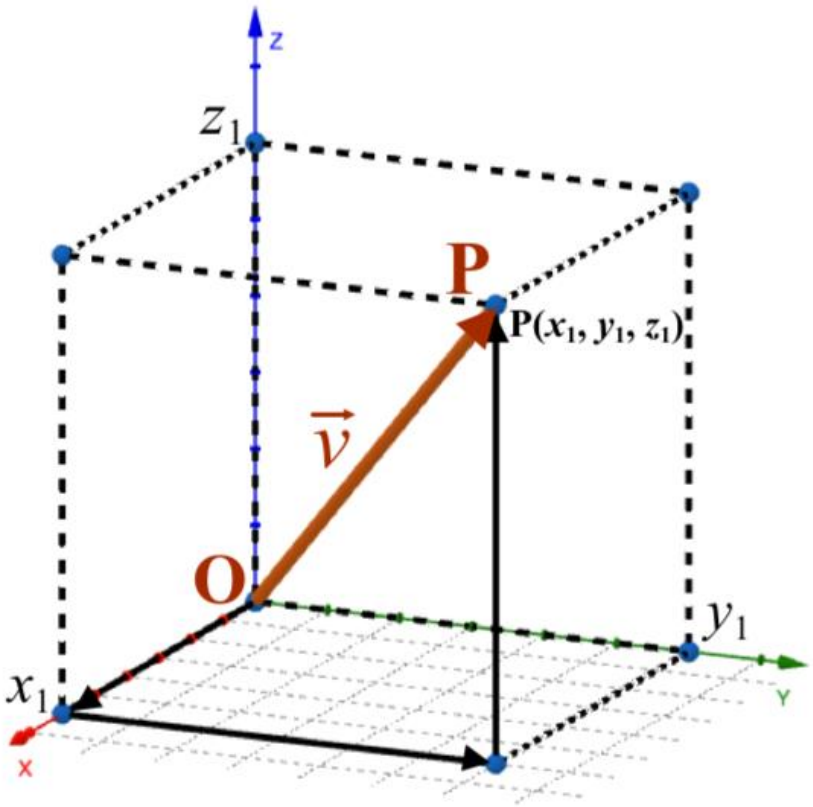
Name:

Unit 5- Cartesian Vectors

WORKBOOK

MCV4U



W1 – Cartesian Vectors Unit 5

MCV4U

Jensen

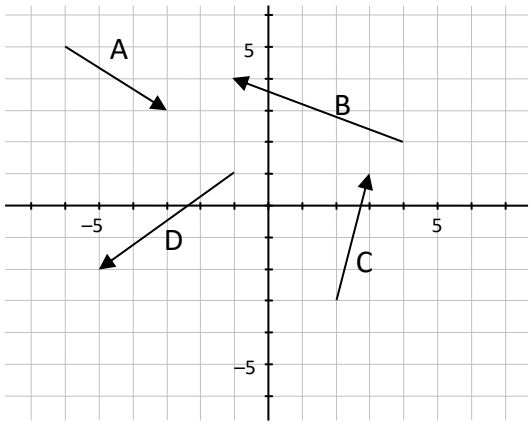
1) Express each vector in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

a) [–2, 0] **b)** [0, 3] **c)** [3, 2] **d)** [–1, 6]

2) Express each vector as a position vector [*a*, *b*].

a) $3\hat{\imath} + 2\hat{\jmath}$ **b)** $4\hat{\jmath}$ **c)** $-7\hat{\imath} + 3\hat{\jmath}$ **d)** $-9\hat{\imath}$

3) Write the coordinates of each Cartesian vector and determine the magnitude.



4) Given the vector $\vec{v}$ = [2, -5].

a) State the vertical and horizontal vector components of $\vec{v}$.

b) Find two vectors that are collinear with $\vec{v}$.

5) If $\vec{u}$ = [–3, 5] and $\vec{v}$ = [2, 9].

a) $\vec{u} + \vec{v}$

b) $\hat{u}$

c) $-3\vec{u} + 4\vec{v}$

d) $7\vec{u} + 6\hat{\imath} - 8\hat{\jmath} - 3\vec{v}$

e) $|\vec{v}|$

f) $|-3\vec{u} - 2\vec{v}|$

6. Write each force as a Cartesian vector.

a) 750 N applied 45° to the horizontal

b) 215 N applied 68° to the vertical

c) 450 N applied upwards

d) 17 N applied downwards

e) 1000 N east

f) 80 N west

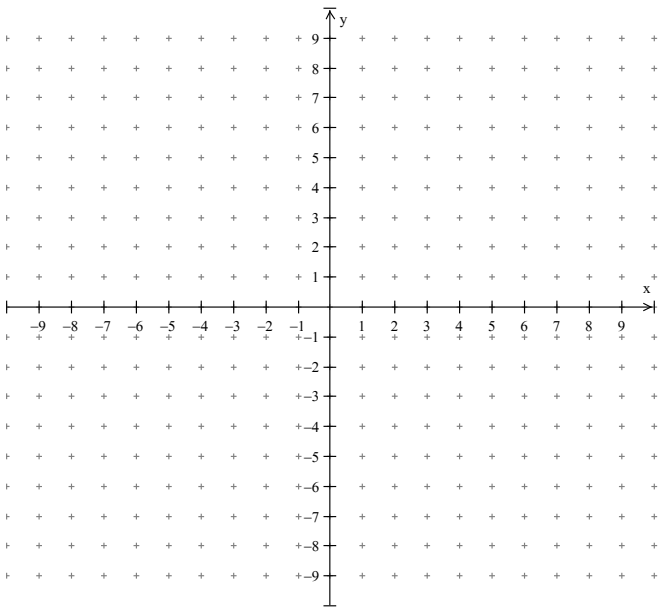
7) An aircraft is travelling at 750 km per hour at an angle of 35° to the level ground below. Find the force in component form as a Cartesian vector.

8) A mom is pulling a sled exerting a force of 220 N along a rope that makes an angle of 20° to the horizontal. Write this force in component form as a Cartesian vector.

9) Let $\vec{a}$ = [–2, 5] and $\vec{b}$ = [5, –7].

a) Plot the two vectors.

b) Which is greater: $\left|\vec{a}+\vec{b}\right|$ or $|\vec{a}|+\left|\vec{b}\right|$?



10) Given the points $P(-6,1)$, $Q(-2,-1)$, and $R(-3,4)$, find…

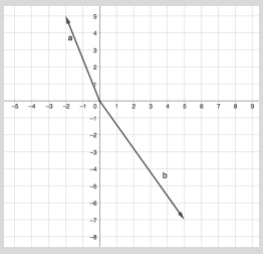
a) $\overrightarrow{QP}$ **b)** $\left|\overrightarrow{RP}\right|$ **c)** perimeter of ΔPQR

11) Which vector is NOT colinear with $\vec{a} = [6,-4]$?

$\vec{b} = [3,-2]$, $\vec{c} = [-6,-4]$, $\vec{d} = [-6,4]$, or $\vec{e} = [-9,6]$

ANSWER KEY:

1)a) $-2\hat{\imath}$ **b)** $3\hat{\jmath}$ **c)** $3\hat{\imath} + 2\hat{\jmath}$ **d)** $-\hat{\imath} + 6\hat{\jmath}$
2)a) [3, 2] **b)** [0, 4] **c)** [–7, 3] **d)** [–9, 0]
3)a) [3, –2]; $\sqrt{13}$ **b)** [–5, 2]; $\sqrt{29}$ **c)** [1, 4]; $\sqrt{17}$ **d)** [–4,–3]; 5
4) a) vertical: 2; horizontal: –5 **b)** e.g., $[4,-10]$, $[-6,15]$
5. a) [–1,14] **b)** $\left[-\frac{3}{\sqrt{34}},\frac{5}{\sqrt{34}}\right]$ **c)** [17, 21] **d)** $[-21,0]$ **e)** $\sqrt{85}$ **f)** $\sqrt{1114}$
6) a) $[750\cos 45\,,\,750\sin 45]$ **b)** $[215\cos 22\,,\,215\sin 22]$ **c)** [0, 450] **d)** [0, –17] **e)** [1000, 0] **f)** [–80, 0]
7) $[750\cos 35\,,\,750\sin 35] \cong [614.36, 430.18]$
8) $[220\cos 20\,,\,220\sin 20] \cong [206.7, 75.2]$
9) a)



b) $|\vec{a}| + \left|\vec{b}\right|$ is greater

10)a) $[-4,2]$ **b)** $3\sqrt{2}$ units **c)** 13.8 units
11) $\vec{c}$

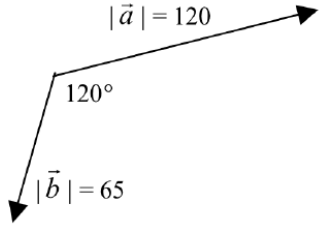
W2 – Dot Product Unit 5

MCV4U

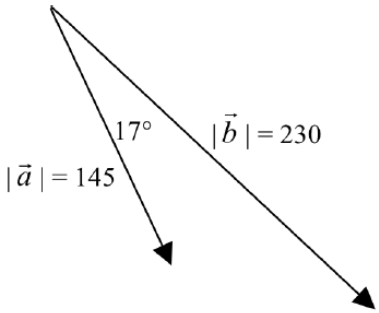
Jensen

1) Calculate the dot product for each pair.

a)



b)



2) Calculate the dot product for each pair of vectors. θ is the angle between the vectors when they are placed tail to tail.

a) $|\vec{u}| = 7$, $|\vec{v}| = 12$, and $\theta = 47°$

b) $|\vec{s}| = 520$, $|\vec{t}| = 745$, and $\theta = 135°$

3) Calculate the dot product of each pair of vectors.

a) $\vec{a} = [5, 8]$, $\vec{b} = [-2, 1]$

b) $\vec{c} = [-1, 8]$, $\vec{d} = [3, -3]$

c) $\vec{l} = 2\hat{\imath} - 3\hat{\jmath}$, $\vec{m} = -9\hat{\imath} + 4\hat{\jmath}$

d) $\vec{u} = -6\hat{\imath} + 7\hat{\jmath}$, $\vec{v} = 3\hat{\imath} - 2\hat{\jmath}$

4) Decide whether the following expressions have meaning or not. If not, explain why.

a) $\vec{u} \cdot (\vec{v} \cdot \vec{w})$ **b)** $|\vec{u} \cdot \vec{v}|$ **c)** $\vec{u}(\vec{v} \cdot \vec{w})$ **d)** $|\vec{u}|^2$ **e)** $\vec{v}^2$ **f)** $(\vec{u} \cdot \vec{v})^2$

5) Let $\vec{a}$ = [1, –2], $\vec{b}$ = [2, 5], and $\vec{c}$ = [4, –1]. Evaluate the following if possible. If not possible, explain why not.

a) $\vec{a} \cdot (\vec{b} + \vec{c})$

b) $(\vec{a} + \vec{b}) \cdot \vec{c}$

c) $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{c})$

d) $(3\vec{a} + 2\vec{b}) \cdot (4\vec{a} - \vec{b})$

e) $\vec{a} \cdot \vec{b} \cdot \vec{c}$

f) $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

g) $4\vec{b} \cdot (-2\vec{c})$

h) $(\vec{a} + \vec{b}) \cdot \vec{c}$

6) Determine a value of t so that $\vec{u}$ = [9, t] and $\vec{v}$ = [–16, t] are perpendicular.

7) Find a vector that is perpendicular to $\vec{a} = [3, -1]$. Verify that the vectors are perpendicular.

8) Which of the following is a right-angled triangle? Identify the right angle in that triangle.

- ΔABC for $A(3,1)$, $B(-2,3)$, and $C(5,6)$
- ΔSTU for $S(4,6)$, $T(-3,7)$, and $U(-5,-4)$

ANSWER KEY:
1)a) –3900 **b)** 31892.76
2)a) 57.29 **b)** –273 933.17
3)a) –2 **b)** –27 **c)** –30 **d)** –32
4)a) no, you cannot dot a vector with a scalar **b)** yes **c)** yes **d)** yes **e)** no, you cannot multiply vectors **f)** yes
5)a) –2 **b)** 9 **c)** 6 **d)** –38 **e)** not possible- you cannot dot a vector with a scalar **f)** -2 **g)** -24 **h)** 9
6) t = 12, –12
7) Answers may vary: $[-1, -3]$, $[1, 3]$, check using the dot product
8) ΔABC is a right triangle; the right angle is $\angle BAC$

W3 – Applications of the Dot Product Unit 5

MCV4U

Jensen

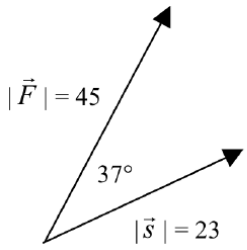
1) Determine the work done by each force $\vec{F}$, in Joules, for each object moving along $\vec{s}$.

a) $\vec{F}$ = [3, –2], $\vec{s}$ = [1, 8]

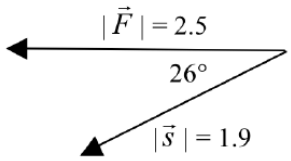
b) $\vec{F}$ = [8, –9], $\vec{s}$ = [–3, 7]

2) Determine the work done by the force $\vec{F}$, in Joules, for each object moving along $\vec{s}$.

a)



b)



3) Determine the angle between the vectors in each pair.

a) $\vec{p}$ = [6, 7] and $\vec{q}$ = [3, 2]

b) $\vec{r}$ = [–1, –7] and $\vec{s}$ = [5, 4]

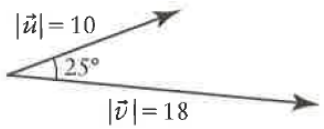
4) Determine the projection of the first vector on the second.

a) $\vec{a}$ = [6, –1], $\vec{b}$ = [3, –4]

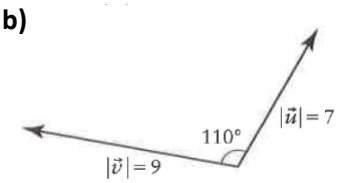
b) $\vec{c}$ = [6, 7], $\vec{d}$ = [3, 2]

5) Determine the projection of $\vec{u}$ on $\vec{v}$

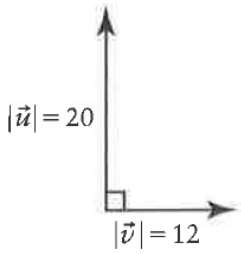
a)



b)



c)



6) For each of the following, find the magnitude of the projection of $\vec{x}$ on $\vec{y}$ and also the vector projection of $\vec{x}$ on $\vec{y}$.

a) $\vec{x} = [1,1], \vec{y} = [1, -1]$

b) $\vec{x} = [2,5], \vec{y} = [-5,12]$

7) ΔDEF has vertices D(–3, 5), E(2, 3), and F(6, 7). Calculate ∠DEF.

8) How much work is done against gravity by the orderly pushing an 85 kg person up a 5 m ramp inclined at an angle of 15° to the horizontal?

9) A stage lamp is dragged 15 m along level ground by a 120 N force applied at an angle of 35° to the ground. It is then dragged up a 12m ramp, inclined at 15° to the ground, onto a stage using the same force. Find the total work done.

10) A box on a wagon pulled a distance of 35 m by a 27 N force applied at an angle of 40° to the ground. The box is then lifted a distance of 1.5 m and placed on a table by exerting a force of 37 N. Find the total work done.

ANSWER KEY

1)a) –13 **b)** –87

2)a) 826.59 **b)** 4.27

3)a) $\theta = 15.71°$ **b)** $\theta = 136.79°$

4)a) $\left[\frac{66}{25}, -\frac{88}{25}\right]$ **b)** $\left[\frac{96}{13}, \frac{64}{13}\right]$

5)a) $9.06\hat{v}$ **b)** $-2.39\hat{v}$ **c)** $\vec{0}$

6) magnitude = 0, vector projection: $\vec{0}$ b) magnitude = $\frac{50}{13}$, vector projection: $\left[\frac{-250}{169}, \frac{600}{169}\right]$

7) 113.2°

8) 1077.98 J

9) 2827.63 J

10) 779.4 J

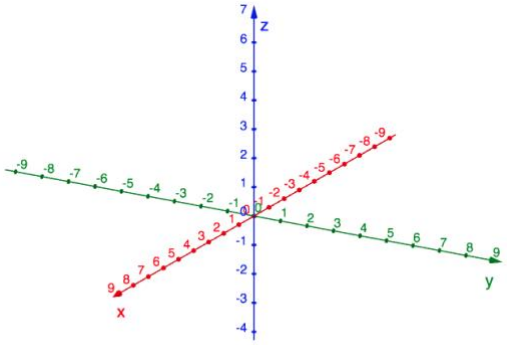
W4 – Vectors in 3-Space Unit 5

MCV4U

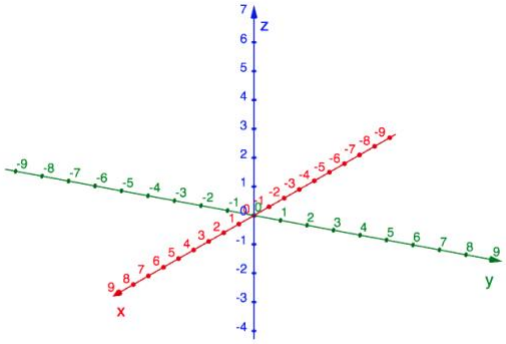
Jensen

1) Draw the position vectors.

a) [–2, 3, –4]



b) [2, –3, 1]



2) Express each vector as the sum of $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$.

a) [2, –1, 7]

b)[–4, –6, 5]

3) Express each vector in the form [*a*, *b*, *c*].

a) $3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$

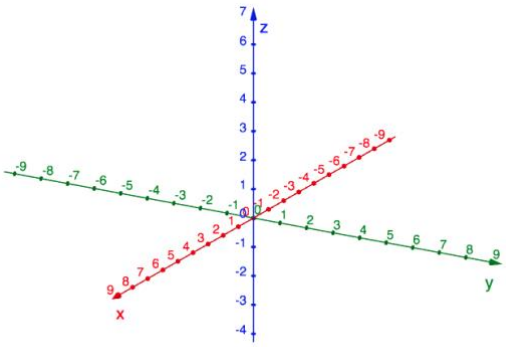
b) $2\hat{\imath} + 3\hat{k}$

c) $-8\hat{\imath} + 9\hat{\jmath} - 4\hat{k}$

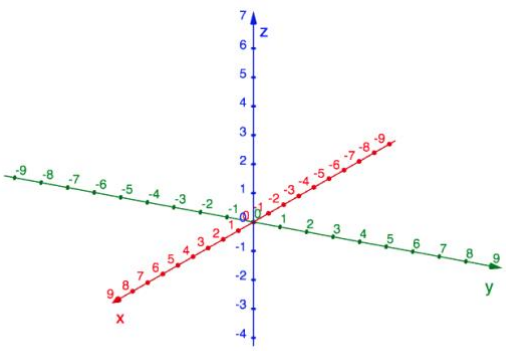
d) $-8\hat{\jmath} - 7\hat{k}$

4) Draw vector $\overrightarrow{AB}$ joining each pair of points. Then write the vector in the form [*a*, *b*, *c*].

a) A(2, –1, 7) and B(0, 2, –1)

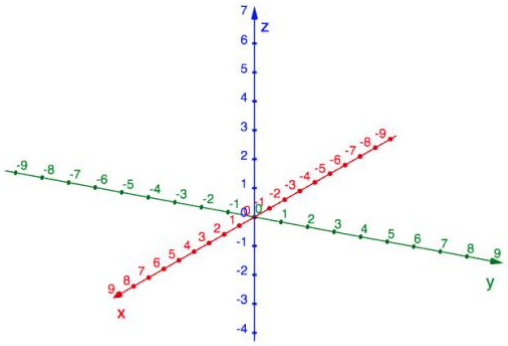


b) A(0, –4, –2) and B(–3, –1, 0)

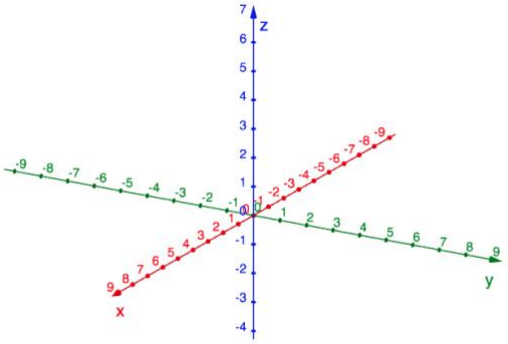


5) Draw each position vector. Then find its magnitude.

a) $[-1,5,-2]$



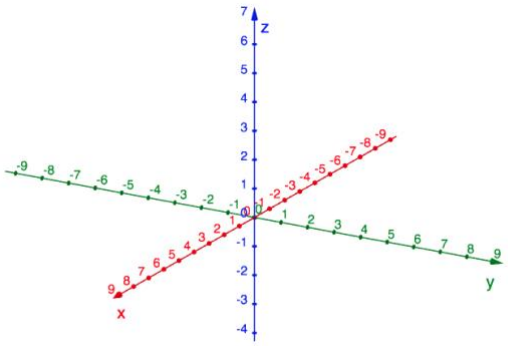
b) $[-2,0,4]$



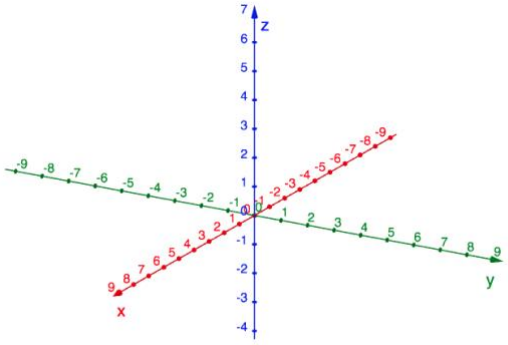
6) Find a and b such that $\vec{u} = [a, 3,6]$ and $\vec{v} = [-8,12, b]$ are collinear.

7) Draw the vector $\overrightarrow{AB}$ joining each pair of points. Write the vector in the form $[x, y, z]$. Then determine the exact magnitude of the vector.

a) $A(2,1,3)$ and $B(5,7,1)$



b) $A(3,-4,1)$ and $B(6,-1,5)$



8) Evaluate each given the vectors $\vec{a}$= [–2, 1, 8], $\vec{b}$= [3, 1, –2], and $\vec{c}$ = [2, –3, 4].

a) $3\vec{b}$ **b)** $\vec{b} - \vec{c}$ **c)** $2\vec{a} - 3\vec{c} + 4\vec{b}$

d) $(\vec{a} + \vec{b}) - (\vec{a} + \vec{c})$ **e)** $\vec{b} \cdot \vec{c}$ **f)** $\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}$

9) Let $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, $\vec{b} = 7\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{c} = -2\hat{\imath} + 5\hat{\jmath} + 9\hat{k}$.

a) $(\vec{a} + \vec{b}) \cdot \vec{c}$ **b)** $2\vec{a} \cdot (4\vec{b} - 3\vec{c})$

10) Determine the values of *k* such that $\vec{u}$ and $\vec{v}$ are orthogonal.

a) $\vec{u}$ = [2, *k*, –1] and $\vec{v}$ = [3, –2, 7] **b)** $\vec{u}$ = [–3, 1, *k*] and $\vec{v}$ = [4, –*k*, *k*]

11) Find a vector orthogonal to each vector.

a) [2, –1, 7]

b) [8, –3, 4]

12) Consider the vectors $\vec{u}$ = [3, –5, 8] and $\vec{v}$ = [3, 1, –2].

a) Find $\vec{u} \cdot \vec{v}$.

b) Calculate the angle between $\vec{u}$ and $\vec{v}$.

13) Determine the projection of $\vec{a}$ on $\vec{b}$.

a) $\vec{a}$ = [2, 1, –3] and $\vec{b}$ = [1, 7, 6]

b) $\vec{a}$ = [3, 4, 7] and $\vec{b}$ = [2, –1, 1]

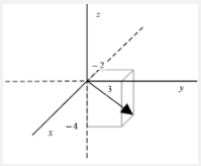
14) The initial point of vector $\overrightarrow{CD} = [2, -9, 1]$ is $C(-3, 2, 2)$ determine the coordinates of D.

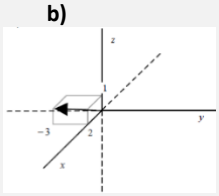
15) Find 2 unit vectors that are parallel to $\vec{a} = [9, -7, 2]$.

16) A triangle has vertices at the points $D = (3, -2, -3)$, $E(7, 0, 1)$ and $F(1, 2, 1)$. What type of triangle is $\triangle DEF$? Explain.

ANSWER KEY:

1. a) **b)**

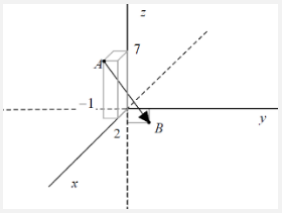


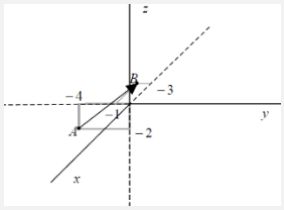


2. a) $2\hat{\imath} - \hat{\jmath} + 7\hat{k}$ **b)** $-4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$

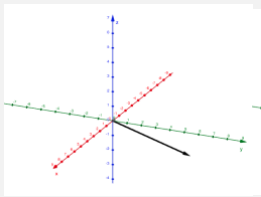
3. a) [3, –4, 5] **b)** [2, 0, 3] **c)** [–8, 9, –4] **d)** [0, –8, –7]

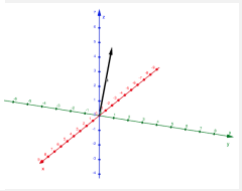
4. a) [–2, 3, –8] **b)** [–3, 3, 2]





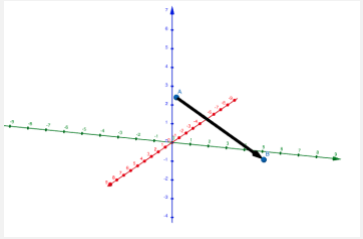
5)a) $\sqrt{30}$ **b)** $2\sqrt{5}$





6) $a = -2, b = 24$

7)a) $[3,6,-2], 7$ **b)** $[3,3,4], \sqrt{34}$



8) a) [9, 3, –6] **b)** [1, 4, –6] **c)** [2, 15, –4] **d)** [1, 4, –6] **e)** –5 **f)** –16

9) a) 17 **b)** -48

10)a) $k = -0.5$ **b)** $k = 4, k = -3$

11)a) [4, 8, 0] **b)** [1,0,–2]

12)a) –12 **b)** 108.9°

13)a) $\left[\frac{-9}{86}, \frac{-63}{86}, \frac{-27}{43}\right]$ **b)** $\left[3, \frac{-3}{2}, \frac{3}{2}\right]$

14) $D(-1, -7, 3)$

15) $\left[\frac{9}{\sqrt{134}}, -\frac{7}{\sqrt{134}}, \frac{2}{\sqrt{134}}\right]$ and $\left[-\frac{9}{\sqrt{134}}, \frac{7}{\sqrt{134}}, -\frac{2}{\sqrt{134}}\right]$

16) This is a non-right isosceles triangle because 2 sides of the triangle are the same length but no 2 vectors that make up the sides of the triangle dot to 0, this tells us there are no perpendicular vectors and therefore no right angles.

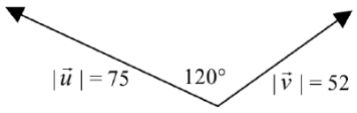
W5 – Cross Product of Vectors Unit 5

MCV4U

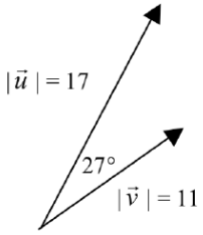
Jensen

1) Determine $\vec{u} \times \vec{v}$.

a)



b)



c) $\vec{u}$= [2, –1, 7], $\vec{v}$ = [2, 1, 3]

d) $\vec{u}$ = [–3, 4, 7], $\vec{v}$ = [4, 3, –5]

e) $\vec{u} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k} \quad \vec{v} = 5\hat{\imath} + \hat{\jmath} - 2\hat{k}$

f) $\vec{u} = 2\hat{\imath} - 3\hat{\jmath} + 7\hat{k} \quad \vec{v} = -\hat{\imath} + \hat{\jmath}$

2) Find a vector perpendicular to each of the following pairs of vectors. Use the dot product to check your answer.

a) [5, 0, 1] and [–2, 5, 8]

b) [1, 4, –2] and [–4, 9, 0]

3) Find a unit vector perpendicular to $\vec{a}$= [6, –2, –3] and $\vec{b}$ = [5, 1, –4].

4) Given $\vec{a}$ = [1, –2, –1], $\vec{b}$ = [2, 2, –1] and $\vec{c}$ = [2, –3, –4], evaluate each of the following:

a) $\vec{a} \times (\vec{b} \times \vec{c}\,)$

b) $(\vec{a} \times \vec{b}\,) \times \vec{c}$

c) $\vec{a} \times \vec{c} - \vec{a} \times \vec{b}$

d) $\vec{b} \times 3\vec{c}$

e) $(\vec{a} \times \vec{c}\,) \cdot \vec{b}$

f) $(\vec{a} \times \vec{b}\,) \cdot \vec{c}$

g) $\left|\vec{a} \times \vec{b}\right|$

h) $\left|\vec{a} \times (\vec{b} - \vec{c}\,)\right|$

5) Use the cross product to determine the angles between the vectors $\vec{a}$ = [2, 1, –3] and $\vec{b}$ = [5, –4, 3]. Consider ambiguous case. Use dot product to confirm or use graphing software to inspect.

6) Determine the area of ΔPQR with vertices of P(3, –2, 7), Q(2, 2, –3), and R(1, 1, 2).

7) Determine the area of the parallelogram ABCD defined by the vertices A(2, -1, -1), B(–4, -2, 3), C(2, 3, 2), and D(8, 4, -2).

ANSWER KEY:
1)a) $-3377.5\hat{n}$ or 3377.5 in to the page **b)** $-84.9\hat{n}$ or 84.9 in to the page **c)** [–10, 8, 4] **d)** [–41, 13, –25] **e)** [–7, 1, –17] **f)** [–7, –7, –1]
2)a) [–5, –42, 25] **b)** [18, 8, 25]
3) $\frac{1}{\sqrt{458}}$ [11, 9, 16]
4)a) [26, 21, –16] **b)** [22, 28, –10] **c)** [1, 3, –5] **d)** [–33, 18, –30] **e)** 13 **f)** –13 **g)** $\sqrt{53}$ **h)** $\sqrt{35}$
5) 96.5°
6) 2.5 $\sqrt{14}$ units^2
7) $\sqrt{1261}$ units^2

W6 – Applications of Dot and Cross Product Unit 5

MCV4U

Jensen

1) Given $\vec{a}$ = [2, 4, –5], $\vec{b}$ = [–1, 3, 7], and $\vec{c}$ = [–2, 7, 3], evaluate each expression.

a) $\vec{a} \times \vec{b} \cdot \vec{c}$

b) $\vec{a} \times \vec{c} \cdot \vec{b}$

2) Determine the projection, and its magnitude of $\vec{u}$ on $\vec{v}$.

a) $\vec{u}$ = [2, 1, 7], $\vec{v}$ = [–7, 2, 6]

b) $\vec{u} = 7\hat{\imath} - 6\hat{\jmath} + 5\hat{k},\ \vec{v} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

3) Determine the work done in the direction of travel.

a) $\vec{F}$ = [200, 150, 75], $\vec{s}$ = [2, –1, 8]

b) $\vec{F} = -3\hat{\imath} + 9\hat{\jmath} + 5\hat{k},\ \vec{s} = 2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$

4) Find the area of the parallelogram with sides consisting of the vectors.

a) $\vec{a}$ = [–4, 5, –8], $\vec{b}$ = [1, –2, 3]

b) $\vec{a}$ = [9, –5, 7], $\vec{b}$ = [3, –2, 5]

5) Find the area of the triangle with the given vertices.

a) A(0, 2, 4), B(3, –2, 1), C(4, –2, 5)

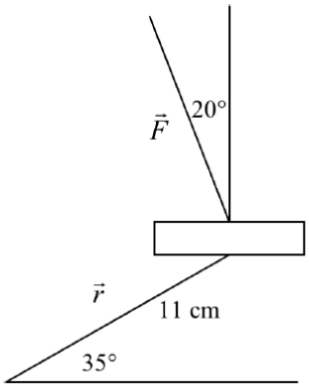
b) A(–2, 4, 5), B(1, 4, 2), C(7, 4, 9)

6) Determine the volume of the parallelepiped determined by the vectors.

a) $\vec{a}$ = [2, 5, –8], $\vec{b}$ = [7, –2, 3], and $\vec{c}$ = [8, 2, –1]

b) $\vec{a}$ = [1, –5, 9], $\vec{b}$ = [3, 4, –7], and $\vec{c}$ = [1, 0, 2]

7) Find the torque produced by a cyclist exerting a force of 85 N on the pedal in the position shown in the diagram, if the shaft of the petal is 11 cm long.



8) A woman pushes her baby stroller a distance of 1500 m by a force of 89 N applied at an angle of 35° to the roadway. Calculate the work done.

9) Determine the work done by gravity in causing a 45 kg child to slide down a 55 m slope, which has an angle of 47° to the horizontal.

10) A force of 75 N is applied to a wrench in a clockwise direction at $52°$ to the handle, 17 cm from the centre of the bolt.

a) Calculate the magnitude of the torque.

b) In what direction does the bolt move?

ANSWER KEY:

1. a) −119 **b)** 119

2. a) $\frac{30}{89}$ [−7, 2, 6]; $\frac{30}{\sqrt{89}}$ **b)** $\frac{38}{14}$ [3, −2, 1]; $\frac{38}{\sqrt{14}}$

3. a) 850 J **b)** 54 J

4. a) $\sqrt{26}$ units^2 **b)** $\sqrt{706}$ units^2

5. a) $\frac{\sqrt{497}}{2}$ units^2 **b)** $\frac{39}{2}$ units^2

6. a) 93 units^3 **b)** 37 units^3

7. 9.03 N•m

8. 109 356.8 J

9. 17 738.98 J

10. a) 10.05 N•m **b)** The bolt is being tightened into the material