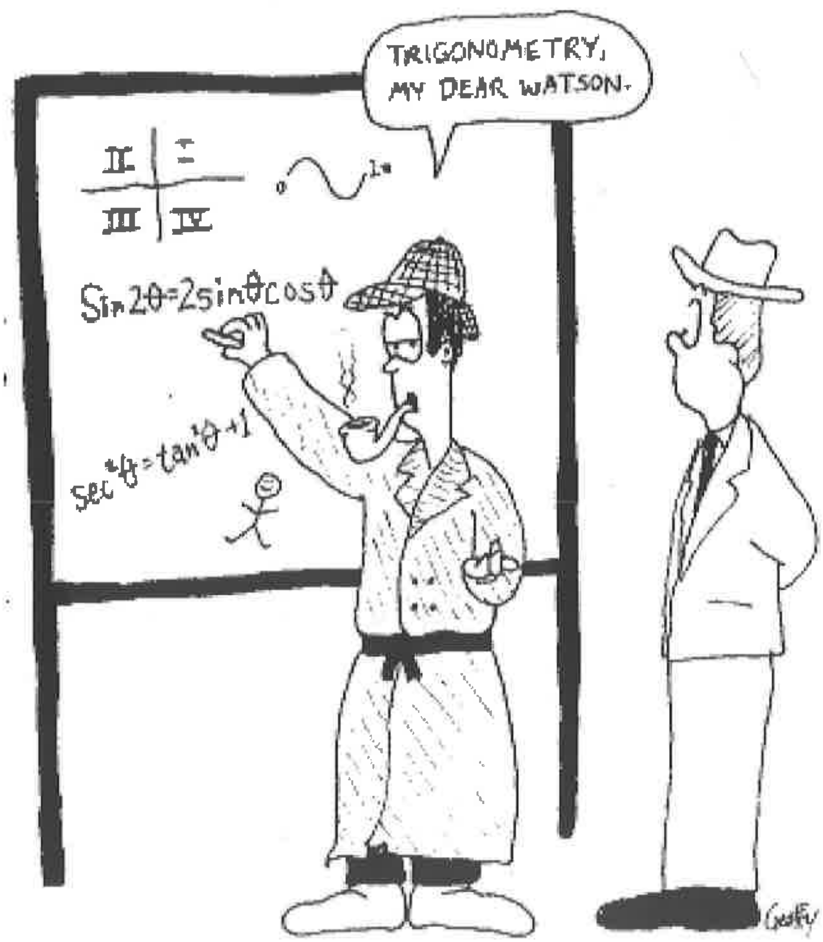
Chapter 4/5 Part 2- Trig Identities and Equations

WORKBOOK

MHF4U



W1 – 4.3 Co-function Identities
MHF4U
Jensen

SOLUTIONS

1) Simplify.

a) $\sin x\left(\frac{1}{\cos x}\right)$

$= \frac{\sin x}{\cos x}$

$= \tan x$

b) $(\cos x)(\sec x)$

$= \cos x\left(\frac{1}{\cos x}\right)$

$= 1$

c) $1 - \cos^2 x$

$= \sin^2 x$

d) $1 - \sin^2 x$

$= \cos^2 x$

e) $\frac{\tan x}{\sin x}$

$= \frac{\left(\frac{\sin x}{\cos x}\right)}{\sin x}$

$= \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$

$= \frac{1}{\cos x}$

$= \sec x$

f) $(1 - \sin x)(1 + \sin x)$

$= 1 - \sin^2 x$

$= \cos^2 x$

g) $\left(\frac{1}{\tan x}\right)\sin x$

$= \cot x(\sin x)$

$= \frac{\cos x}{\sin x}(\sin x)$

$= \cos x$

h) $\frac{1+\tan^2 x}{\tan^2 x}$

$= \frac{\sec^2 x}{\tan^2 x}$

$= \frac{\left(\frac{1}{\cos^2 x}\right)}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}$

$= \frac{1}{\sin^2 x}$

$= \csc^2 x$

i) $\frac{\sin x\cos x}{1-\sin^2 x}$

$= \frac{\sin x\cos x}{\cos^2 x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

j) $\frac{1-\cos^2 x}{\sin x\cos x}$

$= \frac{\sin^2 x}{\sin x\cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

2) Prove the following identities.

a) $\sin^2 x\,(1 + \cot^2 x) = 1$

LS	RS
$= \sin^2 x + \sin^2 x\cot^2 x$	$= 1$
$= \sin^2 x + \sin^2 x\left(\frac{\cos^2 x}{\sin^2 x}\right)$	
$= \sin^2 x + \cos^2 x$	
$= 1$	

LS = RS

b) $1 - \cos^2 x = \tan x\cos x\sin x$

LS	RS
$= 1 - \cos^2 x$	$= \tan x\cos x\sin x$
$= \sin^2 x$	$= \left(\frac{\sin x}{\cos x}\right)(\cos x)(\sin x)$
	$= \sin^2 x$

LS = RS

c) $\cos x \tan^3 x = \sin x \tan^2 x$

LS	RS
$= \cos x \tan^3 x$	$= \sin x \tan^2 x$
$= \cos x \left(\frac{\sin^3 x}{\cos^3 x}\right)$	$= \sin x \left(\frac{\sin^2 x}{\cos^2 x}\right)$
$= \frac{\sin^3 x}{\cos^2 x}$	$= \frac{\sin^3 x}{\cos^2 x}$

LS = RS

d) $1 - 2\cos^2\theta = \sin^4\theta - \cos^4\theta$

LS	RS
$= 1 - 2\cos^2\theta$	$= (\sin^2\theta)^2 - (\cos^2\theta)^2$
	$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$
	$= (\sin^2\theta - \cos^2\theta)(1)$
	$= 1 - \cos^2\theta - \cos^2\theta$
	$= 1 - 2\cos^2\theta$

LS = RS

e) $\cot x + \frac{\sin x}{1+\cos x} = \csc x$

LS	RS
$= \frac{\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$	$= \csc x$
	$= \frac{1}{\sin x}$
$= \frac{(1+\cos x)(\cos x) + \sin x(\sin x)}{\sin x(1+\cos x)}$	
$= \frac{\cos x + \cos^2 x + \sin^2 x}{\sin x(1+\cos x)}$	
$= \frac{\cos x + 1}{\sin x(1+\cos x)}$	
$= \frac{1}{\sin x}$	

LS = RS

f) $\frac{\sec x}{\sin x} + \frac{\csc x}{\cos x} = \frac{2}{\sin x \cos x}$

LS	RS
$= \frac{\left(\frac{1}{\cos x}\right)}{\sin x} + \frac{\left(\frac{1}{\sin x}\right)}{\cos x}$	$= \frac{2}{\sin x \cos x}$
$= \frac{1}{\cos x \sin x} + \frac{1}{\sin x \cos x}$	
$= \frac{2}{\sin x \cos x}$	

LS = RS

g) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

LS	RS
$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x(\cos x + \sin x)}$	$= 1 - \frac{\sin x}{\cos x}$
	$= \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}$
$= \frac{\cos x - \sin x}{\cos x}$	$= \frac{\cos x - \sin x}{\cos x}$

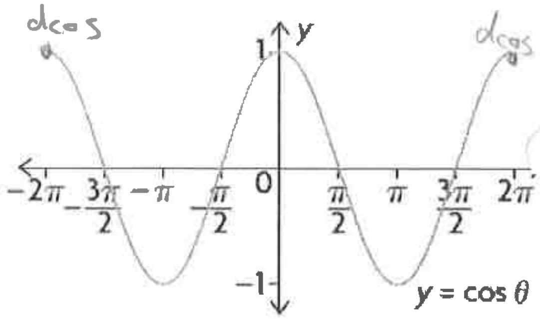
LS = RS

h) $\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\csc^2 x$

LS	RS
$= \frac{1}{1+\cos x} + \frac{1}{1-\cos x}$	$= 2\csc^2 x$
$= \frac{1-\cos x + 1+\cos x}{(1+\cos x)(1-\cos x)}$	$= 2\left(\frac{1}{\sin^2 x}\right)$
$= \frac{2}{1-\cos^2 x}$	$= \frac{2}{\sin^2 x}$
$= \frac{2}{\sin^2 x}$	

LS = RS

3)a) Use transformations and the cosine function to write three equivalent expressions for the following graph:



① $y = \cos(\theta - 2\pi)$

② $y = \cos(\theta + 2\pi)$

③ $y = \cos(\theta - 4\pi)$

b) Transform your 3 equations from part a) to write the equation of 3 sine functions that represent the graph.

$\cos x = \sin\left(x + \frac{\pi}{2}\right)$

① $\cos(\theta - 2\pi) = \sin\left[(\theta - 2\pi) + \frac{\pi}{2}\right] = \sin\left(\theta - \frac{3\pi}{2}\right)$

② $\cos(\theta + 2\pi) = \sin\left[(\theta + 2\pi) + \frac{\pi}{2}\right] = \sin\left(\theta + \frac{5\pi}{2}\right)$

③ $\cos(\theta - 4\pi) = \sin\left[\theta - 4\pi) + \frac{\pi}{2}\right] = \sin\left(\theta - \frac{7\pi}{2}\right)$

4) Use the co-function identities to write an expression that is equivalent to each of the following expressions.

a) $\sin\frac{\pi}{6}$

$= \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= \cos\left(\frac{2\pi}{6}\right)$

$= \cos\left(\frac{\pi}{3}\right)$

b) $\cos\frac{5\pi}{12}$

$= \sin\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)$

$= \sin\left(\frac{\pi}{12}\right)$

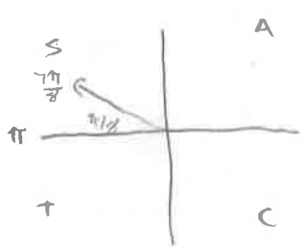
c) $\cos\frac{5\pi}{16}$

$= \sin\left(\frac{\pi}{2} - \frac{5\pi}{16}\right)$

$= \sin\left(\frac{3\pi}{16}\right)$

5) Write an expression that is equivalent to each of the following expressions, using the related acute angle.

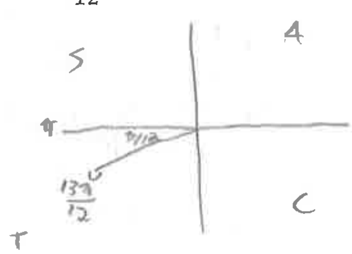
a) $\sin\frac{7\pi}{8}$



$\sin(\pi - x) = \sin x$

$\therefore \sin\left(\pi - \frac{\pi}{8}\right) = \sin\left(\frac{\pi}{8}\right)$

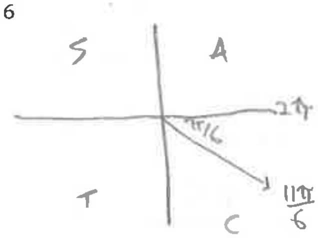
b) $\cos\frac{13\pi}{12}$



$\cos(\pi + x) = -\cos x$

$\therefore \cos\left(\pi + \frac{\pi}{12}\right) = -\cos\left(\frac{\pi}{12}\right)$

c) $\cos\frac{11\pi}{6}$



$\cos(2\pi - x) = \cos x$

$\cos\left(2\pi - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right)$

6) Given that $\sin\frac{\pi}{6} = \frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{\pi}{3} = \frac{1}{2}$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$
$$= \cos\left(\frac{2\pi}{6}\right)$$
$$= \cos\left(\frac{\pi}{3}\right)$$

$\therefore \sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

7) Given that $\sin\frac{\pi}{6} = \frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos\frac{2\pi}{3} = -\frac{1}{2}$

$$\cos\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$$
$$= \sin\left(-\frac{\pi}{6}\right)$$
$$= -\sin\left(\frac{\pi}{6}\right)$$

$\therefore \cos\left(\frac{2\pi}{3}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

8) Given that $\csc\frac{\pi}{4} = \sqrt{2}$, use an equivalent trigonometric expression to show that $\sec\frac{3\pi}{4} = -\sqrt{2}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

$$\cos\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)$$
$$= \sin\left(-\frac{\pi}{4}\right)$$
$$= -\sin\left(\frac{\pi}{4}\right)$$

If $\cos\frac{3\pi}{4} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$,

then $\sec\frac{3\pi}{4} = -\sqrt{2}$

9) Given that $\cos\frac{3\pi}{11} \sim 0.6549$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $\sin\frac{5\pi}{22}$

$$= \cos\left(\frac{\pi}{2} - \frac{5\pi}{22}\right)$$
$$= \cos\left(\frac{6\pi}{22}\right)$$
$$= \cos\left(\frac{3\pi}{11}\right)$$
$$\simeq 0.6549$$

b) $\sin\frac{17\pi}{22}$

$$= \cos\left(\frac{\pi}{2} - \frac{17\pi}{22}\right)$$
$$= \cos\left(-\frac{6\pi}{22}\right)$$
$$= \cos\left(\frac{6\pi}{22}\right)$$
$$= \cos\left(\frac{3\pi}{11}\right)$$
$$\simeq 0.6549$$

Answer Key

1)a) $\tan x$ **b)** 1 **c)** $\sin^2 x$ **d)** $\cos^2 x$ **e)** $\sec x$ **f)** $\cos^2 x$ **g)** $\cos x$ **h)** $\csc^2 x$ **i)** $\tan x$ **j)** $\tan x$

3) Answers will vary depending but possible solutions are:
a) $y = \cos(\theta + 2\pi)$, $y = \cos(\theta - 2\pi)$, $y = \cos(\theta - 4\pi)$
b) $y = \sin(\theta + \frac{5\pi}{2})$, $y = \sin(\theta - \frac{3\pi}{2})$, $y = \sin(\theta - \frac{7\pi}{2})$

4)a) $\cos\frac{\pi}{3}$ **b)** $\sin\frac{\pi}{12}$ **c)** $\sin\frac{3\pi}{16}$

5)a) $\sin\frac{\pi}{8}$ **b)** $-\cos\frac{\pi}{12}$ **c)** $\cos\frac{\pi}{6}$

9)a) $\sin\frac{5\pi}{22} = \cos\frac{3\pi}{11} \sim 0.6549$ **b)** $\sin\frac{17\pi}{22} = \cos\left(-\frac{3\pi}{11}\right) = \cos\left(\frac{3\pi}{11}\right) \sim 0.6549$

W2 – 4.4 Compound Angle Formulas
MHF4U
Jensen

SOLUTIONS

1) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}+\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{12}\right)$

$= \sin\left(\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3}\right)$

b) $\sin\frac{\pi}{4}\cos\frac{\pi}{12}-\cos\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \sin\left(\frac{\pi}{4}-\frac{\pi}{12}\right)$

$= \sin\left(\frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{6}\right)$

c) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3}\right)$

d) $\cos\frac{\pi}{4}\cos\frac{\pi}{12}+\sin\frac{\pi}{4}\sin\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{4}-\frac{\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{6}\right)$

$\cos\frac{2\pi}{9}\cos\frac{5\pi}{18}-\sin\frac{2\pi}{9}\sin\frac{5\pi}{18}$

$= \cos\left(\frac{2\pi}{9}+\frac{5\pi}{18}\right)$

$= \cos\left(\frac{9\pi}{18}\right)$

$= \cos\left(\frac{\pi}{2}\right)$

f) $\cos\frac{10\pi}{9}\cos\frac{5\pi}{18}+\sin\frac{10\pi}{9}\sin\frac{5\pi}{18}$

$= \cos\left(\frac{10\pi}{9}-\frac{5\pi}{18}\right)$

$= \cos\left(\frac{15\pi}{18}\right)$

$= \cos\left(\frac{5\pi}{6}\right)$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4}+\cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

b) $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$ $= \cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)-\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

c) $\cos\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$ $= \cos\frac{2\pi}{3}\cos\frac{\pi}{4}+\sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)+\frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1+\sqrt{3}}{2\sqrt{2}}$

d) $\sin\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$ $= \sin\frac{2\pi}{3}\cos\frac{\pi}{4}-\cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4}-\left(-\cos\frac{\pi}{3}\right)\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

e) $\tan\left(\frac{\pi}{4}+\pi\right) = \dfrac{\tan\frac{\pi}{4}+\tan\pi}{1-\tan\frac{\pi}{4}\tan\pi}$

$= \dfrac{1+0}{1-1(0)}$

$= \dfrac{1}{1}$

$= 1$

f) $\tan\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$

$= \dfrac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)}$

$= \dfrac{\frac{3}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{1+1}$

$= \dfrac{\left(\frac{2}{\sqrt{3}}\right)}{2}$

$= \dfrac{1}{\sqrt{3}}\dfrac{(\sqrt{3})}{(\sqrt{3})}$

$= \dfrac{\sqrt{3}}{3}$

4) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3}+\cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}}$

b) $\sin\frac{5\pi}{12} = \sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6}+\cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

c) $\cos\frac{11\pi}{12}$

$= \cos\left(\frac{3\pi}{12}+\frac{8\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4}+\frac{2\pi}{3}\right)$

$= \cos\frac{\pi}{4}\cos\frac{2\pi}{3}-\sin\frac{\pi}{4}\sin\frac{2\pi}{3}$

$= \frac{1}{\sqrt{2}}\left(\frac{-1}{2}\right)-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{-1-\sqrt{3}}{2\sqrt{2}}$

d) $\cos\frac{5\pi}{12} = \cos\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)-\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

e) $\sin\frac{13\pi}{12} = \sin\left(\frac{4\pi}{12}+\frac{9\pi}{12}\right)$

$= \sin\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{3\pi}{4}+\cos\frac{\pi}{3}\sin\frac{3\pi}{4}$

$= \frac{\sqrt{3}}{2}\left(-\frac{1}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \dfrac{-\sqrt{3}+1}{2\sqrt{2}}$

f) $\cos\frac{17\pi}{12} = \cos\left(\frac{9\pi}{12}+\frac{8\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4}+\frac{2\pi}{3}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{2\pi}{3}-\sin\frac{3\pi}{4}\sin\frac{2\pi}{3}$

$= \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{2}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{1-\sqrt{3}}{2\sqrt{2}}$

g) $\sin\frac{19\pi}{12}$ $= \sin\left(\frac{10\pi}{12}+\frac{9\pi}{12}\right)$

$= \sin\left(\frac{5\pi}{6}+\frac{3\pi}{4}\right)$

$= \sin\frac{5\pi}{6}\cos\frac{3\pi}{4} + \cos\frac{5\pi}{6}\sin\frac{3\pi}{4}$

$= \frac{1}{2}\left(\frac{-1}{\sqrt{2}}\right) + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{-1-\sqrt{3}}{2\sqrt{2}}$

h) $\cos\frac{23\pi}{12}$ $= \cos\left(\frac{8\pi}{12}+\frac{15\pi}{12}\right)$

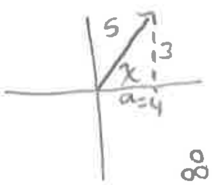
$= \cos\left(\frac{2\pi}{3}+\frac{5\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{5\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{5\pi}{4}$

$= \left(\frac{-1}{2}\right)\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{\sqrt{2}}\right)$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

5) Angles x and y are located in the first quadrant such that $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$. Determine exact values for $\cos x$ and $\sin y$.

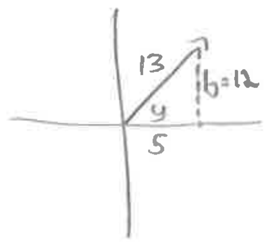


$a^2+3^2=5^2$

$a^2=16$

$a=4$

$\therefore \cos x = \frac{4}{5}$



$5^2+b^2=13^2$

$b^2=144$

$b=12$

$\therefore \sin y = \frac{12}{13}$

6) Refer to the previous question. Determine an exact value for each of the following.

a) $\sin(x+y)$

$= \sin x\cos y + \cos x\sin y$

$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)+\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$= \frac{3}{13}+\frac{48}{65}$

$= \frac{63}{65}$

b) $\sin(x-y)$ $= \sin x\cos y - \cos x\sin y$

$= \frac{3}{13} - \frac{48}{65}$

$= \frac{-33}{65}$

c) $\cos(x+y)$

$= \cos x\cos y - \sin x\sin y$

$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \frac{3}{5}\left(\frac{12}{13}\right)$

$= \frac{4}{13} - \frac{36}{65}$

$= \frac{-16}{65}$

d) $\cos(x-y)$

$= \cos x\cos y + \sin x\sin y$

$= \frac{4}{13}+\frac{36}{65}$

$= \frac{56}{65}$

7) Use a compound angle formula to show that $\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(2x) = \cos(x + x)$

$= \cos x \cdot \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

Answer Key

1)a) $\sin\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ **b)** $\sin\left(\frac{\pi}{4} - \frac{\pi}{12}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$
c) $\cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$ **d)** $\cos\left(\frac{\pi}{4} - \frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
e) $\cos\left(\frac{2\pi}{9} + \frac{5\pi}{18}\right) = \cos\frac{\pi}{2} = 0$ **f)** $\cos\left(\frac{10\pi}{9} - \frac{5\pi}{18}\right) = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

3)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **b)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **c)** $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ **d)** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **e)** 1 **f)** $\frac{\sqrt{3}}{3}$

4)a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **b)** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **c)** $\frac{-1-\sqrt{3}}{2\sqrt{2}}$ **d)** $\frac{-1+\sqrt{3}}{2\sqrt{2}}$ **e)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **f)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **g)** $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ **h)** $\frac{1+\sqrt{3}}{2\sqrt{2}}$

5) $\cos x = \frac{4}{5}$ and $\sin y = \frac{12}{13}$

6)a) $\frac{63}{65}$ **b)** $-\frac{33}{65}$ **c)** $-\frac{16}{65}$ **d)** $\frac{56}{65}$

W3 – 4.5 Double Angle Formulas
MHF4U
Jensen

SOLUTIONS

1) Express each of the following as a single trig ratio.

a) $2\sin(5x)\cos(5x)$

$= \sin[2(5x)]$

$= \sin(10x)$

b) $\cos^2\theta - \sin^2\theta$

$= \cos(2\theta)$

c) $1 - 2\sin^2(3x)$

$= \cos[2(3x)]$

$= \cos(6x)$

d) $\frac{2\tan(4x)}{1-\tan^2(4x)}$

$= \tan[2(4x)]$

$= \tan(8x)$

e) $4\sin\theta\cos\theta$

$= 2(2)\sin\theta\cos\theta$

$= 2\sin(2\theta)$

f) $2\cos^2\frac{\theta}{2} - 1$

$= \cos[2(\frac{\theta}{2})]$

$= \cos\theta$

2) Express each of the following as a single trig ratio and then evaluate

a) $2\sin 45°\cos 45°$

$= \sin(2\times 45°)$

$= \sin 90°$

b) $\cos^2 30° - \sin^2 30°$

$= \cos(2\times 30°)$

$= \cos 60°$

c) $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

$= \sin[2(\frac{\pi}{12})]$

$= \sin(\frac{\pi}{6})$

d) $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$

$= \cos[2(\frac{\pi}{12})]$

$= \cos(\frac{\pi}{6})$

e) $1 - 2\sin^2\frac{3\pi}{8}$

$= \cos[2(\frac{3\pi}{8})]$

$= \cos(\frac{3\pi}{4})$

f) $2\tan 60°\cos^2 60°$

$= 2\left(\frac{\sin 60°}{\cos 60°}\right)\cos^2 60°$

$= 2\sin 60°\cos 60°$

$= \sin[2(60)]$

$= \sin 120°$

3) Use a double angle formula to rewrite each trig ratio

a) $\sin(4\theta) = \sin[2(2\theta)]$

$= 2\sin(2\theta)\cos(2\theta)$

b) $\cos(3x) = \cos[2(\frac{3x}{2})]$

$= 2\cos^2(\frac{3x}{2}) - 1$

c) $\tan x = \tan[2(\frac{x}{2})]$

$= \frac{2\tan(\frac{x}{2})}{1-\tan^2(\frac{x}{2})}$

d) $\cos(6\theta) = \cos[2(3\theta)]$

$= \cos^2(3\theta) - \sin^2(3\theta)$

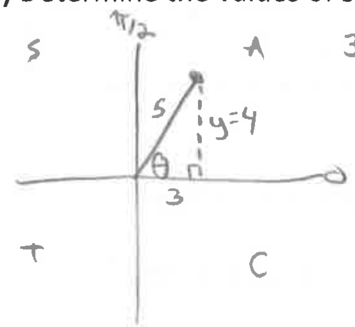
e) $\sin x = \sin[2(\frac{x}{2})]$

$= 2\sin(\frac{x}{2})\cos(\frac{x}{2})$

f) $\tan(5\theta) = \tan[2(\frac{5\theta}{2})]$

$= \frac{2\tan(\frac{5\theta}{2})}{1-\tan^2(\frac{5\theta}{2})}$

4) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos\theta = \frac{3}{5}$ and $0 \le \theta \le \frac{\pi}{2}$



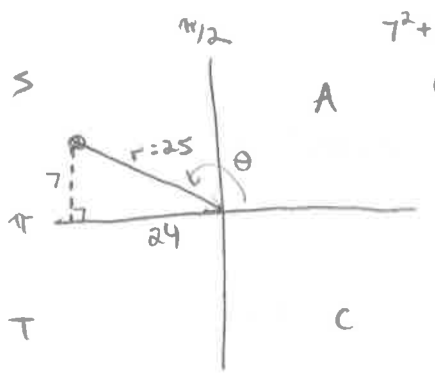
$3^2+y^2=5^2$
$y^2=16$
$y=4$

$\sin(2\theta) = 2\sin\theta\cos\theta$
$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$
$= \frac{24}{25}$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$
$= \frac{9}{25} - \frac{16}{25}$
$= -\frac{7}{25}$

$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$
$= \frac{\left(\frac{24}{25}\right)}{\left(-\frac{7}{25}\right)}$
$= -\frac{24}{7}$

5) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\tan\theta = -\frac{7}{24}$ and $\frac{\pi}{2} \le \theta \le \pi$



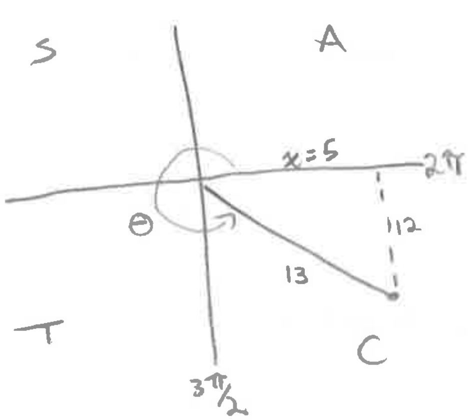
$7^2+24^2=r^2$
$625=r^2$
$r=25$

$\sin(2\theta) = 2\sin\theta\cos\theta$
$= 2\left(\frac{7}{25}\right)\left(\frac{-24}{25}\right)$
$= \frac{-336}{625}$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$= \left(\frac{-24}{25}\right)^2 - \left(\frac{7}{25}\right)^2$
$= \frac{576}{625} - \frac{49}{625}$
$= \frac{527}{625}$

$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$
$= \frac{\left(\frac{-336}{625}\right)}{\left(\frac{527}{625}\right)}$
$= \frac{-336}{527}$

6) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\sin\theta = -\frac{12}{13}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$



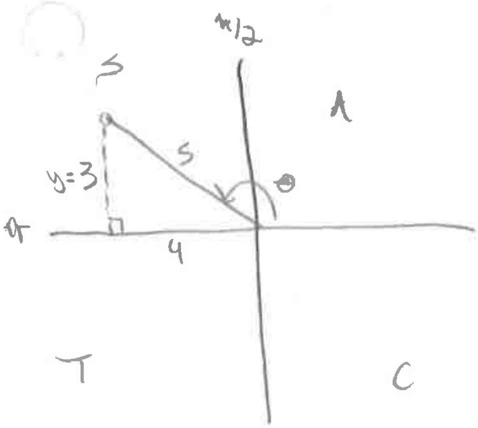
$x^2+12^2=13^2$
$x^2=25$
$x=5$

$\sin(2\theta) = 2\sin\theta\cos\theta$
$= 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right)$
$= \frac{-120}{169}$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$= \left(\frac{5}{13}\right)^2 - \left(\frac{-12}{13}\right)^2$
$= \frac{25}{169} - \frac{144}{169}$
$= \frac{-119}{169}$

$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$
$= \frac{\left(\frac{-120}{169}\right)}{\left(\frac{-119}{169}\right)}$
$= \frac{120}{119}$

7) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos\theta = -\frac{4}{5}$ and $\frac{\pi}{2} \le \theta \le \pi$



$4^2+y^2=5^2$

$y^2=9$

$y=3$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right)$

$= \frac{-24}{25}$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)}$

$= \frac{\left(\frac{-24}{25}\right)}{\left(\frac{7}{25}\right)}$

$= -\frac{24}{7}$

8) Determine the value of a in the equation $2\tan x - \tan(2x) + 2a = 1 - \tan(2x)\tan^2 x$

$2\tan x = \tan(2x)\left[1-\tan^2 x\right] - 2a + 1$

$\frac{2\tan x}{1-\tan^2 x} = \frac{\tan(2x)\left[1-\tan^2 x\right] - 2a + 1}{1-\tan^2 x}$

$\tan(2x) = \tan(2x) + \frac{-2a+1}{1-\tan^2 x}$

$0 = -2a+1$

$-1 = -2a$

$a = \frac{1}{2}$

Answer Key

1)a) $\sin(10x)$ **b)** $\cos(2\theta)$ **c)** $\cos(6x)$ **d)** $\tan(8x)$ **e)** $2\sin(2\theta)$ **f)** $\cos\theta$

2)a) $\sin 90°;\ 1$ **b)** $\cos 60°;\ \frac{1}{2}$ **c)** $\sin\frac{\pi}{6};\ \frac{1}{2}$ **d)** $\cos\frac{\pi}{6};\ \frac{\sqrt{3}}{2}$ **e)** $\cos\frac{3\pi}{4};\ -\frac{1}{\sqrt{2}}$ **f)** $\sin 120°;\ \frac{\sqrt{3}}{2}$

3)a) $2\sin(2\theta)\cos(2\theta)$ **b)** $2\sin^2(1.5x) - 1$ **c)** $\frac{2\tan(0.5x)}{1-\tan^2(0.5x)}$ **d)** $\cos^2(3\theta) - \sin^2(3\theta)$ **e)** $2\sin(0.5x)\cos(0.5x)$ **f)** $\frac{2\tan(2.5\theta)}{1-\tan^2(2.5\theta)}$

4) $\sin(2\theta) = \frac{24}{25}, \cos(2\theta) = -\frac{7}{25}, \tan(2\theta) = -\frac{24}{7}$

5) $\sin(2\theta) = -\frac{336}{625}, \cos(2\theta) = \frac{527}{625}, \tan(2\theta) = -\frac{336}{527}$

in$(2\theta) = -\frac{120}{169}, \cos(2\theta) = -\frac{119}{169}, \tan(2\theta) = \frac{120}{119}$

7) $\sin(2\theta) = -\frac{24}{25}, \cos(2\theta) = \frac{7}{25}, \tan(2\theta) = -\frac{24}{7}$

8) $a = \frac{1}{2}$

W4 – 4.5 Prove Trig Identities
MHF4U
Jensen

SOLUTIONS

Prove each identity using the space on the following pages.

a) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

b) $\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$

c) $\sin(2x) = 2\sin x \cos x$

d) $\cos(2x) = \cos^2 x - \sin^2 x$

e) $\cot\theta - \tan\theta = 2\cot(2\theta)$

f) $\frac{\sin(2\theta)}{1-\cos(2\theta)} = \cot\theta$

g) $\sin x \sec x = \tan x$

h) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

i) $\frac{\sec\theta - 1}{1-\cos\theta} = \sec\theta$

j) $\frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x} = \sec x \csc x$

k) $\frac{1-\sin^2 x \cos^2 x}{\cos^4 x} = \tan^4 x + \tan^2 x + 1$

l) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

m) $\cot\theta - \tan\theta = 2\cot(2\theta)$

n) $(\sin x + \cos x)^2 = 1 + \sin(2x)$

o) $\frac{2\tan x}{1+\tan^2 x} = \sin(2x)$

p) $\sin\left(\frac{\pi}{4}+x\right) + \sin\left(\frac{\pi}{4}-x\right) = \sqrt{2}\cos x$

q) $\cos^4 x - \sin^4 x = \cos(2x)$

r) $\csc(2x) + \cot(2x) = \cot x$

s) $\cos(2x) = 2\cos^2 x - 1$

t) $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

u) $\frac{\cos(2x)+1}{\sin(2x)} = \cot x$

v) $\cot x + \tan x = 2\csc(2x)$

a)

LS	RS
$= \sin(x+y)$	$= \sin x \cos y + \cos x \sin y$
$= \cos\left[\frac{\pi}{2} - (x+y)\right]$	
$= \cos\left[\left(\frac{\pi}{2} - x\right) - y\right]$	
$= \cos\left(\frac{\pi}{2} - x\right)\cos y + \sin\left(\frac{\pi}{2} - x\right)\sin y$	
$= \sin x \cos y + \cos x \sin y$	

LS = RS

b)

LS	RS
$= \tan(x-y)$	$= \frac{\tan x - \tan y}{1+\tan x \tan y}$
$= \frac{\sin(x-y)}{\cos(x-y)}$	
$= \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y + \sin x \sin y}\left(\frac{\frac{1}{\cos x \cos y}}{\frac{1}{\cos x \cos y}}\right)$	
$= \frac{\frac{\sin x \cos y}{\cos x \cos y} - \frac{\cos x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y}{\cos x \cos y} + \frac{\sin x \sin y}{\cos x \cos y}}$	
$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$	

c)

LS	RS
$= \sin(2x)$	$= 2\sin x\cos x$
$= \sin(x+x)$	
$= \sin x\cos x + \cos x\sin x$	
$= 2\sin x\cos x$	

LS = RS

d)

LS	RS
$= \cos(2x)$	$= \cos^2 x - \sin^2 x$
$= \cos(x+x)$	
$= \cos x\cos x - \sin x\sin x$	
$= \cos^2 x - \sin^2 x$	

LS = RS

e)

LS	RS
$= \cot\theta - \tan\theta$	$= 2\cot(2\theta)$
$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$	$= \frac{2\cos(2\theta)}{\sin(2\theta)}$
$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$	$= \frac{\cancel{2}(\cos^2\theta - \sin^2\theta)}{\cancel{2}\sin\theta\cos\theta}$
	$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$

LS = RS

f)

LS	RS
$= \frac{\sin(2\theta)}{1-\cos(2\theta)}$	$= \cot\theta$
$= \frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)}$	
$= \frac{2\sin\theta\cos\theta}{1-1+2\sin^2\theta}$	
$= \frac{\cancel{2}\cancel{\sin\theta}\cos\theta}{\cancel{2}\sin^{\cancel{2}}\theta}$	
$= \frac{\cos\theta}{\sin\theta}$	
$= \cot\theta$	

LS = RS

g)

LS	RS
$= \sin x \sec x$	$= \tan x$
$= \sin x \left(\frac{1}{\cos x}\right)$	$= \frac{\sin x}{\cos x}$
$= \frac{\sin x}{\cos x}$	

LS=RS

h)

LS	RS
$= \frac{1-\sin x}{\cos x}$	$= \frac{\cos x}{1+\sin x} \frac{(1-\sin x)}{(1-\sin x)}$
	$= \frac{\cos x(1-\sin x)}{1-\sin^2 x}$
	$= \frac{\cancel{\cos x}(1-\sin x)}{\cos^2 x}$
	$= \frac{1-\sin x}{\cos x}$

LS=RS

i)

LS	RS
$= \frac{\sec\theta - 1}{1-\cos\theta}$	$= \sec\theta$
$= \frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{1-\cos\theta}$	$= \frac{1}{\cos\theta}$
$= \frac{\left(\frac{1-\cos\theta}{\cos\theta}\right)}{1-\cos\theta}$	
$= \left(\frac{\cancel{1-\cos\theta}}{\cos\theta}\right)\left(\frac{1}{\cancel{1-\cos\theta}}\right)$	
$= \frac{1}{\cos\theta}$	

LS=RS

j)

LS	RS
$= \frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x}$	$= \sec x \csc x$
$= \frac{\sin x(\sin x - \cos x) + \cos x(\sin x + \cos x)}{\cos x \sin x}$	$= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$
$= \frac{\sin^2 x - \sin x\cos x + \cos x \sin x + \cos^2 x}{\cos x \sin x}$	$= \frac{1}{\cos x \sin x}$
$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$	
$= \frac{1}{\cos x \sin x}$	

LS=RS

k) LS | RS

LS

$$= \frac{1 - \sin^2 x \cos^2 x}{\cos^4 x}$$

RS

$$= \tan^4 x + \tan^2 x + 1$$

$$= \frac{\sin^4 x}{\cos^4 x} + \frac{\sin^2 x \,(\cos^2 x)}{\cos^2 x \,(\cos^2 x)} + \frac{\cos^4 x}{\cos^4 x}$$

$$= \frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}$$

$$= \frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}$$

$$= \frac{(\sin^2 x)^2 + \sin^2 x \cos^2 x + (\cos^2 x)^2}{\cos^4 x}$$

$$= \frac{\sin^2 x(1 - \cos^2 x) + \sin^2 x \cos^2 x + \cos^2 x(1 - \sin^2 x)}{\cos^4 x}$$

$$= \frac{\sin^2 x - \sin^2 x \cos^2 x + \sin^2 x \cos^2 x + \cos^2 x - \sin^2 x \cos^2 x}{\cos^4 x}$$

$$= \frac{1 - \sin^2 x \cos^2 x}{\cos^4 x}$$

LS = RS

l)

LS	RS
$= \frac{\cos(2x)+1}{\sin(2x)}$	$= \cot x$
	$= \frac{\cos x}{\sin x}$
$= \frac{2\cos^2 x - 1 + 1}{2\sin x \cos x}$	
$= \frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x \cancel{\cos x}}$	
$= \frac{\cos x}{\sin x}$	

LS = RS

m)

LS	RS
$= \cot\theta - \tan\theta$	$= 2\cot(2\theta)$
$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$	$= \frac{2\cos(2\theta)}{\sin(2\theta)}$
$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$	$= \frac{\cancel{2}\cos(2\theta)}{\cancel{2}\sin\theta\cos\theta}$
$= \frac{\cos(2\theta)}{\sin\theta\cos\theta}$	$= \frac{\cos(2\theta)}{\sin\theta\cos\theta}$

LS = RS

n)

LS	RS
$= (\sin x + \cos x)^2$	$= 1 + \sin(2x)$
$= (\sin x + \cos x)(\sin x + \cos x)$	$= 1 + 2\sin x \cos x$
$= \sin^2 x + 2\sin x\cos x + \cos^2 x$	
$= 1 + 2\sin x \cos x$	

LS = RS

o)

LS	RS
$= \frac{2\tan x}{1+\tan^2 x}$	$= \sin(2x)$
$= \frac{2\left(\frac{\sin x}{\cos x}\right)}{\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}}$	$= 2\sin x \cos x$
$= \frac{\left(\frac{2\sin x}{\cos x}\right)}{\left(\frac{1}{\cos^2 x}\right)}$	
$= \left(\frac{2\sin x}{\cancel{\cos x}}\right)\left(\frac{\cos^{\cancel{2}} x}{1}\right)$	
$= 2\sin x \cos x$	

LS = RS

p)

LS	RS
$= \sin\left(\frac{\pi}{4}+x\right) + \sin\left(\frac{\pi}{4}-x\right)$	$= \sqrt{2}\cos x$
$= \sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$	
$= \frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}$	
$= \frac{\cos x + \cos x}{\sqrt{2}}$	
$= \frac{2\cos x}{\sqrt{2}}\frac{(\sqrt{2})}{(\sqrt{2})}$	
$= \frac{\cancel{2}\sqrt{2}\cos x}{\cancel{2}}$	
$= \sqrt{2}\cos x$	

LS=RS

q)

LS	RS
$= \cos^4 x - \sin^4 x$	$= \cos(2x)$
$= (\cos^2 x)^2 - (\sin^2 x)^2$	$= \cos^2 x - \sin^2 x$
$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$	
$= \cos^2 x - \sin^2 x$	

LS=RS

r)

LS	RS
$= \csc(2x) + \cot(2x)$	$= \cot x$
$= \frac{1}{\sin(2x)} + \frac{\cos(2x)}{\sin(2x)}$	$= \frac{\cos x}{\sin x}$
$= \frac{1 + \cos(2x)}{\sin(2x)}$	
$= \frac{1 + 2\cos^2 x - 1}{2\sin x\cos x}$	
$= \frac{\cancel{2}\cos^2 x}{\cancel{2}\sin x\cos x}$	
$= \frac{\cos x}{\sin x}$	

LS=RS

s)

LS	RS
$= \cos(2x)$	$= 2\cos^2 x - 1$
$= \cos(x+x)$	
$= \cos x\cos x - \sin x\sin x$	
$= \cos^2 x - \sin^2 x$	
$= \cos^2 x - (1-\cos^2 x)$	
$= \cos^2 x - 1 + \cos^2 x$	
$= 2\cos^2 x - 1$	

LS=RS

t)

LS	RS
$= \sin\left(\frac{3\pi}{2} - x\right)$	$= -\cos x$
$= \sin\frac{3\pi}{2}\cos x - \cos\frac{3\pi}{2}\sin x$	
$= (-1)\cos x - 0\sin x$	
$= -\cos x$	

LS=RS

u)

LS	RS
$= \dfrac{\cos(2x)+1}{\sin(2x)}$	$= \cot x$
$= \dfrac{2\cos^2 x - 1 + 1}{2\sin x\cos x}$	$= \dfrac{\cos x}{\sin x}$
$= \dfrac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\,\cancel{\cos x}}$	
$= \dfrac{\cos x}{\sin x}$	

LS=RS

v)

LS	RS
$= \cot x + \tan x$	$= 2\csc(2x)$
$= \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$	$= \dfrac{2}{\sin(2x)}$
$= \dfrac{\cos^2 x + \sin^2 x}{\sin x\cos x}$	$= \dfrac{2}{2\sin x\cos x}$
$= \dfrac{1}{\sin x\cos x}$	$= \dfrac{1}{\sin x\cos x}$

LS=RS

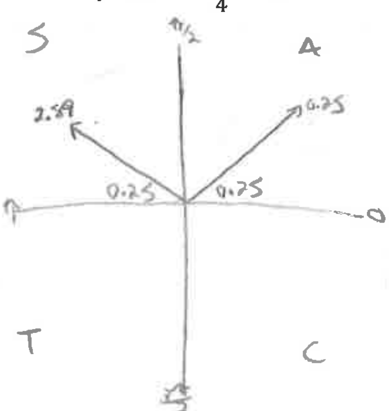
W5 – 5.4 Solve Linear Trigonometric Equations
MHF4U
Jensen

SOLUTIONS

1) Determine approximate solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian.

a) $\sin x - \frac{1}{4} = 0$

$\sin x = \frac{1}{4}$



$x_1 = \sin^{-1}\left(\frac{1}{4}\right)$

$\boxed{\simeq 0.25}$

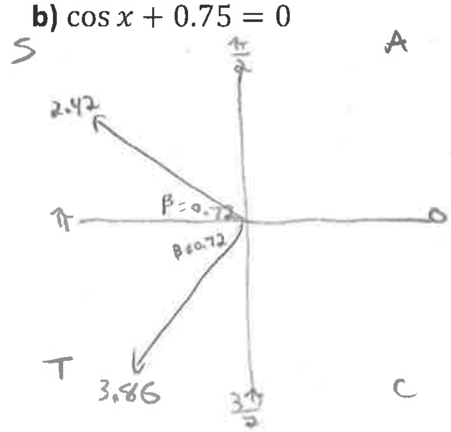
$x_2 = \pi - 0.25$

$\boxed{\simeq 2.89}$

$\boxed{\sin 0.25 = \sin 2.89 = \frac{1}{4}}$

b) $\cos x + 0.75 = 0$

$\cos x = -0.75$



$x_1 = \cos^{-1}(-0.75)$

$\boxed{\simeq 2.42}$

$\beta = \pi - 2.42$
$= 0.72$

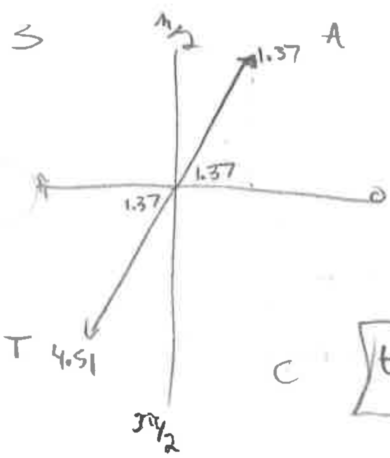
$x_2 = \pi + \beta$
$= \pi + 0.72$

$\boxed{= 3.86}$

$\boxed{\cos 0.72 = \cos 3.86 = -0.75}$

c) $\tan x - 5 = 0$

$\tan x = 5$



$x_1 = \tan^{-1}(5)$

$\boxed{= 1.37}$

$x_2 = \pi + 1.37$

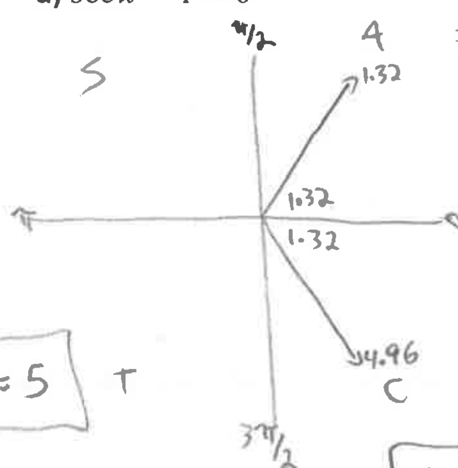
$\boxed{= 4.51}$

$\boxed{\tan 1.37 = \tan 4.51 = 5}$

d) $\sec x - 4 = 0$

$\sec x = 4$

$\cos x = \frac{1}{4}$



$x_1 = \cos^{-1}\left(\frac{1}{4}\right)$

$\boxed{= 1.32}$

$x_2 = 2\pi - 1.32$

$\boxed{x_2 = 4.96}$

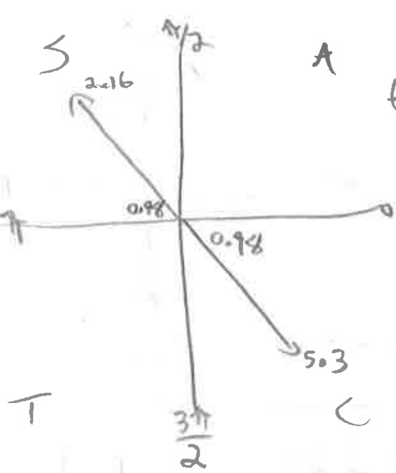
$\boxed{\sec 1.32 = \sec 4.96 = 4}$

e) $3\cot x + 2 = 0$

$3\cot x = -2$

$\cot x = -\frac{2}{3}$

$\tan x = -\frac{3}{2}$



$x_1 = \tan^{-1}\left(-\frac{3}{2}\right)$

$= -0.98 + 2\pi$

$\boxed{= 5.3}$

$x_2 = \pi - 0.98$

$\boxed{= 2.16}$

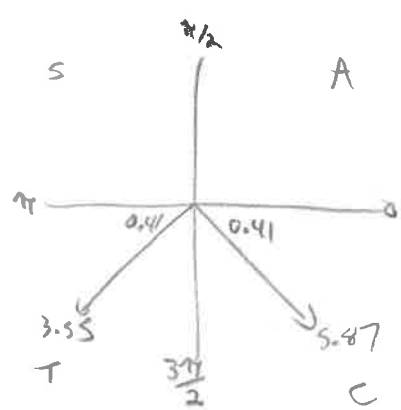
$\boxed{\cot 5.3 = \cot 2.16 = -\frac{2}{3}}$

f) $2\csc x + 5 = 0$

$2\csc x = -5$

$\csc x = -\frac{5}{2}$

$\sin x = -\frac{2}{5}$



$x_1 = \sin^{-1}\left(-\frac{2}{5}\right)$

$= -0.41 + 2\pi$

$\boxed{= 5.87}$

$x_2 = \pi + 0.41$

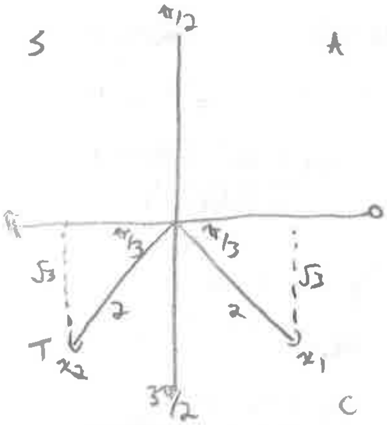
$\boxed{= 3.55}$

$\boxed{\csc 5.87 = \csc 3.55 = -\frac{5}{2}}$

2) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin x + \frac{\sqrt{3}}{2} = 0$

$\sin x = -\frac{\sqrt{3}}{2}$



from special Δ: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q3 + Q4

$x_1 = 2\pi - \frac{\pi}{3}$

$\boxed{x_1 = \frac{5\pi}{3}}$

$x_2 = \pi + \frac{\pi}{3}$

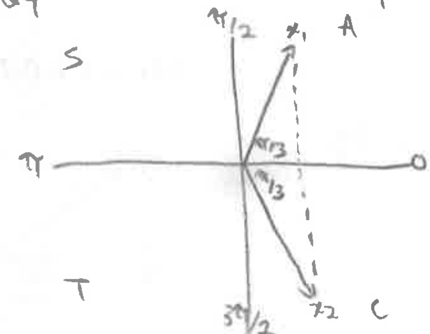
$\boxed{x_2 = \frac{4\pi}{3}}$

b) $\cos x - 0.5 = 0$

$\cos x = \frac{1}{2}$

from special Δ; $\cos\frac{\pi}{3} = \frac{1}{2}$

place $\frac{\pi}{3}$ in Q1 + Q4



$\boxed{x_1 = \frac{\pi}{3}}$

$x_2 = 2\pi - \frac{\pi}{3}$

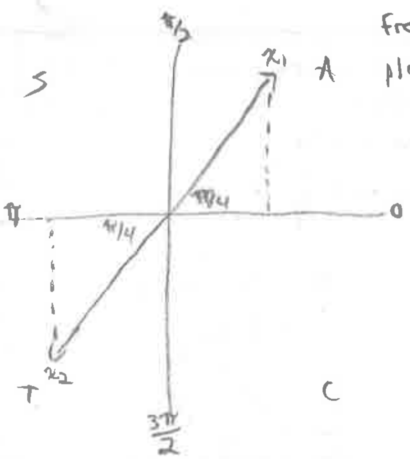
$\boxed{x_2 = \frac{5\pi}{3}}$

c) $\tan x - 1 = 0$

$\tan x = 1$

From special Δ; $\tan\frac{\pi}{4} = 1$

place in Q1 + Q3



$\boxed{x_1 = \frac{\pi}{4}}$

$x_2 = \pi + \frac{\pi}{4}$

$\boxed{x_2 = \frac{5\pi}{4}}$

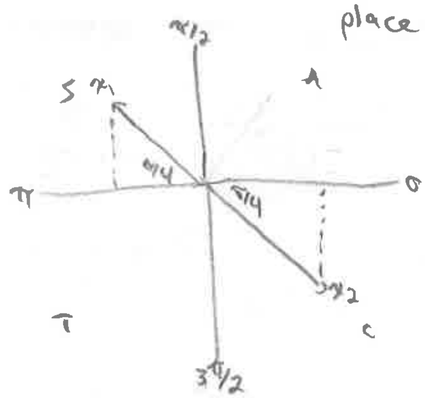
d) $\cot x + 1 = 0$

$\cot x = -1$

$\tan x = -1$

From special Δ; $\tan\frac{\pi}{4} = 1$

place $\frac{\pi}{4}$ in Q2 + Q4



$x_1 = \pi - \frac{\pi}{4}$

$\boxed{x_1 = \frac{3\pi}{4}}$

$x_2 = 2\pi - \frac{\pi}{4}$

$\boxed{x_2 = \frac{7\pi}{4}}$

3) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

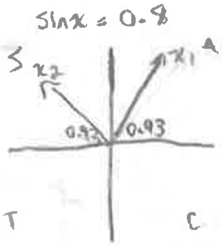
a) $\sin^2 x - 0.64 = 0$

$\sin^2 x = 0.64$

$\sin x = \pm\sqrt{0.64}$

$\sin x = \pm 0.8$

$\sin x = 0.8$



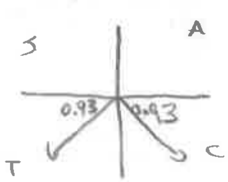
$x_1 = \sin^{-1}(0.8)$

$\boxed{x_1 = 0.93}$

$x_2 = \pi - 0.93$

$\boxed{x_2 = 2.21}$

$\sin x = -0.8$



$x_3 = \sin^{-1}(-0.8)$

$x_3 = -0.93 + 2\pi$

$\boxed{x_3 = 5.36}$

$x_4 = \pi + 0.93$

$\boxed{x_4 = 4.07}$

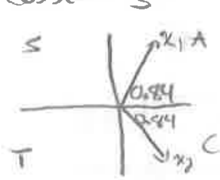
b) $\cos^2 x - \frac{4}{9} = 0$

$\cos^2 x = \frac{4}{9}$

$\cos x = \pm\sqrt{\frac{4}{9}}$

$\cos x = \pm\frac{2}{3}$

$\cos x = \frac{2}{3}$



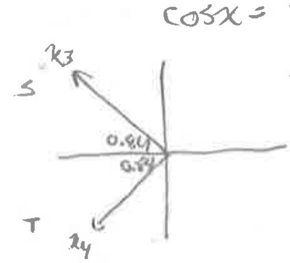
$x_1 = \cos^{-1}\left(\frac{2}{3}\right)$

$\boxed{x_1 = 0.84}$

$x_2 = 2\pi - 0.84$

$\boxed{x_2 = 5.44}$

$\cos x = -\frac{2}{3}$



$x_3 = \cos^{-1}\left(-\frac{2}{3}\right)$

$\boxed{x_3 = 2.30}$

$x_4 = \pi + 0.84$

$\boxed{x_4 = 3.98}$

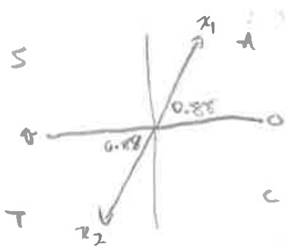
c) $\tan^2 x - 1.44 = 0$

$\tan^2 x = 1.44$

$\tan x = \pm\sqrt{1.44}$

$\tan x = \pm 1.2$

$\tan x = 1.2$



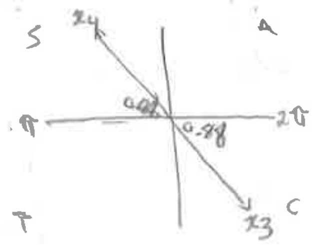
$x_1 = \tan^{-1}(1.2)$

$\boxed{x_1 = 0.88}$

$x_2 = \pi + 0.88$

$\boxed{x_2 = 4.02}$

$\tan x = -1.2$



$x_3 = \tan^{-1}(-1.2)$

$x_3 = -0.88 + 2\pi$

$\boxed{x_3 = 5.4}$

$x_4 = \pi - 0.88$

$\boxed{x_4 = 2.26}$

d) $\sec^2 x - 2.5 = 0$

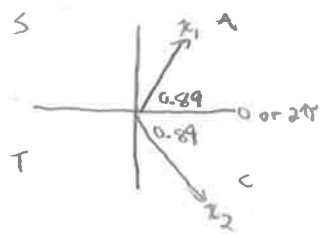
$\sec^2 x = 2.5$

$\cos^2 x = \frac{1}{2.5}$

$\cos x = \pm\sqrt{\frac{1}{2.5}}$

$\cos x = \pm 0.63$

$\cos x = 0.63$



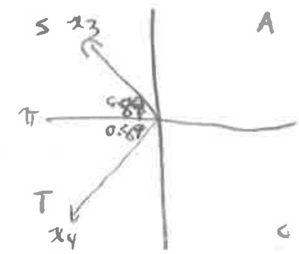
$x_1 = \cos^{-1}(0.63)$

$\boxed{x_1 = 0.89}$

$x_2 = 2\pi - 0.89$

$\boxed{x_2 = 5.39}$

$\cos x = -0.63$



$x_3 = \cos^{-1}(-0.63)$

$\boxed{x_3 = 2.25}$

$x_4 = \pi + 0.89$

$\boxed{x_4 = 4.03}$

Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin^2 x - \frac{1}{4} = 0$

$\sin^2 x = \frac{1}{4}$

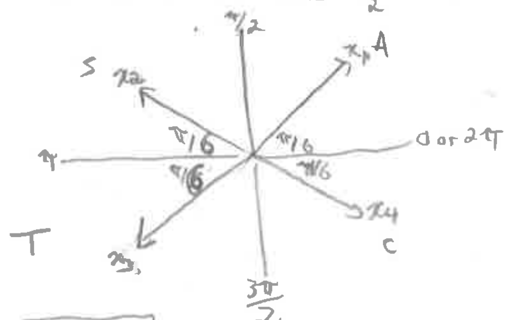
$\sin x = \pm\sqrt{\frac{1}{4}}$

$\sin x = \pm\frac{1}{2}$

from special Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2 for $\sin x = \frac{1}{2}$

Place in Q3 + Q4 for $\sin x = -\frac{1}{2}$



$\boxed{x_1 = \frac{\pi}{6}}$

$x_2 = \pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{5\pi}{6}}$

$x_3 = \pi + \frac{\pi}{6}$

$\boxed{x_3 = \frac{7\pi}{6}}$

$x_4 = 2\pi - \frac{\pi}{6}$

$\boxed{x_4 = \frac{11\pi}{6}}$

b) $\cos^2 x - \frac{3}{4} = 0$

$\cos^2 x = \frac{3}{4}$

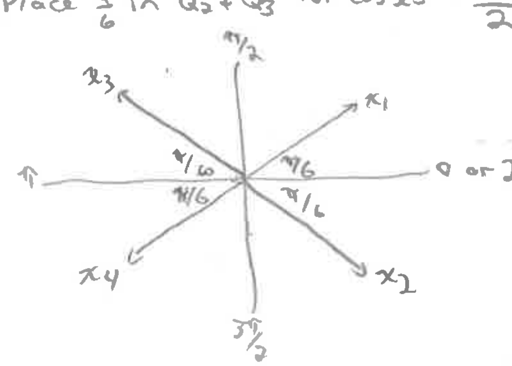
$\cos x = \pm\sqrt{\frac{3}{4}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

From special Δ; $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{6}$ in $Q_1 + Q_4$ for $\cos x = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{6}$ in $Q_2 + Q_3$ for $\cos x = -\frac{\sqrt{3}}{2}$



$\boxed{x_1 = \frac{\pi}{6}}$

$x_2 = 2\pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{11\pi}{6}}$

$x_3 = \pi - \frac{\pi}{6}$

$\boxed{x_3 = \frac{5\pi}{6}}$

$x_4 = \pi + \frac{\pi}{6}$

$\boxed{x_4 = \frac{7\pi}{6}}$

c) $\tan^2 x - 3 = 0$

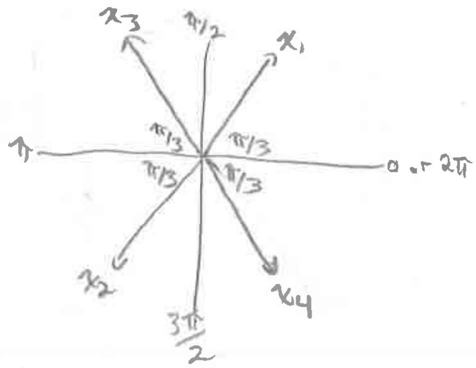
$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

From special Δ; $\tan\frac{\pi}{3} = \sqrt{3}$

place $\frac{\pi}{3}$ in Q1 + Q3 for $\tan x = \sqrt{3}$

place $\frac{\pi}{3}$ in Q2 + Q4 for $\tan x = -\sqrt{3}$



$\boxed{x_1 = \frac{\pi}{3}}$

$x_2 = \pi + \frac{\pi}{3}$

$\boxed{x_2 = \frac{4\pi}{3}}$

$x_3 = \pi - \frac{\pi}{3}$

$\boxed{x_3 = \frac{2\pi}{3}}$

$x_4 = 2\pi - \frac{\pi}{3}$

$\boxed{x_4 = \frac{5\pi}{3}}$

d) $3\csc^2 x - 4 = 0$

$\csc^2 x = \frac{4}{3}$

$\sin^2 x = \frac{3}{4}$

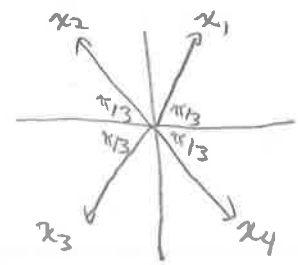
$\sin x = \pm\sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

From special Δ; $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q1 + Q2 for $\sin x = \frac{\sqrt{3}}{2}$

Place $\frac{\pi}{3}$ in Q3 + Q4 for $\sin x = -\frac{\sqrt{3}}{2}$



$$\boxed{x_1 = \frac{\pi}{3} \quad x_3 = \frac{4\pi}{3} \quad x_2 = \frac{2\pi}{3} \quad x_4 = \frac{5\pi}{3}}$$

5) Determine solutions for each equation in the interval $0 \le x \le 2\pi$.

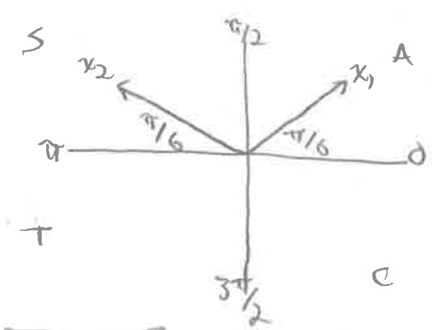
a) $3\sin x = \sin x + 1$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

From special Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2



$\boxed{x_1 = \frac{\pi}{6}}$

$x_2 = \pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{5\pi}{6}}$

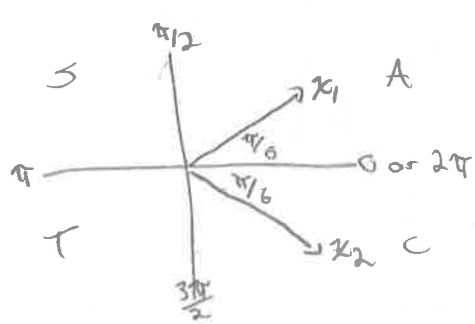
b) $5\cos x - \sqrt{3} = 3\cos x$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

From special Δ; $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Place in Q1 + Q4



$\boxed{x_1 = \frac{\pi}{6}}$

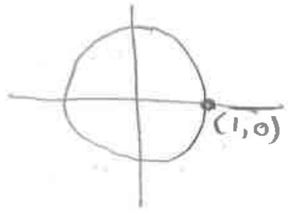
$x_2 = 2\pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{11\pi}{6}}$

c) $7 \sec x = 7$ $\sec x = 1$

$\cos x = 1$

use unit circle
where each point is $(\cos x, \sin x)$



$x_1 = 0$

$x_2 = 2\pi$

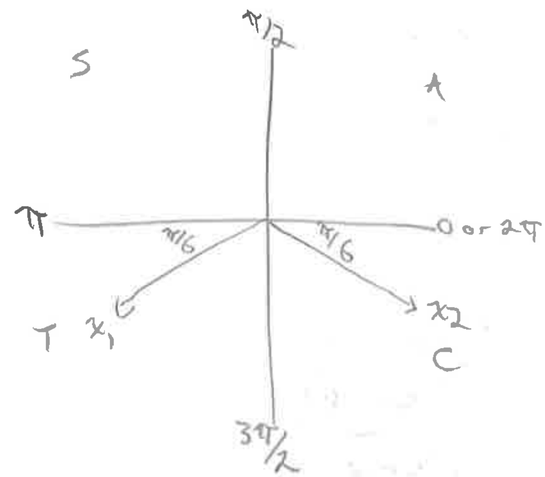
d) $2 \csc x + 17 = 15 + \csc x$

$\csc x = -2$

$\sin x = -\frac{1}{2}$

From special Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place $\frac{\pi}{6}$ in Q3 + Q4



$x_1 = \pi + \frac{\pi}{6}$ $x_2 = 2\pi - \frac{\pi}{6}$

$\boxed{x_1 = \frac{7\pi}{6}}$ $\boxed{x_2 = \frac{11\pi}{6}}$

Answer Key

1)a) 0.25, 2.89 **b)** 2.42, 3.86 **c)** 1.37, 4.51 **d)** 1.32, 4.97 **e)** 2.16, 5.3 **f)** 3.55, 5.87
2)a) $\frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $\frac{\pi}{3}, \frac{5\pi}{3}$ **c)** $\frac{\pi}{4}, \frac{5\pi}{4}$ **d)** $\frac{3\pi}{4}, \frac{7\pi}{4}$
) 0.93, 2.21, 4.07, 5.36 **b)** 0.84, 2.3, 3.98, 5.44 **c)** 0.88, 2.27, 4.02, 5.41 **d)** 0.89, 2.26, 4.03, 5.4
4)a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **c)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **d)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
5)a) $\frac{\pi}{6}, \frac{5\pi}{6}$ **b)** $\frac{\pi}{6}, \frac{11\pi}{6}$ **c)** 0 or 2π **d)** 3.67 or 5.76

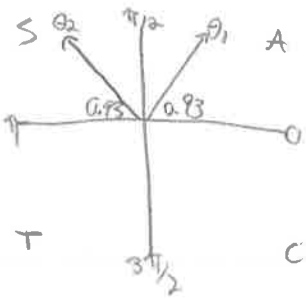
W6 – 5.4 Solve Double Angle Trigonometric Equations
MHF4U
Jensen

SOLUTIONS

Determine solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible.

a) $\sin(2x) - 0.8 = 0$ Let $\theta = 2x$

$\sin\theta = 0.8$



$\theta_1 = \sin^{-1}(0.8)$

$\theta_1 = 0.93$

$\theta_2 = \pi - 0.93$

$\theta_2 = 2.21$

$2x = \theta$

$2x = 0.93$

$\boxed{x_1 = 0.47}$

$2x = 2.21$

$\boxed{x_2 = 1.11}$

*Add the period of π to find other solutions

$x_3 = x_1 + \pi$

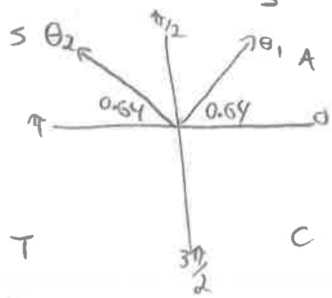
$\boxed{x_3 = 3.61}$

$x_4 = x_2 + \pi$

$\boxed{x_4 = 4.25}$

b) $5\sin(2x) - 3 = 0$ Let $\theta = 2x$

$\sin\theta = \frac{3}{5}$



$\theta_1 = \sin^{-1}\left(\frac{3}{5}\right)$

$\theta_1 = 0.64$

$\theta_2 = \pi - 0.64$

$\theta_2 = 2.5$

$2x = \theta$

$2x = 0.64$

$\boxed{x_1 = 0.32}$

$2x = 2.5$

$\boxed{x_2 = 1.25}$

* Add period of π to find other solutions *

$x_3 = x_1 + \pi$

$\boxed{x_3 = 3.46}$

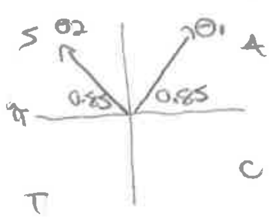
$x_4 = x_2 + \pi$

$\boxed{x_4 = 4.39}$

c) $-4\sin(2x) + 3 = 0$

Let $\theta = 2x$

$\sin\theta = \frac{3}{4}$



$\theta_1 = \sin^{-1}\left(\frac{3}{4}\right)$

$\theta_1 = 0.85$

$\theta_2 = \pi - 0.85$

$\theta_2 = 2.29$

$2x = \theta$

$2x = 0.85$

$\boxed{x_1 = 0.43}$

$2x = 2.29$

$\boxed{x_2 = 1.15}$

* add period of π to find other solutions *

$x_3 = x_1 + \pi$

$\boxed{x_3 = 3.57}$

$x_4 = x_2 + \pi$

$\boxed{x_4 = 4.29}$

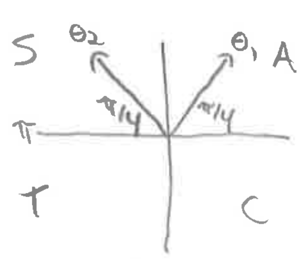
d) $\sin(2x) = \frac{1}{\sqrt{2}}$

Let $\theta = 2x$

$\sin\theta = \frac{1}{\sqrt{2}}$

from Δ; $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Place in Q1+Q2



$\theta_1 = \frac{\pi}{4}$

$\theta_2 = \pi - \frac{\pi}{4}$

$\theta_2 = \frac{3\pi}{4}$

$2x = \theta$

$2x = \frac{\pi}{4}$

$\boxed{x_1 = \frac{\pi}{8}}$

$2x = \frac{3\pi}{4}$

$\boxed{x_2 = \frac{3\pi}{8}}$

$x_3 = x_1 + \pi$

$\boxed{x_3 = \frac{9\pi}{8}}$

$x_4 = x_2 + \pi$

$\boxed{x_4 = \frac{11\pi}{8}}$

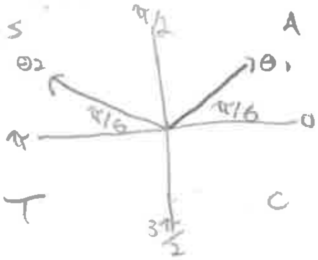
Let $\theta = 4x$

e) $\sin(4x) = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

From Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place in Q1+Q2



$\theta_1 = \frac{\pi}{6}$

$\theta_2 = \pi - \frac{\pi}{6}$

$\theta_2 = \frac{5\pi}{6}$

$4x = \theta$

$4x = \frac{\pi}{6}$ $\quad$ $4x = \frac{5\pi}{6}$

$\boxed{x_1 = \frac{\pi}{24}}$ $\quad$ $\boxed{x_2 = \frac{5\pi}{24}}$

* add period of $\frac{\pi}{2} = \frac{12\pi}{24}$ to find other solutions *

$\boxed{x_3 = \frac{13\pi}{24}, \; x_4 = \frac{25\pi}{24}, \; x_5 = \frac{37\pi}{24}}$ $\quad$ $\boxed{x_6 = \frac{17\pi}{24}, \; x_7 = \frac{29\pi}{24}, \; x_8 = \frac{41\pi}{24}}$

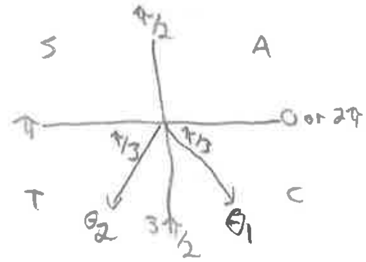
Let $3x = \theta$

f) $\sin(3x) = -\frac{\sqrt{3}}{2}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

from Δ; $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Place in Q3+Q4



$\theta_1 = 2\pi - \frac{\pi}{3}$

$\theta_1 = \frac{5\pi}{3}$

$\theta_2 = \pi + \frac{\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$

$3x = \theta$

$3x = \frac{5\pi}{3}$ $\quad$ $3x = \frac{4\pi}{3}$

$\boxed{x_1 = \frac{5\pi}{9}}$ $\quad$ $\boxed{x_2 = \frac{4\pi}{9}}$

* add period of $\frac{2\pi}{3} = \frac{6\pi}{9}$ to find other solutions *

$\boxed{x_3 = \frac{11\pi}{9}}$ $\quad$ $\boxed{x_5 = \frac{10\pi}{9}}$

$\boxed{x_4 = \frac{17\pi}{9}}$ $\quad$ $\boxed{x_6 = \frac{16\pi}{9}}$

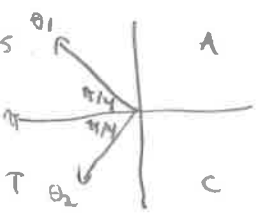
g) $\cos(4x) = -\frac{1}{\sqrt{2}}$

Let $\theta = 4x$

$\cos\theta = -\frac{1}{\sqrt{2}}$

From Δ; $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

place in Q2+Q3



$\theta_1 = \pi - \frac{\pi}{4}$

$\theta_1 = \frac{3\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

$4x = \theta$

$4x = \frac{3\pi}{4}$ $\quad$ $4x = \frac{5\pi}{4}$

$\boxed{x_1 = \frac{3\pi}{16}}$ $\quad$ $\boxed{x_2 = \frac{5\pi}{16}}$

* add period of $\frac{\pi}{2} = \frac{8\pi}{16}$ to find other solutions *

$\boxed{x_3 = \frac{11\pi}{16}, \; x_4 = \frac{19\pi}{16}, \; x_5 = \frac{27\pi}{6}}$ $\quad$ $\boxed{x_6 = \frac{13\pi}{16}, \; x_7 = \frac{21\pi}{16}, \; x_8 = \frac{29\pi}{16}}$

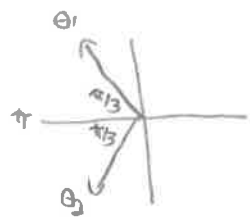
h) $\cos(2x) = -\frac{1}{2}$

let $\theta = 2x$

$\cos\theta = -\frac{1}{2}$

From Δ; $\cos\frac{\pi}{3} = \frac{1}{2}$

place in Q2+Q3



$\theta_1 = \pi - \frac{\pi}{3}$

$\theta_1 = \frac{2\pi}{3}$

$\theta_2 = \pi + \frac{\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$

$2x = \theta$

$2x = \frac{2\pi}{3}$ $\quad$ $2x = \frac{4\pi}{3}$

$\boxed{x_1 = \frac{\pi}{3}}$ $\quad$ $\boxed{x_2 = \frac{2\pi}{3}}$

* add period of $\pi = \frac{3\pi}{3}$ to find other solutions *

$\boxed{x_3 = \frac{4\pi}{3}}$ $\quad$ $\boxed{x_4 = \frac{5\pi}{3}}$

swer Key

a) 0.46, 1.11 **b)** 0.32, 1.25 **c)** 0.42, 1.15 **d)** $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$ **e)** $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{37\pi}{24}, \frac{41\pi}{24}$

f) $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$ **g)** $\frac{3\pi}{16}, \frac{5\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}, \frac{19\pi}{16}, \frac{21\pi}{16}, \frac{27\pi}{16}, \frac{29\pi}{16}$ **h)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

W7 – 5.4 Solve Quadratic Trigonometric Equations
MHF4U
Jensen

SOLUTIONS

1) Solve $\sin^2 x - 2\sin x - 3 = 0$ on the interval $0 \le x \le 2\pi$

$(\sin x - 3)(\sin x + 1) = 0$

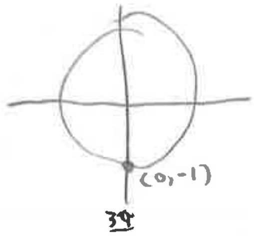
$\sin x - 3 = 0$

$\sin x = 3$

No solutions

$\sin x = -1$

use unit circle where each point is $(\cos x, \sin x)$



$\boxed{\therefore x = \frac{3\pi}{2}}$

2) Solve $\csc^2 x - \csc x - 2 = 0$ on the interval $0 \le x \le 2\pi$

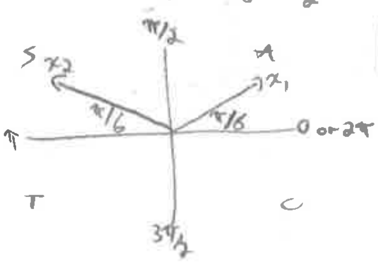
$(\csc x - 2)(\csc x + 1) = 0$

$\csc x - 2 = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$

from Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$



$\boxed{x_1 = \frac{\pi}{6}}$

$x_2 = \pi - \frac{\pi}{6}$

$\boxed{x_2 = \frac{5\pi}{6}}$

$\csc x + 1 = 0$

$\csc x = -1$

$\sin x = -1$

* refer to part a) *

$\boxed{x_3 = \frac{3\pi}{2}}$

3) Solve $2\sec^2 x - \sec x - 1 = 0$ on the interval $0 \le x \le 2\pi$

$2\sec^2 x - 2\sec + 1\sec x - 1 = 0$

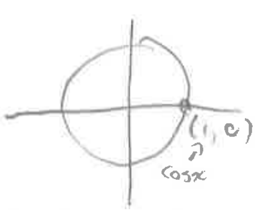
$2\sec x(\sec x - 1) + 1(\sec x - 1) = 0$

$(\sec x - 1)(2\sec x + 1) = 0$

$\sec x - 1 = 0$

$\sec x = 1$

$\cos x = 1$



$\boxed{\begin{array}{l} x_1 = 0 \\ x_2 = 2\pi \end{array}}$

$2\sec x + 1 = 0$

$\sec x = -\frac{1}{2}$

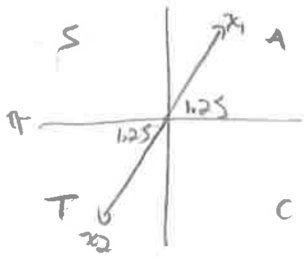
$\cos x = -2$

No solutions

4) Solve $\tan^2 x - \tan x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$. Round answers to the nearest hundredth of a radian.

$(\tan x - 3)(\tan x + 2) = 0$

$\tan x = 3$



$x_1 = \tan^{-1}(3)$

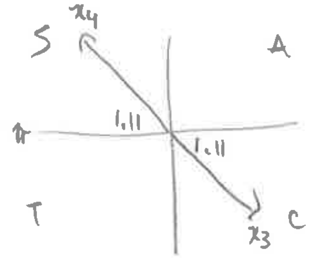
$\boxed{x_1 = 1.25}$

$x_2 = \pi + 1.25$

$\boxed{x_2 = 4.39}$

$\tan x + 2 = 0$

$\tan x = -2$



$x_3 = \tan^{-1}(-2)$

$x_3 = -1.107 + 2\pi$

$\boxed{x_3 = 5.18}$

$x_4 = \pi - 1.11$

$\boxed{x_4 = 2.03}$

5) Solve $6\cos^2 x + 5\cos x - 6 = 0$ on the interval $0 \leq x \leq 2\pi$

$6\cos^2 x + 9\cos x - 4\cos x - 6 = 0$

$3\cos x(2\cos x + 3) - 2(2\cos x + 3) = 0$

$(2\cos x + 3)(3\cos x - 2) = 0$

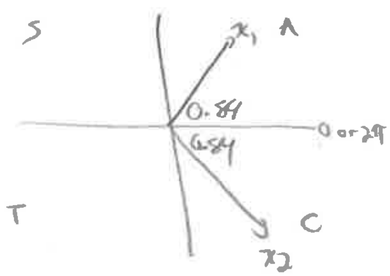
$2\cos x + 3 = 0$

$\cos x = -\frac{3}{2}$

No solutions

$3\cos x - 2 = 0$

$\cos x = \frac{2}{3}$



$x_1 = \cos^{-1}\left(\frac{2}{3}\right)$

$\boxed{x_1 = 0.84}$

$x_2 = 2\pi - 0.84$

$\boxed{x_2 = 5.44}$

6) Solve $3\csc^2 x - 5\csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

$3\csc^2 x - 6\csc x + 1\csc x - 2 = 0$

$3\csc x(\csc x - 2) + 1(\csc x - 2) = 0$

$(\csc x - 2)(3\csc x + 1) = 0$

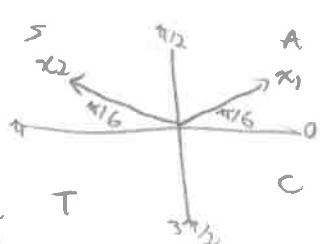
$\csc x - 2 = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$

From Δ; $\sin\frac{\pi}{6} = \frac{1}{2}$

Place in Q1 + Q2



$\boxed{x_1 = \frac{\pi}{6}}$ $\boxed{x_2 = \frac{5\pi}{6}}$

$3\csc x + 1 = 0$

$\csc x = -\frac{1}{3}$

$\sin x = -3$

No solutions

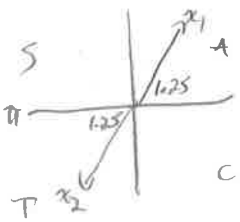
7) Solve $2\tan^2 x - 5\tan x - 3 = 0$ on the interval $0 \le x \le 2\pi$

$2\tan^2 x - 6\tan x + 1\tan x - 3 = 0$

$2\tan x(\tan x - 3) + 1(\tan x - 3) = 0$

$(\tan x - 3)(2\tan x + 1) = 0$

$\tan x = 3$



$x_1 = \tan^{-1}(3)$

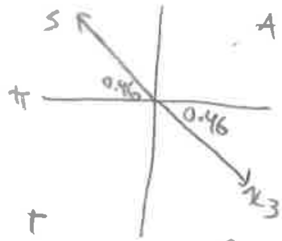
$\boxed{x_1 = 1.25}$

$x_2 = \pi + 1.25$

$\boxed{x_2 = 4.39}$

$2\tan x + 1 = 0$

$\tan x = -\frac{1}{2}$



$x_3 = \tan^{-1}\left(-\frac{1}{2}\right)$

$x_3 = -0.463647609 + 2\pi$

$\boxed{x_3 = 5.82}$

$x_4 = \pi - 0.46$

$\boxed{x_4 = 2.68}$

8) Solve $\cot x \csc^2 x = 2\cot x$ on the interval $0 \le x \le 2\pi$

$\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\sin^2 x}\right) = 2\left(\frac{\cos x}{\sin x}\right)$

$\frac{\cos x}{\sin^2 x} = 2\cos x$

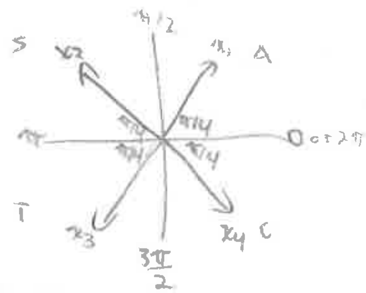
$\frac{\cos x}{\cos x} = 2\sin^2 x$

$\frac{1}{2} = \sin^2 x$

$\sin x = \pm\frac{1}{\sqrt{2}}$

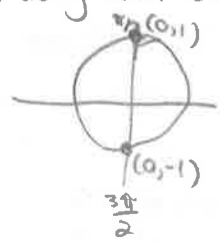
from Δ; $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Place in all 4 quadrants



* Notice in 1st line of solution; if $\cos x = 0$, then LS = RS as well.

OR using unit circle where a point is $(\cos x, \sin x)$



$\boxed{x_1 = \frac{\pi}{4},\ x_2 = \frac{3\pi}{4},\ x_3 = \frac{5\pi}{4},\ x_4 = \frac{7\pi}{4},\ x_5 = \frac{\pi}{2}^{*},\ x_6 = \frac{3\pi}{2}^{*}}$

9) Solve for θ to the nearest hundredth, where $0 \le \theta \le 2\pi$

a) $3\tan^2\theta - 2\tan\theta = 1$

$3\tan^2\theta - 2\tan\theta - 1 = 0$

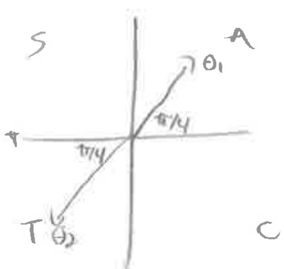
$3\tan^2\theta - 3\tan\theta + 1\tan\theta - 1 = 0$

$3\tan\theta(\tan\theta - 1) + 1(\tan\theta - 1) = 0$

$(\tan\theta - 1)(3\tan\theta + 1) = 0$

$\tan\theta - 1 = 0$

$\tan\theta = 1$



from Δ; $\tan\frac{\pi}{4} = 1$

Place in Q1 + Q3

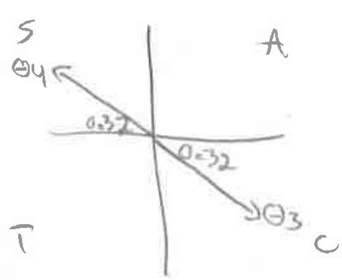
$\boxed{\theta_1 = \frac{\pi}{4}}$

$\theta_2 = \pi + \frac{\pi}{4}$

$\boxed{\theta_2 = \frac{5\pi}{4}}$

$3\tan\theta + 1 = 0$

$\tan\theta = -\frac{1}{3}$



$\theta_3 = \tan^{-1}\left(-\frac{1}{3}\right)$

$\theta_3 = -0.32175 + 2\pi$

$\boxed{\theta_3 = 5.96}$

$\theta_4 = \pi - 0.32$

$\boxed{\theta_4 = 2.82}$

b) $12\sin^2\theta + \sin\theta - 6 = 0$

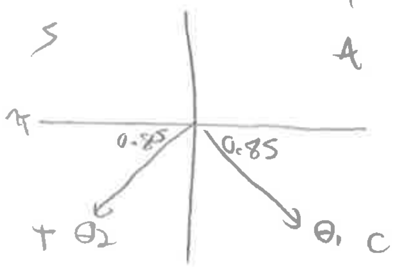
$12\sin^2\theta + 9\sin\theta - 8\sin\theta - 6 = 0$

$3\sin\theta(4\sin\theta + 3) - 2(4\sin\theta + 3) = 0$

$(4\sin\theta + 3)(3\sin\theta - 2) = 0$

$4\sin\theta + 3 = 0$

$\sin\theta = -\frac{3}{4}$



$\theta_1 = \sin^{-1}\left(-\frac{3}{4}\right)$

$\theta_1 = -0.848062 + 2\pi$

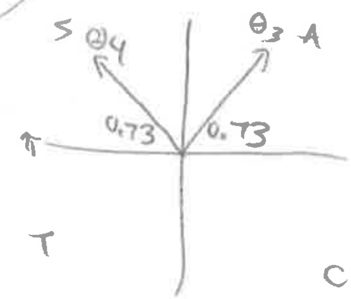
$\boxed{\theta_1 = 5.44}$

$\theta_2 = \pi + 0.85$

$\boxed{\theta_2 = 3.99}$

$3\sin\theta - 2 = 0$

$\sin\theta = \frac{2}{3}$



$\theta_3 = \sin^{-1}\left(\frac{2}{3}\right)$

$\boxed{\theta_3 = 0.73}$

$\theta_4 = \pi - 0.73$

$\boxed{\theta_4 = 2.41}$

c) $5\cos(2\theta) - \cos\theta + 3 = 0$

$5(2\cos^2\theta - 1) - \cos\theta + 3 = 0$

$10\cos^2\theta - 5 - \cos\theta + 3 = 0$

$10\cos^2\theta - \cos\theta - 2 = 0$

$10\cos^2\theta - 5\cos\theta + 4\cos\theta - 2 = 0$

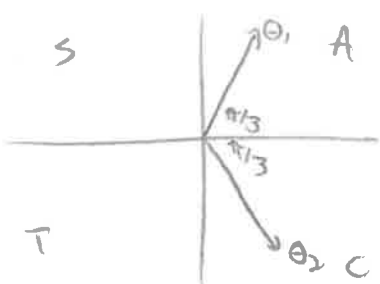
$5\cos\theta(2\cos\theta - 1) + 2(2\cos\theta - 1) = 0$

$(2\cos\theta - 1)(5\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2}$

from Δ; $\cos\frac{\pi}{3} = \frac{1}{2}$

place in Q1 + Q4

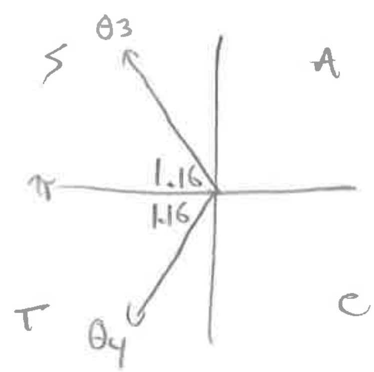


$\boxed{\theta_1 = \frac{\pi}{3}}$

$\theta_2 = 2\pi - \frac{\pi}{3}$

$\boxed{\theta_2 = \frac{5\pi}{3}}$

$\cos\theta = -\frac{2}{5}$



$\theta_3 = \cos^{-1}\left(-\frac{2}{5}\right)$

$\boxed{\theta_3 = 1.98}$

$\theta_4 = \pi + 1.16$

$\boxed{\theta_4 = 4.3}$

Answer Key

$\frac{3\pi}{2}$ **2)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **3)** π **4)** 1.11, 1.89, 4.25, 5.03 **5)** 0.84, 5.44 **6)** $\frac{\pi}{6}, \frac{5\pi}{6}$ **7)** 1.25, 2.68, 4.39, 5.82
8) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$ **9)a)** $\frac{\pi}{4}$, 2.82, $\frac{5\pi}{4}$, 5.96 **b)** 0.73, 2.41, 3.99, 5.44 **c)** $\frac{\pi}{3}$, 1.98, 4.3, $\frac{5\pi}{3}$