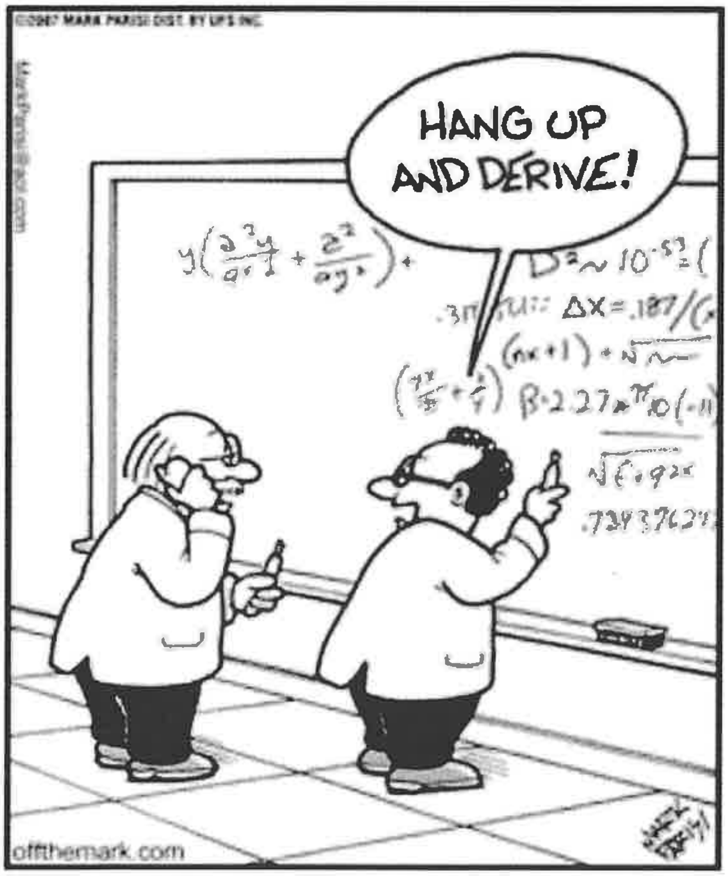
UNIT 1- Derivative Rules

WORKBOOK

SOLUTIONS

MCV4U



SOLUTIONS

1) Circle the functions that have a derivative of zero:

A) $y = 8.7$ B) $y = -4 + x$ C) $y = \frac{5}{9}x$ D) $y = \sqrt{7}$ E) $y = -7.1\pi$

2) For each function, determine $\frac{dy}{dx}$

a) $y = x$

$\frac{dy}{dx} = 1$

b) $y = \frac{1}{4}x^2$

$\frac{dy}{dx} = 2\left(\frac{1}{4}\right)x$

$\frac{dy}{dx} = \frac{1}{2}x$

c) $y = -3x^4$

$\frac{dy}{dx} = 4(-3)x^3$

$\frac{dy}{dx} = -12x^3$

d) $y = \sqrt[5]{x^3}$

$y = x^{3/5}$

$\frac{dy}{dx} = \frac{3}{5}x^{3/5-1}$

$\frac{dy}{dx} = \frac{3}{5}x^{-2/5}$

$\frac{dy}{dx} = \frac{3}{5\sqrt[5]{x^2}}$

e) $y = \frac{5}{x}$

$y = 5x^{-1}$

$\frac{dy}{dx} = -5x^{-2}$

$\frac{dy}{dx} = \frac{-5}{x^2}$

f) $y = \frac{4}{\sqrt{x}}$

$y = 4x^{-1/2}$

$\frac{dy}{dx} = -\frac{1}{2}(4)x^{-1/2-1}$

$\frac{dy}{dx} = -2x^{-3/2}$

$\frac{dy}{dx} = \frac{-2}{\sqrt{x^3}}$

3) Determine the slope of the tangent to the graph of each function at the indicated value.

a) $y = 6$ at $x = 12$

$\frac{dy}{dx} = 0$

$\left.\frac{dy}{dx}\right|_{x=12} = 0$

b) $f(x) = 2x^5$ at $x = \sqrt{3}$

$f'(x) = 10x^4$

$f'(\sqrt{3}) = 10(\sqrt{3})^4$

$f'(\sqrt{3}) = 10(3)^2$

$f'(\sqrt{3}) = 90$

c) $y = \frac{1}{3x}$ at $x = -2$

$y = \frac{1}{3}x^{-1}$

$\frac{dy}{dx} = -\frac{1}{3}x^{-2}$

$\frac{dy}{dx} = \frac{-1}{3x^2}$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-1}{3(-2)^2}$

$\left.\frac{dy}{dx}\right|_{x=-2} = \frac{-1}{12}$

4) Find the derivative of each function

a) $f(x) = 2x^2 + x^3$

$f'(x) = 4x + 3x^2$

b) $y = \frac{4}{5}x^5 - 3x$

$y' = 5(\frac{4}{5})x^4 - 3$

$y' = 4x^4 - 3$

c) $h(t) = -1.1x^4 + 78$

$h'(t) = -4.4x^3$

d) $p(a) = \frac{a^5}{15} - 2\sqrt{a}$

$p'(a) = 5(\frac{1}{15})a^4 - \frac{1}{2}(2)a^{-1/2}$

$p'(a) = \frac{1}{3}a^4 - \frac{1}{\sqrt{a}}$

e) $k(s) = -\frac{1}{s^2} + 7s^4$

$k(s) = -1s^{-2} + 7s^4$

$k'(s) = 2s^{-3} + 28s^3$

$k'(s) = \frac{2}{s^3} + 28s^3$

5)a) Determine the point at which the slope of the tangent to each parabola is zero.

i) $y = 6x^2 - 3x + 4$

$y' = 12x - 3$

$0 = 12x - 3$

$x = \frac{1}{4}$

$y = 6(\frac{1}{4})^2 - 3(\frac{1}{4}) + 4$

$y = \frac{29}{8}$

$(\frac{1}{4}, \frac{29}{8})$

ii) $y = \frac{3}{4}x^2 + 2x + 3$

$y' = \frac{3}{2}x + 2$

$0 = \frac{3}{2}x + 2$

$x = \frac{-4}{3}$

$y = \frac{3}{4}(\frac{-4}{3})^2 + 2(\frac{-4}{3}) + 3$

$y = 5/3$

$(\frac{-4}{3}, \frac{5}{3})$

b) Use technology to look at the graph of each parabola. What does the point found in part a) correspond to on each of these graphs?

The vertex.

6) Simplify and then differentiate

a) $f(x) = \frac{10x^4 - 6x^3}{2x^2}$

$f(x) = \frac{10x^4}{2x^2} - \frac{6x^3}{2x^2}$

$f(x) = 5x^2 - 3x$

$f'(x) = 10x - 3$

b) $(5x + 2)^2$

$f(x) = 25x^2 + 20x + 4$

$f'(x) = 50x + 20$

7) A skydiver jumps out of a plane that is flying 2500 meters above the ground. The skydiver's height, h, in meters, above th4e ground after t seconds is $h(t) = 2500 - 4.9t^2$.

a) Determine the rate of change of the height of the skydiver at $t = 5$s

$h'(t) = -9.8t$

$h'(5) = -9.8(5)$

$h'(5) = -49$ m/s

b) The skydiver's parachute opens at 1000m above the ground. After how many seconds does this happen?

$1000 = 2500 - 4.9t^2$

$t^2 = \frac{1500}{4.9}$

$t = \sqrt{\frac{1500}{4.9}}$

$t \simeq 17.5$ seconds

c) What is the rate of change of the height of the skydiver at the time found in part b)?

$h'(17.5) \simeq -9.8(17.5)$

$h'(17.5) \simeq -171.5$ m/s

8) Determine the equation of the tangent line to the graph of $y = -6x^4 + 2x^3 + 5$ at the point $(-1, -3)$.

$y' = -24x^3 + 6x^2$

$y'(-1) = -24(-1)^3 + 6(-1)^2$

$y'(-1) = 30$

$m = 30$

$y = mx + b$

$-3 = 30(-1) + b$

$-3 = -30 + b$

$b = 27$

$\boxed{y = 30x + 27}$

9) Determine the equation of the tangent line to the graph of $y = -1.5x^3 + 3x - 2$ at the point $(2, -8)$.

$y' = -4.5x^2 + 3$

$y'(2) = -4.5(2)^2 + 3$

$y'(2) = -18 + 3$

$y'(2) = -15$

$m = -15$

$y = mx + b$

$-8 = -15(2) + b$

$b = 22$

$\boxed{y = -15x + 22}$

10) A flaming arrow is shot into the air to mark the beginning of a festival. Its height, h, in meters, after t seconds can be modelled by the function $h(t) = -4.9t^2 + 24.5t + 2$.

a) Determine the height of the arrow at $t = 2$s.

$h(2) = -4.9(2)^2 + 24.5(2) + 2$

$h(2) = 31.4\text{ m}$

b) How long does it take the arrow to land on the ground?

$0 = -4.9t^2 + 24.5t + 2$

$t = \dfrac{-24.5 \pm \sqrt{(24.5)^2 - 4(-4.9)(2)}}{2(-4.9)}$

$\cancel{t_1 \doteq -0.08}$

$\boxed{t_2 \doteq 5.08 \text{ seconds}}$

c) How fast is the arrow travelling when it hits the ground?

$h'(t) = -9.8t + 24.5$

$h'(5.08) = -9.8(5.08) + 24.5$

$h'(5.08) = -25.284 \text{ m/s}$

Answers:

1) A, D, E

2)a) $\frac{dy}{dx} = 1$ **b)** $\frac{dy}{dx} = \frac{1}{2}x$ **c)** $\frac{dy}{dx} = -12x^3$ **d)** $\frac{dy}{dx} = \frac{3}{5\sqrt[5]{x^2}}$ **e)** $\frac{dy}{dx} = -\frac{5}{x^2}$ **f)** $\frac{dy}{dx} = -\frac{2}{\sqrt{x^3}}$

3)a) 0 **b)** 90 **c)** $-\frac{1}{12}$

4)a) $f'(x) = 4x + 3x^2$ **b)** $\frac{dy}{dx} = 4x^4 - 3$ **c)** $h'(t) = -4.4t^3$ **d)** $p'(a) = \frac{1}{3}a^4 - \frac{1}{\sqrt{a}}$ **e)** $k'(s) = \frac{2}{s^3} + 28s^3$

5)a)i) (0.25, 3.625) **ii)** $(-0.53, -1.93)$ **b)** the vertex

ı) $f'(x) = 10x - 3$ **b)** $f'(x) = 50x + 20$

, a) -49m/s **b)** 17.5 seconds **c)** -171.5m/s

8) $y = 30x + 27$

9) $y = -15x + 22$

10)a) 31.4m **b)** 5.08s **c)** velocity is -25.28m/s

W2 – The Product Rule Unit 1
MCV4U
Jensen

1) Use the product rule to differentiate each function

a) $f(x) = (5x + 2)(8x - 6)$

$f'(x) = 5(8x-6) + 8(5x+2)$

$f'(x) = 40x - 30 + 40x + 16$

$f'(x) = 80x - 14$

b) $h(t) = (-t + 4)(2t + 1)$

$h'(t) = -1(2t+1) + 2(-t+4)$

$h'(t) = -2t - 1 - 2t + 8$

$h'(t) = -4t + 7$

c) $p(x) = (-2x + 3)(x - 9)$

$p'(x) = -2(x-9) + 1(-2x+3)$

$p'(x) = -2x + 18 - 2x + 3$

$p'(x) = -4x + 21$

d) $g(x) = (x^2 + 2)(4x - 5)$

$g'(x) = 2x(4x-5) + 4(x^2+2)$

$g'(x) = 8x^2 - 10x + 4x^2 + 8$

$g'(x) = 12x^2 - 10x + 8$

e) $f(x) = (1 - x)(x^2 - 5)$

$f'(x) = -1(x^2-5) + 2x(1-x)$

$f'(x) = -x^2 + 5 + 2x - 2x^2$

$f'(x) = -3x^2 + 2x + 5$

f) $h(t) = (t^2 + 3)(3t^2 - 7)$

$h'(t) = 2t(3t^2-7) + 6t(t^2+3)$

$h'(t) = 6t^3 - 14t + 6t^3 + 18t$

$h'(t) = 12t^3 + 4t$

2) Determine $f'(-2)$ for each function.

a) $f(x) = (x^2 - 2x)(3x + 1)$

$f'(x) = (2x-2)(3x+1) + 3(x^2-2x)$

$f'(x) = 6x^2 - 4x - 2 + 3x^2 - 6x$

$f'(x) = 9x^2 - 10x - 2$

$f'(-2) = 9(-2)^2 - 10(-2) - 2$

$f'(-2) = 54$

b) $f(x) = (1 - x^3)(-x^2 + 2)$

$f'(x) = -3x^2(-x^2+2) + (-2x)(1-x^3)$

$f'(x) = 3x^4 - 6x^2 - 2x + 2x^4$

$f'(x) = 5x^4 - 6x^2 - 2x$

$f'(-2) = 5(-2)^4 - 6(-2)^2 - 2(-2)$

$f'(-2) = 60$

3) Determine an equation for the tangent to each curve at the indicated value.

a) $f(x) = (x^2 - 3)(x^2 + 1)$ at $x = -4$

Slope

$f'(x) = 2x(x^2+1) + 2x(x^2-3)$

$f'(x) = 2x^3 + 2x + 2x^3 - 6x$

$f'(x) = 4x^3 - 4x$

$f'(-4) = 4(-4)^3 - 4(-4)$

$f'(-4) = -240$

$m = -240$

Point

$f(-4) = [(-4)^2 - 3][(-4)^2 + 1]$

$f(-4) = 221$

Eqⁿ

$y = mx + b$

$221 = -240(-4) + b$

$b = -739$

$\boxed{y = -240x - 739}$

b) $h(x) = (x^4 + 4)(2x^2 - 6)$ at $x = -1$

Slope

$h'(x) = 4x^3(2x^2-6) + 4x(x^4+4)$

$h'(-1) = 4(-1)^3[2(-1)^2 - 6] + 4(-1)[(-1)^4 + 4]$

$h'(-1) = -4(-4) - 4(5)$

$h'(-1) = -4$

Point

$h(-1) = [(-1)^4 + 4][2(-1)^2 - 6]$

$h(-1) = (5)(-4)$

$h(-1) = -20$

Eqⁿ

$y = mx + b$

$-20 = -4(-1) + b$

$b = -24$

$\boxed{y = -4x - 24}$

4) Determine the point(s) on each curve that correspond to the given slope of the tangent.

a) $(-4x+3)(x+3),\ m=0$

b) $(x^2-2)(2x+1),\ m=-2$

5) Differentiate using the product rule.

a) $(5x^2-x+1)(x+2)$

b) $y=-x^2(4x-1)(x^3+2x+3)$

6) The owner of a local hair salon is planning to raise the price for a haircut and blow dry. The current rate is \$30 for this service, with the salon averaging 550 clients a month. A survey indicates that the salon will lose 5 clients for every incremental price increase of \$2.50.

a) Write an equation to model the salon's monthly revenue, R, in dollars, as a function of x, where x represents the number of \$2.50 increases in the price.

$R(x) = (30 + 2.5x)(550 - 5x)$

b) Use the product rule to determine $R'(x)$

$R'(x) = 2.5(550 - 5x) + [illegible](30 + 2.5x)$

$R'(x) = 1375 - 12.5x - 150 - 12.5x$

$R'(x) = -25x + 1225$

c) Evaluate $R'(3)$ and interpret it for this situation.

$R'(3) = -25(3) + 1225$

$R'(3) = 1150$

At an increase of \$7.50, the rate of change of revenue is \$1150

d) Solve $R'(x) = 0$.

$0 = -25x + 1225$

$x = 49$

e) Explain how the owner can use the result of part d).

$R(49) = (30 + 2.5(49))(550 - 5(49))$

$= (152.50)(305)$

$= \$46\ [illegible]$

Increasing the price [illegible] results in a max revenue of \$[illegible]

Answers:

1)a) $f'(x) = 80x - 14$ **b)** $h'(t) = -4t + 7$ **c)** $p'(x) = -4x + 21$ **d)** $g'(x) = 12x^2 - 10x + 8$
e) $f'(x) = -3x^2 + 2x + 5$ **f)** $h'(t) = 12t^3 + 4t$
2)a) 54 **b)** 60
3)a) $y = -240x - 739$ **b)** $y = -4x - 24$
4)a) $\left(-\frac{9}{8}, \frac{225}{16}\right)$ **b)** $(0.43, -3.38)$ and $(-0.77, 0.76)$
5)a) $15x^2 + 18x - 1$ **b)** $-24x^5 + 5x^4 - 32x^3 - 30x^2 + 6x$
6)a) $R(x) = (30 + 2.50x)(550 - 5x)$ **b)** $R'(x) = 1225 - 25x$ **c)** 1150; this is the rate of change of revenue at a \$7.50 increase **d)** $x = 49$ **e)** The owner could maximize the revenue by making 49 increases of \$2.50. A visit to the hair salon would cost \$152.50 and would generate a max revenue of \$46 512.50.

W3 – Velocity, Acceleration, and Second Derivatives Unit 1

MCV4U

Jensen

SOLUTIONS

1) Determine the second derivative of each function.

a) $y = 2x^3 + 21$

$\frac{dy}{dx} = 6x^2$

$\frac{d^2y}{dx^2} = 12x$

b) $s(t) = -t^4 + 5t^3 - 2t^2 + t$

$s'(t) = -4t^3 + 15t^2 - 4t + 1$

$s''(t) = -12t^2 + 30t - 4$

c) $h(x) = \frac{1}{6}x^6 - \frac{1}{5}x^5$

$h'(x) = x^5 - x^4$

$h''(x) = 5x^4 - 4x^3$

2) Determine $f''(3)$ for each function:

a) $f(x) = 4x^3 - 5x + 6$

$f'(x) = 12x^2 - 5$

$f''(x) = 24x$

$f''(3) = 24(3)$

$f''(3) = 72$

b) $f(x) = (3x^2 + 2)(1 - x)$

$f'(x) = 6x(1-x) + (-1)(3x^2+2)$

$f'(x) = 6x - 6x^2 - 3x^2 - 2$

$f'(x) = -9x^2 + 6x - 2$

$f''(x) = -18x + 6$

$f''(3) = -18(3) + 6$

$f''(3) = -48$

3) Determine the velocity and acceleration functions for each position function $s(t)$.

a) $s(t) = 5 + 7t - 8t^3$

$v(t) = s'(t) = 7 - 24t^2$

$a(t) = s''(t) = -48t$

b) $s(t) = (2t + 3)(4 - 5t)$

$v(t) = s'(t) = 2(4-5t) + (-5)(2t+3)$

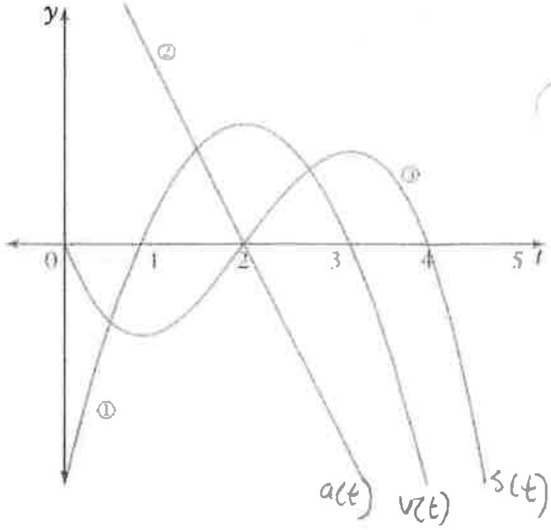
$= 8 - 10t - 10t - 15$

$= -20t - 7$

$a(t) = s''(t) = -20$

4) For the following graph,

a) Identify which curve or line represents $s(t)$, $v(t)$, and $a(t)$.



s(t) is curve 3 ; highest degree (3)

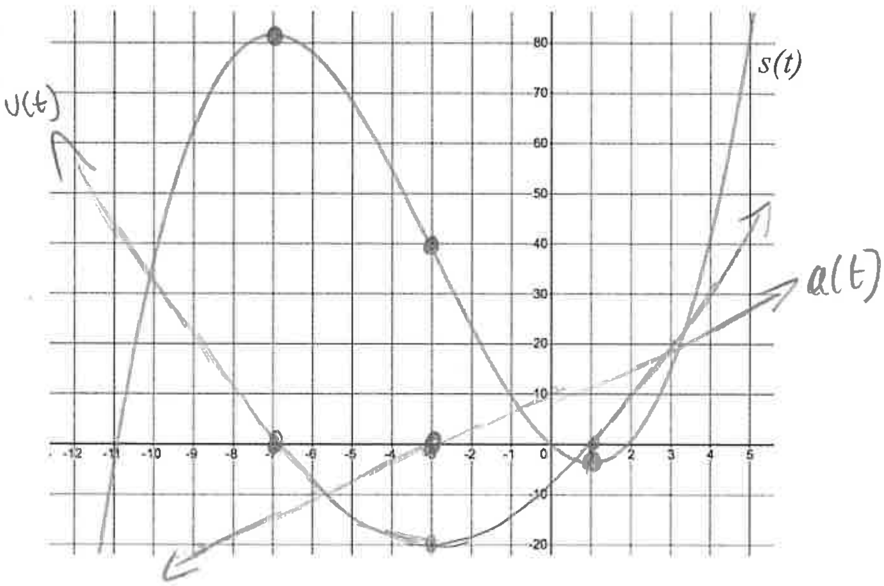
v(t) is curve 1 ; degree 2

a(t) is line 2 ; degree 1

b) Complete the table to determine the motion of the object.

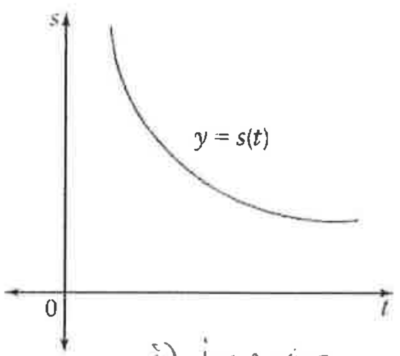
Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	Slope of $s(t)$	Motion of particle
$(0,1)$	−	+	−	negative slope that is increasing	slowing down and moving in reverse
$(1,2)$	+	+	+	positive slope that is increasing	speeding up and moving forward
$(2,3)$	+	−	−	positive slope that is decreasing	slowing down and moving forward.
$(3,\infty)$	−	−	+	negative slope that is decreasing	speeding up and moving in reverse

5) For the graph of $s(t)$ given, sketch possible graphs of $v(t)$ and $a(t)$.



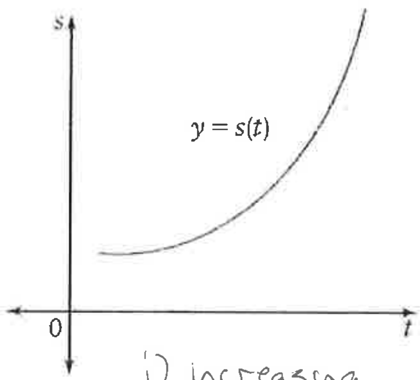
6) For each of the following graphs of $s(t)$,
i) Is the velocity increasing, decreasing, or constant?
ii) Is the acceleration positive, negative, or zero?

a)



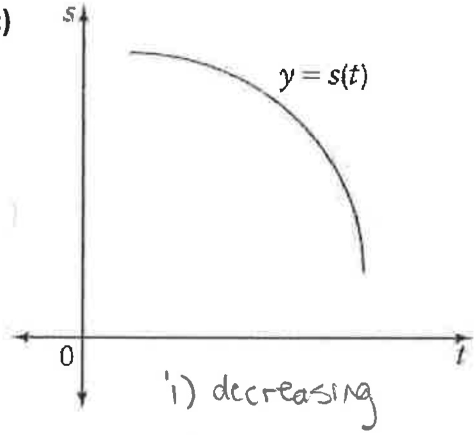
i) increasing
ii) positive

b)



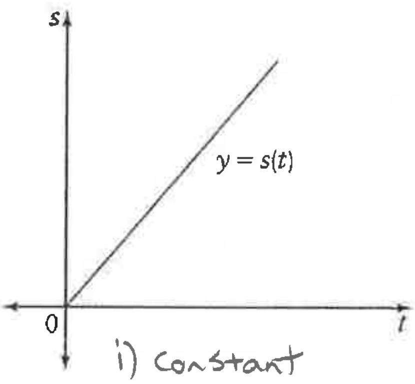
i) increasing
ii) positive

c)



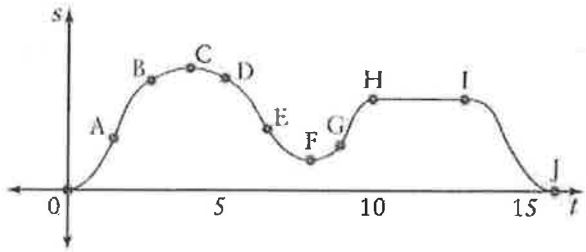
i) decreasing
ii) negative

d)



i) constant
ii) zero

7) The graph shows the position function of a bus during a 15-minute trip.



a) What is the initial velocity of the bus?

zero

b) What is the bus's velocity at C and at F?

zero

c) Is the bus going faster at A or at B? Explain.

A; steeper slope.

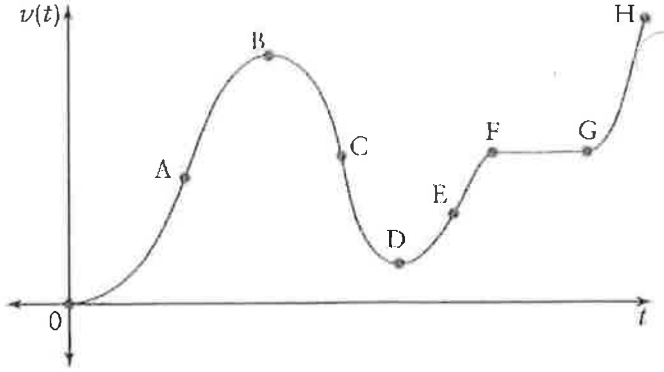
d) What happens to the motion of the bus between H and I?

doesn't move.

e) Is the bus speeding up or slowing down at B and D?

B - slowing down
D - speeding up

8) The graph shows a velocity function. State whether the acceleration is positive or negative for the following intervals:



a) 0 to B +

b) B to D −

c) D to F +

d) F to G 0

e) G to H +

f) at B and at D 0

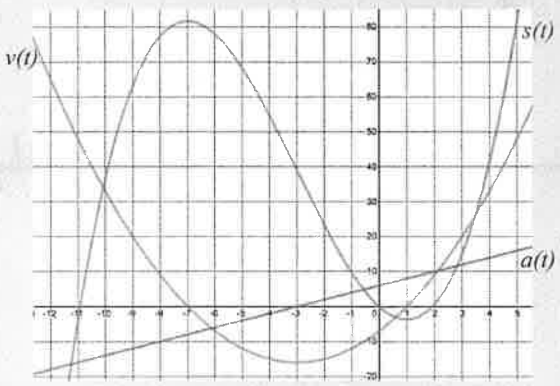
Answers:

1)a) $\frac{d^2y}{dx^2} = 12x$ **b)** $s''(t) = -12t^2 + 30t - 4$ **c)** $h''(x) = 5x^4 - 4x^3$
2)a) 72 **b)** -48
3)a) $v(t) = 7 - 24t^2 \quad a(t) = -48t$ **b)** $v(t) = -20t - 7 \quad a(t) = -20$
4)a) curve (3) is the position function since it is a cubic with the highest exponent. Curve (1) is the velocity function since it is a quadratic with and exponent one less than the position function. Line (2) is the acceleration function since it is linear and its exponent is one less than the velocity function.

b)

Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	Slope of $s(t)$	Motion of particle
(0,1)	−	+	−	Negative slope that is increasing	Slowing down and moving in reverse
(1,2)	+	+	+	Positive slope that is increasing	Speeding up and moving forward
(2,3)	+	−	−	positive slope that is decreasing	Slowing down and moving forward
(3, ∞)	−	−	+	Negative slope that is decreasing	Speeding up and moving in reverse

5)



6)a)i) increasing **ii)** positive **b)i)** increasing **ii)** positive **c)i)** decreasing **ii)** negative **d)i)** constant **ii)** zero
7a) 0 **b)** 0 **c)** A; slope is steeper **d)** the bus is stopped **e)** B – slowing down, D – speeding up
8a) + **b)** - **c)** + **d)** 0 **e)** + **f)** 0

W4 – The Quotient Rule Unit 1

MCV4U

Jensen

1) Use the quotient rule to differentiate each function

a) $h(x) = \frac{x}{x+1}$

$$h'(x) = \frac{1(x+1) - 1(x)}{(x+1)^2}$$

$$h'(x) = \frac{x+1-x}{(x+1)^2}$$

$$h'(x) = \frac{1}{(x+1)^2}$$

b) $h(t) = \frac{2t-3}{t+5}$

$$h'(t) = \frac{2(t+5) - 1(2t-3)}{(t+5)^2}$$

$$h'(t) = \frac{2t+10-2t+3}{(t+5)^2}$$

$$h'(t) = \frac{13}{(t+5)^2}$$

c) $h(x) = \frac{x^3}{2x^2-1}$

$$h'(x) = \frac{3x^2(2x^2-1) - 4x(x^3)}{(2x^2-1)^2}$$

$$h'(x) = \frac{6x^4 - 3x^2 - 4x^4}{(2x^2-1)^2}$$

$$h'(x) = \frac{2x^4 - 3x^2}{(2x^2-1)^2}$$

d) $h(x) = \frac{1}{x^2+3}$

$$h'(x) = \frac{0(x^2+3) - 2x(1)}{(x^2+3)^2}$$

$$h'(x) = \frac{-2x}{(x^2+3)^2}$$

e) $y = \frac{x(3x+5)}{1-x^2} = \frac{3x^2+5x}{1-x^2}$

$$y' = \frac{(6x+5)(1-x^2) - (-2x)(3x^2+5x)}{(1-x^2)^2}$$

$$y' = \frac{6x - 6x^3 + 5 - 5x^2 + 6x^3 + 10x^2}{(1-x^2)^2}$$

$$y' = \frac{5x^2+6x+5}{(1-x^2)^2}$$

f) $y = \frac{x^2-x+1}{x^2+3}$

$$y' = \frac{(2x-1)(x^2+3) - (2x)(x^2-x+1)}{(x^2+3)^2}$$

$$y' = \frac{2x^3+6x-x^2-3-2x^3+2x^2-2x}{(x^2+3)^2}$$

$$y' = \frac{x^2+4x-3}{(x^2+3)^2}$$

2) Determine $\frac{dy}{dx}$ at each given value of x.

a) $y = \frac{3x+2}{x+5}$ at $x = -3$

$$\frac{dy}{dx} = \frac{3(x+5) - 1(3x+2)}{(x+5)^2}$$

$$\frac{dy}{dx} = \frac{3x+15-3x-2}{(x+5)^2}$$

$$\frac{dy}{dx} = \frac{13}{(x+5)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=-3} = \frac{13}{(-3+5)^2} = \frac{13}{4}$$

b) $y = \frac{x^3}{x^2+9}$ at $x = 1$

$$\frac{dy}{dx} = \frac{3x^2(x^2+9) - 2x(x^3)}{(x^2+9)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{3(1)^2[(1)^2+9] - 2(1)[(1)^3]}{[(1)^2+9]^2}$$

$$= \frac{30-2}{100}$$

$$= \frac{7}{25}$$

3) Find the point(s) at which the tangent to the curve is horizontal.

a) $y = \frac{2x^2}{x-4}$

$$y' = \frac{4x(x-4) - 1(2x^2)}{(x-4)^2}$$

$$y' = \frac{4x^2-16x-2x^2}{(x-4)^2}$$

$$0 = 2x^2 - 16x$$

$$0 = 2x(x-8)$$

$x_1 = 0$ $\qquad$ $x_2 = 8$

$y_1 = \frac{2(0)^2}{0-4}$ $\qquad$ $y_2 = \frac{2(8)^2}{8-4}$

$y_1 = 0$ $\qquad$ $y_2 = 32$

$(0, 0)$ $\qquad$ $(8, 32)$

b) $y = \frac{x^2-1}{x^2+x-2}$

Note: could have simplified first

$y = \frac{(x-1)(x+1)}{(x-1)(x+2)} = \frac{x+1}{x+2}$; $x \neq -2, 1$

VA $\qquad$ hole

$$y' = \frac{2x(x^2+x-2) - (2x+1)(x^2-1)}{(x^2+x-2)^2}$$

$$y' = \frac{2x^3+2x^2-4x-(2x^3-2x+x^2-1)}{(x^2+x-2)^2}$$

$$y' = \frac{x^2-2x+1}{(x^2+x-2)^2}$$

$$y' = \frac{\cancel{(x-1)^2}}{\cancel{(x-1)^2}(x+2)^2}$$

$$0 = \frac{1}{(x+2)^2}$$

$\therefore$ No Solutions

Note: hole in graph when $x = 1$

4) Determine the equation of the tangent to the curve $y = \frac{x^2-1}{3x}$ at $x = 2$.

$$y' = \frac{2x(3x) - 3(x^2-1)}{(3x)^2}$$

$$y' = \frac{6x^2-3x^2+3}{9x^2}$$

$$y' = \frac{3x^2+3}{9x^2}$$

$$y' = \frac{x^2+1}{3x^2}$$

Slope:

$$y'(2) = \frac{(2)^2+1}{3(2)^2}$$

$$= \frac{5}{12}$$

Point:

$$y(2) = \frac{(2)^2-1}{3(2)}$$

$$y(2) = \frac{1}{2}$$

Eq^n:

$$y = mx + b$$

$$\frac{1}{2} = \frac{5}{12}(2) + b$$

$$\frac{1}{2} = \frac{5}{6} + b$$

$$\frac{3}{6} - \frac{5}{6} = b$$

$$-\frac{1}{3} = b$$

$$\boxed{y = \frac{5}{12}x - \frac{1}{3}}$$

Answers:

1a) $h'(x) = \frac{1}{(x+1)^2}$ **b)** $h'(t) = \frac{13}{(t+5)^2}$ **c)** $h'(x) = \frac{2x^4-3x^2}{(2x^2-1)^2}$ **d)** $h'(x) = \frac{-2x}{(x^2+3)^2}$ **e)** $y' = \frac{5x^2+6x+5}{(1-x^2)^2}$ **f)** $\frac{dy}{dx} = \frac{x^2+4x-3}{(x^2+3)^2}$

2)a) $\frac{13}{4}$ **b)** $\frac{7}{25}$

3)a) (0,0) and (8,32) **b)** no horizontal tangents

4) $y = \frac{5}{12}x - \frac{1}{3}$

W5 – The Chain Rule **Unit 1**

MCV4U

Jensen

SOLUTIONS

1) Differentiate using the chain rule.

a) $f(x) = (-4x^2)^2$

$f'(x) = 2(-4x^2)(-8x)$

$f'(x) = 64x^3$

b) $f(x) = (16x^2)^{\frac{3}{4}}$

$f'(x) = \frac{3}{4}(16x^2)^{-1/4}(32x)$

$f'(x) = \dfrac{96x}{4(16x^2)^{1/4}}$

$f'(x) = \dfrac{96x}{8x^{1/2}}$ $\rightarrow$ $f'(x) = 12\sqrt{x}$

c) $y = (4x+1)^2$

$y' = 2(4x+1)(4)$

$y' = 8(4x+1)$

OR $y' = 32x+8$

d) $y = (x^3 - x)^{-3}$

$y' = -3(x^3-x)^{-4}(3x^2-1)$

$y' = \dfrac{-3(3x^2-1)}{(x^3-x)^4}$

OR $y' = \dfrac{-3(3x^2-1)}{[x(x^2-1)]^4} = \dfrac{-3(3x^2-1)}{x^4(x^2-1)^4}$

e) $y = \sqrt{2x - 3x^5}$

$y = (2x-3x^5)^{1/2}$

$y' = \frac{1}{2}(2x-3x^5)^{-1/2}(2-15x^4)$

$y^2 = \dfrac{2-15x^4}{2(2x-3x^5)^{1/2}}$

f) $y = \sqrt[5]{2 + 3x^2 - x^3}$

$y = (2+3x^2-x^3)^{1/5}$

$y' = \frac{1}{5}(2+3x^2-x^3)^{-4/5}(6x-3x^2)$

$y' = \dfrac{3x(2-x)}{5(2+3x^2-x^3)^{4/5}}$

2) Determine $f'(1)$.

a) $f(x) = (4x^2 - x + 1)^2$

$f'(x) = 2(4x^2 - x + 1)(8x - 1)$

$f'(1) = 2[4(1)^2 - 1 + 1][8(1) - 1]$

$f'(1) = 2(4)(7)$

$f'(1) = 56$

b) $f(x) = \frac{5}{\sqrt[3]{2x - x^2}}$

$$f'(x) = \frac{0(\sqrt[3]{2x - x^2}) - \frac{1}{3}(2x - x^2)^{-2/3}(2 - 2x)(5)}{[(2x - x^2)^{1/3}]^2}$$

$$f'(x) = \frac{-5(2 - 2x)}{3(2x - x^2)^{2/3}(2x - x^2)^{2/3}}$$

$$f'(x) = \frac{-5(2 - 2x)}{3(2x - x^2)^{4/3}}$$

$$f'(1) = \frac{-5[2 - 2(1)]}{3[2(1) - (1)^2]^{4/3}}$$

$f'(1) = 0$

3) Determine an equation for the tangent to the curve $y = (x^3 - 4x^2)^3$ at $x = 3$

$y' = 3(x^3 - 4x^2)^2(3x^2 - 8x)$

Slope:

$y'(3) = 3[(3)^3 - 4(3)^2]^2[3(3)^2 - 8(3)]$

$y'(3) = 3(-9)^2(3)$

$y'(3) = 729$

Point:

$y(3) = [(3)^3 - 4(3)^2]^3$

$y(3) = (-9)^3$

$y(3) = -729$

Eq^n:

$y = mx + b$

$-729 = 729(3) + b$

$b = -2916$

$\boxed{y = 729x - 2916}$

4) Determine the point(s) on the curve $y = x^2(x^3 - x)^2$ where the tangent line is horizontal.

$y' = 2x(x^3-x)^2 + 2(x^3-x)(3x^2-1)(x^2)$

$y' = 2x(x^3-x)[(x^3-x) + x(3x^2-1)]$

$y' = 2x(x^3-x)(4x^3-2x)$

$y' = 2x(x)(x^2-1)(2x)(2x^2-1)$

$y' = 4x^3(x-1)(x+1)(2x^2-1)$

$0 = 4x^3(x-1)(x+1)(2x^2-1)$

$0 = 4x^3$ $\quad 0 = x-1$ $\quad 0 = x+1$ $\quad 0 = 2x^2-1$

$x_1 = 0$ $\quad x_2 = 1$ $\quad x_3 = -1$ $\quad x^2 = \frac{1}{2}$

$y_1 = 0$ $\quad y_2 = 0$ $\quad y_3 = 0$

$(0,0)$ $\quad (1,0)$ $\quad (-1,0)$ $\quad x_4 = \frac{1}{\sqrt{2}}$ $\quad x_5 = -\frac{1}{\sqrt{2}}$

$y_4 = \frac{1}{16}$ $\quad y_5 = \frac{1}{16}$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$ $\quad \left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$

5) Differentiate each of the following.

a) $f(x) = (x+4)^3(x-3)^6$

$f'(x) = 3(x+4)^2(1)(x-3)^6 + 6(x-3)^5(1)(x+4)^3$

$f'(x) = 3(x+4)^2(x-3)^5[(x-3) + 2(x+4)]$

$f'(x) = 3(x+4)^2(x-3)^5(3x+5)$

b) $y = \left(\frac{x^2-3}{x^2+3}\right)^4$

$y' = 4\left(\frac{x^2-3}{x^2+3}\right)^3\left[\frac{2x(x^2+3) - 2x(x^2-3)}{(x^2+3)^2}\right]$

$y' = \dfrac{4(x^2-3)^3(2x)[(x^2+3)-(x^2-3)]}{(x^2+3)^3(x^2+3)^2}$

$y' = \dfrac{8x(x^2-3)^3(6)}{(x^2+3)^5}$

$y' = \dfrac{48x(x^2-3)^3}{(x^2+3)^5}$

Answers:

1a) $f'(x) = 64x^3$ b) $f'(x) = \frac{-24x}{(-16x^2)^{\frac{1}{4}}}$ c) $y' = 8(4x+1)$ d) $y' = \frac{-3(3x^2-1)}{x^4(x^2-1)^4}$ e) $\frac{dy}{dx} = \frac{2-15x^4}{2(2x-3x^5)^{\frac{1}{2}}}$ f) $\frac{dy}{dx} = \frac{6x-3x^2}{5(2+3x^2-x^3)^{\frac{4}{5}}}$

2)a) 56 b) 0

3) $y = 729x - 291$

4) $(-1,0)$, $(1,0)$, $(0,0)$, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$, and $\left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$

5)a) $(x+4)^2(x-3)^5(9x+15)$ b) $\frac{48x(x^2-3)^3}{(x^2+3)^5}$

W6 – Applications of Rates of Change | Unit 1
MCV4U
Jensen

SOLUTIONS

1) The demand function for a DVD player is $p(x) = \frac{575}{\sqrt{x}} - 3$, where x is the number of DVD players sold and p is the price, in dollars. Determine…

a) the revenue function

$$R(x) = x \cdot p(x)$$
$$R(x) = x\left(\frac{575}{\sqrt{x}} - 3\right)$$
$$R(x) = \frac{575x}{\sqrt{x}} - 3x$$
$$R(x) = 575\sqrt{x} - 3x$$

b) the marginal revenue function

$$R'(x) = \frac{1}{2}(575)(x)^{-1/2} - 3$$
$$R'(x) = \frac{575}{2\sqrt{x}} - 3$$

c) the marginal revenue when 200 DVD players are sold

$$R'(200) = \frac{575}{2\sqrt{200}} - 3$$

$R'(200) = \$17.33$ per DVD player

2) Refer to question 1. If the cost, C, in dollars, of producing x DVD players is $C(x) = 2000 + 150x - 0.002x^2$, determine…

a) the profit function

$$P(x) = R(x) - C(x)$$
$$= 575\sqrt{x} - 3x - (2000 + 150x - 0.002x^2)$$
$$= 0.002x^2 - 153x + 575\sqrt{x} - 2000$$

b) the marginal profit function

$$P'(x) = 0.004x - 153 + \frac{575}{2\sqrt{x}}$$

c) the marginal profit for the sale of 500 DVD players

$$P'(500) = 0.004(500) - 153 + \frac{575}{2\sqrt{500}}$$

$$= \$ -138.14 \text{ per DVD player}$$

3) A paint store sells 270 cans of paint per month at a price of $32 each. A customer survey indicates that for each $1.20 decrease in price, sales will increase by six cans of paint.

a) Determine the demand, or price, function.

sold $= x = 270 + 6n$

$n = \frac{x - 270}{6}$

price $= p = 32 - 1.20n$

$p(x) = 32 - 1.2\left(\frac{x-270}{6}\right)$

$p(x) = 32 - \left(\frac{x-270}{5}\right)$

$p(x) = 32 - \frac{x}{5} + \frac{270}{5}$

$p(x) = -0.2x + 86$

b) Determine the revenue function.

$R(x) = x \cdot p(x)$

$= x(-0.2x + 86)$

$= -0.2x^2 + 86x$

c) Determine the marginal revenue function.

$R'(x) = -0.4x + 86$

d) Solve $R'(x) = 0$. Interpret this value for this situation.

$0 = -0.4x + 86$

$x = 215$

Selling 215 cans per month maximizes the revenue.

e) What price corresponds to the value found in part d)? How can the paint store use this information.

$p(215) = -0.2(215) + 86$

$p(215) = \$43$ per dvd player

Charging $43 per DVD player will maximize the revenue.

4) A yogurt company estimates that the revenue from selling x containers of yogurt is $4.5x$. Its cost, C, in dollars, for producing this number of containers of yogurt is $C(x) = 0.0001x^2 + 2x + 3200$.

a) Determine the marginal cost of producing 4000 containers of yogurt.

$C'(x) = 0.0002x + 2$

$C'(4000) = 0.0002(4000) + 2$

$C'(4000) = \$2.80$ per yogurt container

b) Determine the marginal profit from selling 4000 containers of yogurt.

$P(x) = R(x) - C(x)$

$= 4.5x - (0.0001x^2 + 2x + 3200)$

$= -0.0001x^2 + 2.5x - 3200$

$P'(x) = -0.0002x + 2.5$

$P'(4000) = -0.0002(4000) + 2.5$

$P'(4000) = \$1.70$ per yogurt

c) What is the selling price of a container of yogurt?

$4.50

5) The cost, C, in dollars, of producing x hot tubs can be modelled by the function $C(x) = 3450x - 1.02x^2, 0 \leq x \leq 1500$.

a) Determine the marginal cost at a production level of 750 hot tubs. Explain what this means to the manufacturer.

$C'(x) = 3450 - 2.04x$

$C'(750) = 3450 - 2.04(750)$

$C'(750) = \$1920$

The negative slope of the linear equation shows that the rate of change of cost will decrease as you produce more hot tubs.

b) Find the cost of producing the 751st hot tub.

$C(750) = 3450(750) - 1.02(750)^2$

$= \$2\,013\,750$

$C(751) = 3450(751) - 1.02(751)^2$

$= \$2\,015\,668.98$

Cost of 751st hot tub $= C(751) - C(750)$

$= \$1918.98$

c) Compare and comment on the values you found in parts a) and b).

The marginal cost of producing x items is approximately equal to the cost of producing 1 more item.

d) Each hot tub is sold for \$9200. Write an expression to model the total revenue from the sale of x hot tubs.

$R(x) = 9200x$

e) Determine the rate of change of profit for the sale of 750 hot tubs.

$P(x) = 9200x - (3450x - 1.02x^2)$

$P(x) = 5750x + 1.02x^2$

$P'(x) = 5750 + 2.04x$

$P'(750) = 5750 + 2.04(750)$

$P'(750) = \$7280$ per hot tub.

6) The mass, in grams, of the first x meters of a wire can be modelled by the function $f(x) = \sqrt{2x-1}$.

a) Determine the average linear density of the part of the wire from $x = 1$ to $x = 8$.

average LD $= \dfrac{f(8) - f(1)}{8-1}$

$= \dfrac{\sqrt{2(8)-1} - \sqrt{2(1)-1}}{7}$

$= \dfrac{\sqrt{15} - 1}{7}$ $\simeq 0.41$ g/m

b) Determine the linear density at $x = 5$ and at $x = 8$, and compare the densities at the two points. What do these values confirm about the wire?

$f'(x) = \frac{1}{2}(2x-1)^{-1/2}(2)$

$f'(x) = \dfrac{1}{\sqrt{2x-1}}$

$f'(5) = \dfrac{1}{\sqrt{2(5)-1}}$

$= \frac{1}{3}$ g/m

$f'(8) = \dfrac{1}{\sqrt{2(8)-1}}$

$= \dfrac{1}{\sqrt{15}}$

$\simeq 0.26$ g/m

The density is decreasing as the the length of the wire increases.

The material is non-homogeneous.

7) A train leaves the station at 10:00 a.m. and travels due south at a speed of 60 km/h. Another train has been heading due west at 45 km/h and reaches the same station at 11:00 a.m. At what time were the two trains closest together?

Answers:

1)a) $R(x) = 575\sqrt{x} - 3x$ **b)** $R'(x) = \frac{575}{2\sqrt{x}} - 3$ **c)** $17.33 per DVD
2)a) $P(x) = 0.002x^2 - 153x + 575\sqrt{x} - 2000$ **b)** $P'(x) = 0.004x + \frac{575}{2\sqrt{x}} - 153$ **c)** −$138.14 per DVD
3)a) $p(x) = 86 - 0.2x$ **b)** $R(x) = 86x - 0.2x^2$ **c)** $R'(x) = 86 - 0.4x$ **d)** if 215 cans per month are sold, revenue is at a maximum **e)** charging $43 per can will maximize the revenue
4)a) $2.80 per container **b)** $1.70 per container **c)** $4.50
5)a) $1920 per hot tub. The equation shows that the rate of change in cost of producing x hot tubs reduces for greater values of x **b)** $1918.98 **c)** the marginal cost when producing x items is approximately equal to the cost of producing one more item **d)** $R(x) = 9200x$ **e)** $7280 per hot tub
6)a) 0.41 g/m **b)** $\frac{1}{3}$ g/m at $x = 5$ and $\frac{1}{\sqrt{15}}$ at $x = 8$. The density of the wire decreases as the distance increases.
~~7) 0.36 hours after the first train left the station (10:22 am)~~