac{\pi}{4}=\sqrt{2}$, use an equivalent trigonometric expression to show that $\sec\frac{3\pi}{4}=-\sqrt{2}$

9) Given that $\cos\frac{3\pi}{11}\sim 0.6549$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $\sin\frac{5\pi}{22}$

b) $\sin\frac{17\pi}{22}$

Answer Key

1)a) $\tan x$ **b)** 1 **c)** $\sin^2 x$ **d)** $\cos^2 x$ **e)** $\sec x$ **f)** $\cos^2 x$ **g)** $\cos x$ **h)** $\csc^2 x$ **i)** $\tan x$ **j)** $\tan x$

3) Answers will vary depending but possible solutions are:
a) $y = \cos(\theta + 2\pi), y = \cos(\theta - 2\pi), y = \cos(\theta - 4\pi)$
b) $y = \sin(\theta + \frac{5\pi}{2}), y = \sin(\theta - \frac{3\pi}{2}), y = \sin(\theta - \frac{7\pi}{2})$

4)a) $\cos\frac{\pi}{3}$ **b)** $\sin\frac{\pi}{12}$ **c)** $\sin\frac{3\pi}{16}$ **5)a)** $\sin\frac{\pi}{8}$ **b)** $-\cos\frac{\pi}{12}$ **c)** $\cos\frac{\pi}{6}$

9)a) $\sin\frac{5\pi}{22} = \cos\frac{3\pi}{11} \sim 0.6549$ **b)** $\sin\frac{17\pi}{22} = \cos\left(-\frac{3\pi}{11}\right) = \cos\left(\frac{3\pi}{11}\right) \sim 0.6549$

W2 – 4.4 Compound Angle Formulas

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1) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}+\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

b) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}-\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

c) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

d) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}+\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

e) $\cos\frac{2\pi}{9}\cos\frac{5\pi}{18}-\sin\frac{2\pi}{9}\sin\frac{5\pi}{18}$

f) $\cos\frac{10\pi}{9}\cos\frac{5\pi}{18}+\sin\frac{10\pi}{9}\sin\frac{5\pi}{18}$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

b) $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

c) $\cos\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$

d) $\sin\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$

e) $\tan\left(\frac{\pi}{4}+\pi\right)$

f) $\tan\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$

4) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin\frac{7\pi}{12}$

b) $\sin\frac{5\pi}{12}$

c) $\cos\frac{11\pi}{12}$

d) $\cos\frac{5\pi}{12}$

e) $\sin\frac{13\pi}{12}$

f) $\cos\frac{17\pi}{12}$

g) $\sin\frac{19\pi}{12}$

h) $\cos\frac{23\pi}{12}$

5) Angles x and y are located in the first quadrant such that $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$. Determine exact values for $\cos x$ and $\sin y$.

6) Refer to the previous question. Determine an exact value for each of the following.

a) $\sin(x+y)$

b) $\sin(x-y)$

c) $\cos(x+y)$

d) $\cos(x-y)$

7) Use a compound angle formula to show that $\cos(2x) = \cos^2 x - \sin^2 x$

Answer Key

1)a) $\sin\left(\frac{\pi}{4}+\frac{\pi}{12}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ **b)** $\sin\left(\frac{\pi}{4}-\frac{\pi}{12}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$
c) $\cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$ **d)** $\cos\left(\frac{\pi}{4}-\frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
e) $\cos\left(\frac{2\pi}{9}+\frac{5\pi}{18}\right) = \cos\frac{\pi}{2} = 0$ **f)** $\cos\left(\frac{10\pi}{9}-\frac{5\pi}{18}\right) = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

3)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **b)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **c)** $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ **d)** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **e)** 1 **f)** $\frac{\sqrt{3}}{3}$

4)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **b)** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **c)** $\frac{-1-\sqrt{3}}{2\sqrt{2}}$ **d)** $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ **e)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **f)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **g)** $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ **h)** $\frac{1+\sqrt{3}}{2\sqrt{2}}$

5) $\cos x = \frac{4}{5}$ and $\sin y = \frac{12}{13}$

6)a) $\frac{63}{65}$ **b)** $-\frac{33}{65}$ **c)** $-\frac{16}{65}$ **d)** $\frac{56}{65}$

W3 – 4.5 Double Angle Formulas
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1) Express each of the following as a single trig ratio.

a) $2\sin(5x)\cos(5x)$ **b)** $\cos^2\theta - \sin^2\theta$ **c)** $1 - 2\sin^2(3x)$

d) $\frac{2\tan(4x)}{1-\tan^2(4x)}$ **e)** $4\sin\theta\cos\theta$ **f)** $2\cos^2\frac{\theta}{2} - 1$

2) Express each of the following as a single trig ratio and then evaluate

a) $2\sin 45°\cos 45°$ **b)** $\cos^2 30° - \sin^2 30°$ **c)** $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

d) $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$ **e)** $1 - 2\sin^2\frac{3\pi}{8}$ **f)** $2\tan 60°\cos^2 60°$

3) Use a double angle formula to rewrite each trig ratio

a) $\sin(4\theta)$ **b)** $\cos(3x)$ **c)** $\tan x$

d) $\cos(6\theta)$ **e)** $\sin x$ **f)** $\tan(5\theta)$

4) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos\theta = \frac{3}{5}$ and $0 \leq \theta \leq \frac{\pi}{2}$

5) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\tan\theta = -\frac{7}{24}$ and $\frac{\pi}{2} \leq \theta \leq \pi$

6) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\sin\theta = -\frac{12}{13}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$

7) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos\theta = -\frac{4}{5}$ and $\frac{\pi}{2} \leq \theta \leq \pi$

8) Determine the value of a in the equation $2\tan x - \tan(2x) + 2a = 1 - \tan(2x)\tan^2 x$

Answer Key

1)a) $\sin(10x)$ **b)** $\cos(2\theta)$ **c)** $\cos(6x)$ **d)** $\tan(8x)$ **e)** $2\sin(2\theta)$ **f)** $\cos\theta$

2)a) $\sin 90°; 1$ **b)** $\cos 60°; \frac{1}{2}$ **c)** $\sin\frac{\pi}{6}; \frac{1}{2}$ **d)** $\cos\frac{\pi}{6}; \frac{\sqrt{3}}{2}$ **e)** $\cos\frac{3\pi}{4}; -\frac{1}{\sqrt{2}}$ **f)** $\sin 120°; \frac{\sqrt{3}}{2}$

3)a) $2\sin(2\theta)\cos(2\theta)$ **b)** $2\cos^2(1.5x) - 1$ **c)** $\frac{2\tan(0.5x)}{1-\tan^2(0.5x)}$ **d)** $\cos^2(3\theta) - \sin^2(3\theta)$ **e)** $2\sin(0.5x)\cos(0.5x)$ **f)** $\frac{2\tan(2.5\theta)}{1-\tan^2(2.5\theta)}$

4) $\sin(2\theta) = \frac{24}{25}, \cos(2\theta) = -\frac{7}{25}, \tan(2\theta) = -\frac{24}{7}$

5) $\sin(2\theta) = -\frac{336}{625}, \cos(2\theta) = \frac{527}{625}, \tan(2\theta) = -\frac{336}{527}$

6) $\sin(2\theta) = -\frac{120}{169}, \cos(2\theta) = -\frac{119}{169}, \tan(2\theta) = \frac{120}{119}$

7) $\sin(2\theta) = -\frac{24}{25}, \cos(2\theta) = \frac{7}{25}, \tan(2\theta) = -\frac{24}{7}$

8) $a = \frac{1}{2}$

W4 – 4.5 Prove Trig Identities

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Prove each identity using the space on the following pages.

a) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

b) $\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$

c) $\sin(2x) = 2\sin x \cos x$

d) $\cos(2x) = \cos^2 x - \sin^2 x$

e) $\cot\theta - \tan\theta = 2\cot(2\theta)$

f) $\frac{\sin(2\theta)}{1-\cos(2\theta)} = \cot\theta$

g) $\sin x \sec x = \tan x$

h) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

i) $\frac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

j) $\frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x} = \sec x \csc x$

k) $\frac{1-\sin^2 x \cos^2 x}{\cos^4 x} = \tan^4 x + \tan^2 x + 1$

l) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

m) $\cot\theta - \tan\theta = 2\cot(2\theta)$

n) $(\sin x + \cos x)^2 = 1 + \sin(2x)$

o) $\frac{2\tan x}{1+\tan^2 x} = \sin(2x)$

p) $\sin\left(\frac{\pi}{4}+x\right) + \sin\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos x$

q) $\cos^4 x - \sin^4 x = \cos(2x)$

r) $\csc(2x) + \cot(2x) = \cot x$

s) $\cos(2x) = 2\cos^2 x - 1$

t) $\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$

u) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

v) $\cot x + \tan x = 2\csc(2x)$

W5 – 5.4 Solve Linear Trigonometric Equations
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1) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\sin x - \frac{1}{4} = 0$

b) $\cos x + 0.75 = 0$

c) $\tan x - 5 = 0$

d) $\sec x - 4 = 0$

e) $3 \cot x + 2 = 0$

f) $2 \csc x + 5 = 0$

2) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin x + \frac{\sqrt{3}}{2} = 0$

b) $\cos x - 0.5 = 0$

c) $\tan x - 1 = 0$

d) $\cot x + 1 = 0$

3) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\sin^2 x - 0.64 = 0$

b) $\cos^2 x - \frac{4}{9} = 0$

c) $\tan^2 x - 1.44 = 0$

d) $\sec^2 x - 2.5 = 0$

4) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin^2 x - \frac{1}{4} = 0$

b) $\cos^2 x - \frac{3}{4} = 0$

c) $\tan^2 x - 3 = 0$

d) $3\csc^2 x - 4 = 0$

5) Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $3\sin x = \sin x + 1$

b) $5\cos x - \sqrt{3} = 3\cos x$

c) $7\sec x = 7$

d) $2\csc x + 17 = 15 + \csc x$

Answer Key

1)a) 0.25, 2.89 **b)** 2.42, 3.86 **c)** 1.37, 4.51 **d)** 1.32, 4.97 **e)** 2.16, 5.3 **f)** 3.55, 5.87

2)a) $\frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $\frac{\pi}{3}, \frac{5\pi}{3}$ **c)** $\frac{\pi}{4}, \frac{5\pi}{4}$ **d)** $\frac{3\pi}{4}, \frac{7\pi}{4}$

3)a) 0.93, 2.21, 4.07, 5.36 **b)** 0.84, 2.3, 3.98, 5.44 **c)** 0.88, 2.27, 4.02, 5.41 **d)** 0.89, 2.26, 4.03, 5.4

4)a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **c)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **d)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

5)a) $\frac{\pi}{6}, \frac{5\pi}{6}$ **b)** $\frac{\pi}{6}, \frac{11\pi}{6}$ **c)** 0 or 2π **d)** $\frac{7\pi}{6}, \frac{11\pi}{6}$

W6 – 5.4 Solve Double Angle Trigonometric Equations

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Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible.

a) $\sin(2x) - 0.8 = 0$

b) $5\sin(2x) - 3 = 0$

c) $-4\sin(2x) + 3 = 0$

d) $\sin(2x) = \frac{1}{\sqrt{2}}$

e) $\sin(4x) = \frac{1}{2}$

f) $\sin(3x) = -\frac{\sqrt{3}}{2}$

g) $\cos(4x) = -\frac{1}{\sqrt{2}}$

h) $\cos(2x) = -\frac{1}{2}$

Answer Key

a) 0.46, 1.11, 3.61, 4.25 **b)** 0.32, 1.25, 3.46, 4.39 **c)** 0.42, 1.15, 3.57, 4.29 **d)** $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$ **e)** $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{37\pi}{24}, \frac{41\pi}{24}$

f) $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$ **g)** $\frac{3\pi}{16}, \frac{5\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}, \frac{19\pi}{16}, \frac{21\pi}{16}, \frac{27\pi}{16}, \frac{29\pi}{16}$ **h)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

W7 – 5.4 Solve Quadratic Trigonometric Equations
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1) Solve $\sin^2 x - 2\sin x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

2) Solve $\csc^2 x - \csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

3) Solve $2\sec^2 x - \sec x - 1 = 0$ on the interval $0 \leq x \leq 2\pi$

4) Solve $\tan^2 x - \tan x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$. Round answers to the nearest hundredth of a radian.

5) Solve $6\cos^2 x + 5\cos x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$

6) Solve $3\csc^2 x - 5\csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

7) Solve $2\tan^2 x - 5\tan x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

8) Solve $\cot x \csc^2 x = 2\cot x$ on the interval $0 \leq x \leq 2\pi$

9) Solve for θ to the nearest hundredth, where $0 \leq \theta \leq 2\pi$

a) $3\tan^2 \theta - 2\tan \theta = 1$

b) $12\sin^2\theta + \sin\theta - 6 = 0$

c) $5\cos(2\theta) - \cos\theta + 3 = 0$

Answer Key

1) $\frac{3\pi}{2}$ **2)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **3)** 0, 2π **4)** 1.25, 2.03, 4.39, 5.18 **5)** 0.84, 5.44 **6)** $\frac{\pi}{6}, \frac{5\pi}{6}$ **7)** 1.25, 2.68, 4.39, 5.82
8) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$ **9)a)** $\frac{\pi}{4}, 2.82, \frac{5\pi}{4}, 5.96$ **b)** 0.73, 2.41, 3.99, 5.44 **c)** $\frac{\pi}{3}, 1.98, 4.3, \frac{5\pi}{3}$