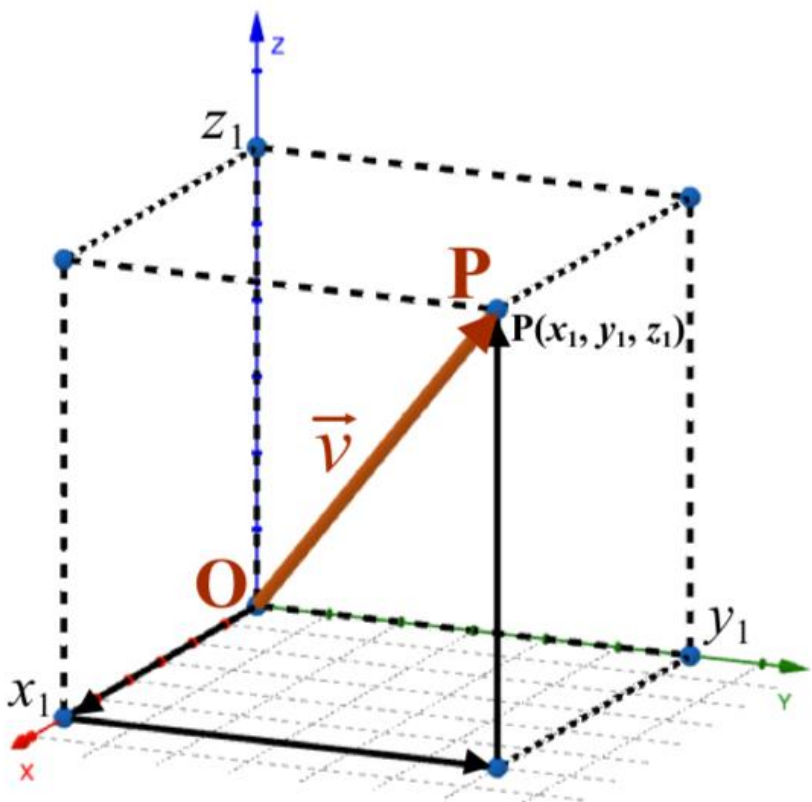
Name:

Unit 5- Cartesian Vectors

WORKBOOK

MCV4U



W1 – Cartesian Vectors Unit 5

MCV4U

Jensen

SOLUTIONS

1) Express each vector in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

a) [–2, 0] $= -2\hat{\imath}$

b) [0, 3] $= 3\hat{\jmath}$

c) [3, 2] $= 3\hat{\imath} + 2\hat{\jmath}$

d) [–1, 6] $= -\hat{\imath} + 6\hat{\jmath}$

2) Express each vector as a position vector $[a, b]$.

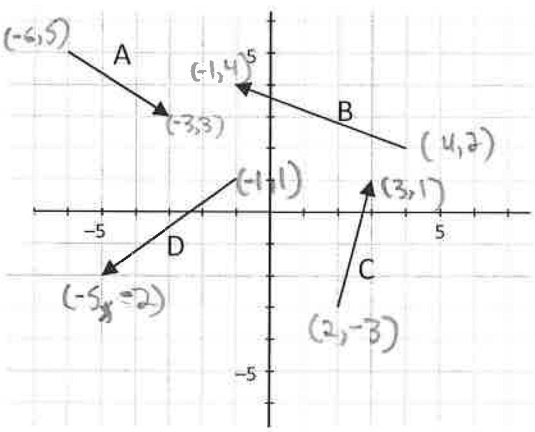
a) $3\hat{\imath} + 2\hat{\jmath}$ $[3, 2]$

b) $4\hat{\jmath}$ $[0, 4]$

c) $-7\hat{\imath} + 3\hat{\jmath}$ $[-7, 3]$

d) $-9\hat{\imath}$ $[-9, 0]$

3) Write the coordinates of each Cartesian vector and determine the magnitude.



$\vec{A} = [-3-(-6), 3-5] = [3, -2]$

$|\vec{A}| = \sqrt{(3)^2 + (-2)^2} = \sqrt{13}$

$\vec{B} = [-1-4, 4-2] = [-5, 2]$

$|\vec{B}| = \sqrt{(-5)^2 + (2)^2} = \sqrt{29}$

$\vec{C} = [3-2, 1-(-3)] = [1, 4]$

$|\vec{C}| = \sqrt{(1)^2 + (4)^2} = \sqrt{17}$

$\vec{D} = [-5-(-1), -2-1] = [-4, -3]$

$|\vec{D}| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$

4) Given the vector $\vec{v}$ = [2, -5].

a) State the vertical and horizontal vector components of $\vec{v}$.

horizontal: $v_x = 2$

vertical: $v_y = -5$

b) Find two vectors that are collinear with $\vec{v}$.

$2\vec{v} = [2(2), 2(-5)] = [4, -10]$

$3\vec{v} = [3(2), 3(-5)] = [6, -15]$

5) If $\vec{u} = [-3, 5]$ and $\vec{v} = [2, 9]$.

a) $\vec{u} + \vec{v}$

$= [-3+2, 5+9]$

$= [-1, 14]$

b) $\hat{u}$

$= \frac{1}{|\vec{u}|}\vec{u}$

$= \frac{1}{\sqrt{34}}[-3, 5]$

$= \left[\frac{-3}{\sqrt{34}}, \frac{5}{\sqrt{34}}\right]$

c) $-3\vec{u} + 4\vec{v}$

$= -3[-3,5] + 4[2,9]$

$= [9,-15] + [8, 36]$

$= [17, 21]$

d) $7\vec{u} + 6\hat{\imath} - 8\hat{\jmath} - 3\vec{v}$

$= 7(-3\hat{\imath}+5\hat{\jmath}) + 6\hat{\imath} - 8\hat{\jmath} - 3(2\hat{\imath}+9\hat{\jmath})$

$= -21\hat{\imath} + 35\hat{\jmath} + 6\hat{\imath} - 8\hat{\jmath} - 6\hat{\imath} - 27\hat{\jmath}$

$= -21\hat{\imath} + 0\hat{\jmath}$

$= [-21, 0]$

e) $|\vec{v}|$

$= \sqrt{(2)^2+(9)^2}$

$= \sqrt{85}$

f) $|-3\vec{u} - 2\vec{v}|$

$-3\vec{u} - 2\vec{v} = -3[-3,5] - 2[2,9]$

$= [9,-15] - [4,18]$

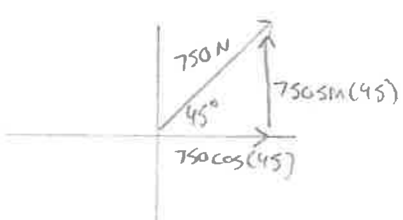
$= [5, -33]$

$|-3\vec{u} - 2\vec{v}| = \sqrt{(5)^2+(-33)^2}$

$= \sqrt{1114}$

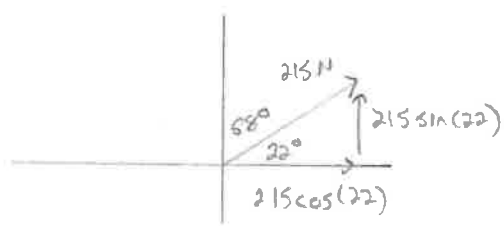
6. Write each force as a Cartesian vector.

a) 750 N applied 45° to the horizontal



$\vec{F} = [750\cos(45), 750\sin(45)]$

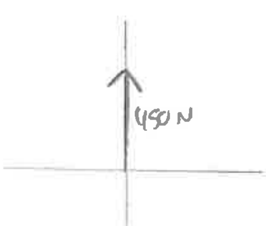
b) 215 N applied 68° to the vertical



$\vec{F} = [215\cos(22), 215\sin(22)]$

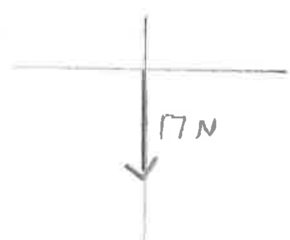
c) 450 N applied upwards

$[0, 450]$

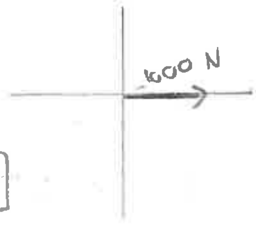


d) 17 N applied downwards

$[0, -17]$

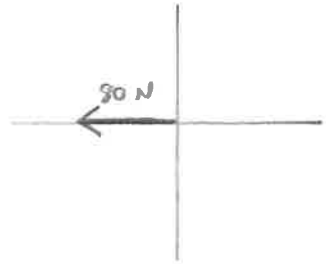


e) 1000 N east



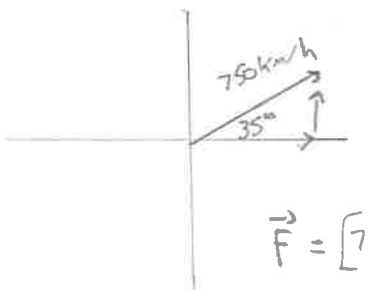
$[1000, 0]$

f) 80 N west



$[-80, 0]$

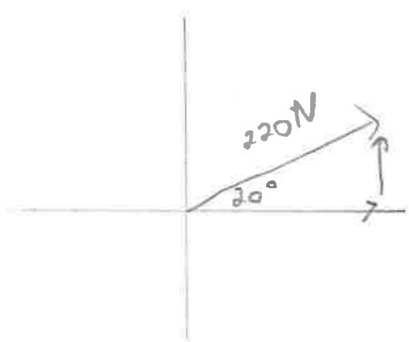
7) An aircraft is travelling at 750 km per hour at an angle of 35° to the level ground below. Find the force in component form as a Cartesian vector.



$\vec{F} = [750\cos(35), 750\sin(35)] \approx [614.36, 430.18]$

A mom is pulling a sled exerting a force of 220 N along a rope that makes an angle of 20° [redacted] to the horizontal. Write this force in component form as a Cartesian vector.

$\vec{F} = [220\cos 20, 220\sin 20] \simeq [206.73, 75.24]$



9) Let $\vec{a} = [-2, 5]$ and $\vec{b} = [5, -7]$.

a) Plot the two vectors.

b) Which is greater: $|\vec{a} + \vec{b}|$ or $|\vec{a}| + |\vec{b}|$?

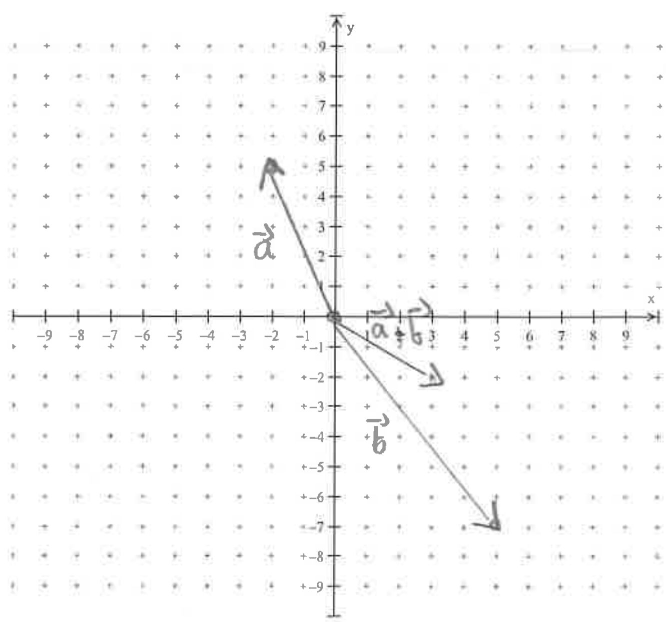
$\vec{a} + \vec{b} = [-2+5, 5+(-7)]$

$= [3, -2]$

$|\vec{a}+\vec{b}| = \sqrt{13}$

$|\vec{a}| + |\vec{b}| = \sqrt{29} + \sqrt{74}$

$|\vec{a}| + |\vec{b}| > |\vec{a}+\vec{b}|$



10) Given the points $P(-6,1)$, $Q(-2,-1)$, and $R(-3,4)$, find...

a) $\overrightarrow{QP}$

$\overrightarrow{QP} = [-6-(-2), 1-(-1)]$

$= [-4, 2]$

b) $|\overrightarrow{RP}|$

$= \sqrt{[-6-(-3)]^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{18}$

$= 3\sqrt{2}$

c) perimeter of ΔPQR

$|\overrightarrow{RP}| = 3\sqrt{2}$

$|\overrightarrow{QP}| = \sqrt{(-4)^2 + (2)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$|\overrightarrow{QR}| = \sqrt{(-1)^2 + (5)^2}$

$= \sqrt{26}$

Perimeter $= 3\sqrt{2} + 2\sqrt{5} + \sqrt{26}$

$\simeq 13.8$ units

11) Which vector is NOT colinear with $\vec{a} = [6, -4]$?

$\vec{b} = [3, -2]$, $\vec{c} = [-6, -4]$, $\vec{d} = [-6,4]$, or $\vec{e} = [-9,6]$

$\frac{a_x}{b_x} = \frac{a_y}{b_y}$ $\quad \frac{6}{3} = \frac{-4}{-2}$ $\quad 2 = 2$ $\quad \vec{a} = 2\vec{b}$

$\frac{a_x}{c_x} = \frac{a_y}{c_y}$ $\quad \frac{6}{-6} = \frac{-4}{-4}$ $\quad -1 \neq 1$ $\quad \therefore$ NOT collinear

$\frac{a_x}{d_x} = \frac{a_y}{d_y}$ $\quad \frac{6}{-6} = \frac{-4}{4}$ $\quad -1 = -1$ $\quad \vec{a} = -\vec{d}$

$\frac{a_x}{e_x} = \frac{a_y}{e_y}$ $\quad \frac{6}{-9} = \frac{-4}{6}$ $\quad -2/3 = -2/3$ $\quad \vec{a} = -2/3\, \vec{e}$

ANSWER KEY:

1)a) $-2\hat{\imath}$ **b)** $3\hat{\jmath}$ **c)** $3\hat{\imath} + 2\hat{\jmath}$ **d)** $-\hat{\imath} + 6\hat{\jmath}$

2)a) [3, 2] **b)** [0, 4] **c)** [−7, 3] **d)** [−9, 0]

3)a) (3, −2); $\sqrt{13}$ **b)** (−5, 2); $\sqrt{29}$ **c)** (1, 4); $\sqrt{17}$ **d)** (−4,−3); 5

4) a) vertical: 2; horizontal: −5 **b)** e.g., [4, −10], [−6, 15]

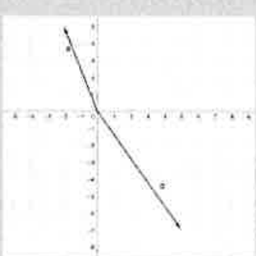
5. a) [−1,14] **b)** [illegible] **c)** [17, 21] **d)** [−21, 0] **e)** $\sqrt{85}$ **f)** $\sqrt{1114}$

6) a) [530.3, 530.33] **b)** [199, 80.5] **c)** [0, 450] **d)** [0, −17] **e)** [1000, 0] **f)** [−80, 0]

7) [614.36, 430.18]

8) [206.7, 75.2]

9) a)



b) $|\vec{a}| + |\vec{b}|$ is greater

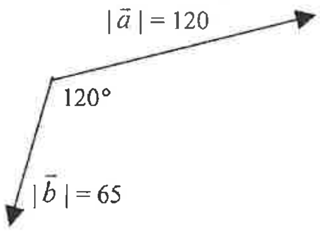
10)a) [−4,2] **b)** $3\sqrt{2}$ units **c)** 13.8 units

11) $\vec{c}$

W2 – Dot Product | Unit 5
MCV4U
Jensen

1) Calculate the dot product for each pair.

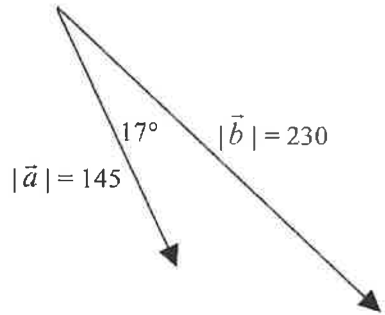
a)



$\vec{a} \cdot \vec{b} = 120(65)\cos(120)$

$= -3900$

b)



$\vec{a} \cdot \vec{b} = 145(230)\cos(17)$

$\simeq 31892.76$

2) Calculate the dot product for each pair of vectors. θ is the angle between the vectors when they are placed tail to tail.

$|\vec{u}| = 7$, $|\vec{v}| = 12$, and $\theta = 47°$

$\vec{u} \cdot \vec{v} = 7(12)\cos(47)$

$\simeq 57.29$

b) $|\vec{s}| = 520$, $|\vec{t}| = 745$, and $\theta = 135°$

$\vec{s} \cdot \vec{t} = 520(745)\cos(135)$

$\simeq -273933.17$

3) Calculate the dot product of each pair of vectors.

a) $\vec{a} = [5, 8]$, $\vec{b} = [-2, 1]$

$\vec{a} \cdot \vec{b} = 5(-2) + 8(1)$

$= -2$

b) $\vec{c} = [-1, 8]$, $\vec{d} = [3, -3]$

$\vec{c} \cdot \vec{d} = -1(3) + 8(-3)$

$= -27$

c) $\vec{l} = 2\hat{\imath} - 3\hat{\jmath}$, $\vec{m} = -9\hat{\imath} + 4\hat{\jmath}$

$l = [2, -3]$ $m = [-9, 4]$

$\vec{l} \cdot \vec{m} = 2(-9) + (-3)(4)$

$= -30$

d) $\vec{u} = -6\hat{\imath} + 7\hat{\jmath}$, $\vec{v} = 3\hat{\imath} - 2\hat{\jmath}$

$\vec{u} = [-6, 7]$ $\vec{v} = [3, -2]$

$\vec{u} \cdot \vec{v} = -6(3) + 7(-2)$

$= -32$

4) Decide whether the following expressions have meaning or not. If not, explain why.

a) $\vec{u}\cdot(\vec{v}\cdot\vec{w})$ **b)** $|\vec{u}\cdot\vec{v}|$ **c)** $\vec{u}(\vec{v}\cdot\vec{w})$ **d)** $|\vec{u}|^2$ **e)** $\vec{v}^2$ **f)** $(\vec{u}\cdot\vec{v})^2$

a) NO — can't dot vector with scalar

b) YES

c) YES

d) YES

e) NO — can't multiply vectors

f) YES

5) Let $\vec{a}$ = [1, −2], $\vec{b}$ = [2, 5], and $\vec{c}$ = [4, −1]. Evaluate the following if possible. If not possible, explain why not.

a) $\vec{a}\cdot(\vec{b}+\vec{c})$

$= [1,-2]\cdot([2,5]+[4,-1])$

$= [1,-2]\cdot[6,4]$

$= 1(6)+(-2)(4)$

$= -2$

b) $(\vec{a}+\vec{b})\cdot\vec{c}$

$= ([1,-2]+[2,5])\cdot[4,-1]$

$= [3,3]\cdot[4,-1]$

$= 3(4)+3(-1)$

$= 9$

c) $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{c})$

$= ([1,-2]+[2,5])\cdot([1,-2]+[4,-1])$

$= [3,3]\cdot[5,-3]$

$= 3(5)+3(-3)$

$= 6$

d) $(3\vec{a}+2\vec{b})\cdot(4\vec{a}-\vec{b})$

$= (3[1,-2]+2[2,5])\cdot(4[1,-2]-[2,5])$

$= ([3,-6]+[4,10])\cdot([4,-8]-[2,5])$

$= [7,4]\cdot[2,-13]$

$= 7(2)+4(-13)$

$= -38$

e) $\vec{a}\cdot\vec{b}\cdot\vec{c}$

Not possible.

$\vec{a}\cdot\vec{b}$ is a scalar.

Can't do $\vec{c}$ with a scalar.

f) $\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

$= [1,-2]\cdot[2,5]+[1,-2]\cdot[4,-1]$

$= 1(2)+(-2)(5)+1(4)+(-2)(-1)$

$= -2$

g) $4\vec{b}\cdot(-2\vec{c})$

$= 4[2,5]\cdot(-2[4,-1])$

$= [8,20]\cdot[-8,2]$

$= 8(-8)+20(2)$

$= -24$

h) $(\vec{a}+\vec{b})\cdot\vec{c}$

$= [1,-2]+[2,5]\cdot[4,-1]$

$= [3,3]\cdot[4,-1]$

$= 3(4)+3(-1)$

$= 9$

6) Determine a value of t so that $\vec{u}$ = [9, t] and $\vec{v}$ = [−16, t] are perpendicular.

If perpendicular, $\vec{u} \cdot \vec{v} = 0$

$[9,t] \cdot [-16,t] = 0$

$9(-16) + t(t) = 0$

$t^2 = 144$

$t = \pm 12$

7) Find a vector that is perpendicular to $\vec{a} = [3, -1]$. Verify that the vectors are perpendicular.

$\vec{a} = [3,-1]$ $\quad \vec{b} = [5,K]$

$\vec{a} \cdot \vec{b} = 0$

$3(5) + (-1)(k) = 0$

$k = 15$

$\boxed{[5,15]}$

Shortcut:

$[3,-1]$

$= [-1,3]$ change 1 sign

$\boxed{= [1,3]}$

check $[1,3] \cdot [3,-1]$

$= 3 + (-3)$

$= 0$

8) Which of the following is a right-angled triangle? Identify the right angle in that triangle.

- ΔABC for $A(3,1)$, $B(-2,3)$, and $C(5,6)$
- ΔSTU for $S(4,6)$, $T(-3,7)$, and $U(-5,-4)$

$\vec{AB} = [-5,2]$

$\vec{BC} = [7,3]$

$\vec{AC} = [2,5]$

$\vec{AB} \cdot \vec{BC} = -5(7) + 2(3) = -30$

$\vec{AB} \cdot \vec{AC} = -5(2) + 2(5) = 0$

$\therefore$ $\vec{AB}$ and $\vec{AC}$ are perpendicular.

$\angle A = 90°$

$\vec{ST} = [-7,1]$

$\vec{SU} = [-9,-10]$

$\vec{TU} = [-2,-11]$

$\vec{ST} \cdot \vec{SU} = -7(-9) + 1(-10) = 53$

$\vec{ST} \cdot \vec{TU} = -7(-2) + 1(-11) = 3$

$\vec{SU} \cdot \vec{TU} = -9(-2) + (-10)(-11) = 128$

$\therefore$ NOT a right triangle.

ANSWER KEY:

1)a) −3900 **b)** 31892.76

2)a) 57.28 **b)** −273 933.16

3)a) −2 **b)** −27 **c)** −30 **d)** −32

4)a) no, you cannot dot a vector with a scalar **b)** yes **c)** yes **d)** yes **e)** no, you cannot multiply vectors **f)** yes

5)a) −2 **b)** 9 **c)** 6 **d)** −38 **e)** not possible- you cannot dot a vector with a scalar **f)** −2 **g)** −24 **h)** 9

6) t = 12, −12

7) Answers may vary: $[-1, -3]$, $[1, 3]$, check using the dot product

8) ΔABC is a right triangle; the right angle is $\angle BAC$

W3 – Applications of the Dot Product Unit 5

MCV4U

Jensen

1) Determine the work done by each force $\vec{F}$, in Newtons, for each object moving along $\vec{s}$.

a) $\vec{F} = [3, -2]$, $\vec{s} = [1, 8]$

$W = \vec{F} \cdot \vec{s}$

$= 3(1) + (-2)(8)$

$= -13$ Joules

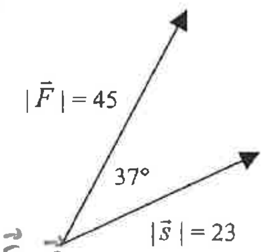
b) $\vec{F} = [8, -9]$, $\vec{s} = [-3, 7]$

$W = 8(-3) + (-9)(7)$

$= -87$ Joules

2) Determine the work done by the force $\vec{F}$, in Newtons, for each object moving along $\vec{s}$.

a)



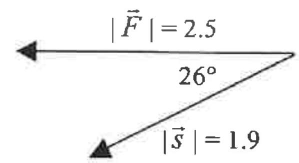
$W = \vec{F} \cdot \vec{s}$

$W = |\vec{F}||\vec{s}|\cos(37)$

$= 45(23)\cos(37)$

≈ 826.59 Joules

b)



$W = \vec{F} \cdot \vec{s}$

$W = |\vec{F}||\vec{s}|\cos(26)$

$W = 2.5(1.9)\cos(26)$

$W \approx 4.27$ Joules.

3) Determine the angle between the vectors in each pair.

a) $\vec{p} = [6, 7]$ and $\vec{q} = [3, 2]$

$\cos\theta = \dfrac{\vec{p} \cdot \vec{q}}{|\vec{p}||\vec{q}|}$

$\cos\theta = \dfrac{6(3) + 7(2)}{(\sqrt{6^2+7^2})(\sqrt{(3)^2+(2)^2})}$

$\cos\theta = \dfrac{32}{\sqrt{85}\sqrt{13}}$

$\boxed{\theta \approx 15.71^\circ}$

b) $\vec{r} = [-1, -7]$ and $\vec{s} = [5, 4]$

$\cos\theta = \dfrac{\vec{r} \cdot \vec{s}}{|\vec{r}||\vec{s}|}$

$\cos\theta = \dfrac{-1(5) + (-7)(4)}{(\sqrt{(-1)^2+(-7)^2})(\sqrt{(5)^2+(4)^2})}$

$\cos\theta = \dfrac{-33}{\sqrt{50}\sqrt{41}}$

$\boxed{\theta \approx 136.79^\circ}$

4) Determine the projection of the first vector on the second.

a) $\vec{a} = [6, -1]$, $\vec{b} = [3, -4]$

$proj_{\vec{b}}\vec{a} = \dfrac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}(\vec{b})$

$= \dfrac{6(3) + (-1)(-4)}{3^2 + (-4)^2}[3, -4]$

$= \dfrac{22}{25}[3, -4]$

$= \left[\dfrac{66}{25}, \dfrac{-88}{25}\right]$

b) $\vec{c} = [6, 7]$, $\vec{d} = [3, 2]$

$proj_{\vec{d}}\vec{c} = \dfrac{\vec{c} \cdot \vec{d}}{\vec{d} \cdot \vec{d}}\vec{d}$

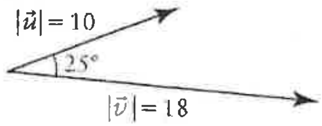
$= \dfrac{6(3) + 7(2)}{3^2 + 2^2}[3, 2]$

$= \dfrac{32}{13}[3, 2]$

$= \left[\dfrac{96}{13}, \dfrac{64}{13}\right]$

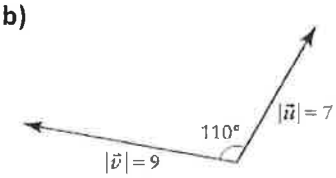
5) Determine the projection of $\vec{u}$ on $\vec{v}$

a)



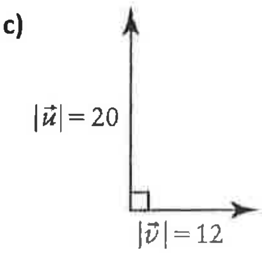
$$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\,(\hat{v})$$
$$= 10\cos(25)\,\hat{v}$$
$$\simeq 9.06\,\hat{v}$$

b)



$$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\,(\hat{v})$$
$$= 7\cos(110)\,\hat{v}$$
$$\simeq -2.39\,\hat{v}$$

c)



$$= \vec{0}$$

6) For each of the following, find the magnitude of the projection of $\vec{x}$ on $\vec{y}$ and also the vector projection of $\vec{x}$ on $\vec{y}$.

a) $\vec{x} = [1,1]$, $\vec{y} = [1,-1]$

$$|\text{proj}_{\vec{y}}\vec{x}| = \left|\frac{\vec{x}\cdot\vec{y}}{|\vec{y}|}\right|$$
$$= \left|\frac{1(1)+1(-1)}{\sqrt{(1)^2+(-1)^2}}\right|$$
$$= 0$$
$$\text{proj}_{\vec{y}}\vec{x} = 0\left(\frac{\vec{y}}{|\vec{y}|}\right)$$
$$= \vec{0}$$

b) $\vec{x} = [2,5]$, $\vec{y} = [-5,12]$

$$|\text{proj}_{\vec{y}}\vec{x}| = \left|\frac{\vec{x}\cdot\vec{y}}{|\vec{y}|}\right|$$
$$= \frac{2(-5)+5(12)}{\sqrt{(-5)^2+(12)^2}}$$
$$= \frac{50}{13}$$

$$\text{proj}_{\vec{y}}\vec{x} = \frac{50}{13}\left(\frac{\vec{y}}{|\vec{y}|}\right)$$
$$= \frac{50}{13}\left(\frac{[-5,12]}{13}\right)$$
$$= \frac{50}{169}[-5,12]$$
$$= \left[\frac{-250}{169}, \frac{600}{169}\right]$$

7) ΔDEF has vertices D(−3, 5), E(2, 3), and F(6, 7). Calculate $\angle$DEF.

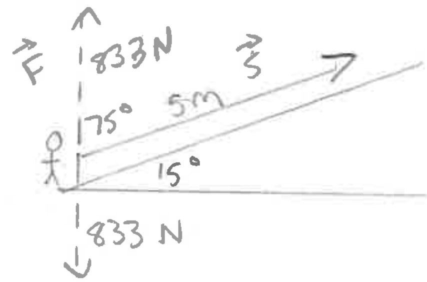
$$\vec{ED} = [-5,2]$$
$$\vec{EF} = [4,4]$$

$$\cos\theta = \frac{\vec{ED}\cdot\vec{EF}}{|\vec{ED}||\vec{EF}|}$$
$$\cos\theta = \frac{-5(4)+(2)(4)}{(\sqrt{29})(\sqrt{32})}$$
$$\cos\theta = \frac{-12}{\sqrt{928}}$$
$$\theta \simeq 113.2^\circ$$

against gravity

8) How much work is done by the orderly pushing an 85 kg person up a 5 m ramp inclined at an angle of 15° to the horizontal?

$\vec{F} = 85(9.8) = 833$ N



$W = \vec{F} \cdot \vec{s}$

$W = 833(5)\cos(75)$

$W \simeq 1077.98$ J

9) A stage lamp is dragged 15 m along level ground by a 120 N force applied at an angle of 35° to the ground. It is then dragged up a 12m ramp onto a stage by the same force. If the ramp is inclined at 15° to the ground. Find the total work done.

$W = 120(15)\cos(35°) + 120(12)\cos(20°)$

$W \simeq 2827.63$ J

10) A box on a wagon pulled a distance of 35 m by a 27 N force applied at an angle of 40° to the ground. The box is then lifted a distance of 1.5 m and placed on a table by exerting a force of 37 N. Find the total work done.

$W = 27(35)\cos(40°) + 37(1.5)\cos(0)$

$W \simeq 779.4$ J

ANSWER KEY

1)a) -13 **b)** -87

2)a) 826.58 **b)** 4.269

3)a) $\theta = 15.7°$ **b)** $\theta = 136.8°$

4)a) $[2.64, -3.52]$ **b)** $[7.38, 4.92]$

5)a) $9.1\hat{v}$ **b)** $-2.4\hat{v}$ **c)** $\vec{0}$

6) magnitude = 0, vector projection: $\vec{0}$ **b)** magnitude = $\frac{50}{13}$, vector projection: $\left[\frac{-250}{169}, \frac{600}{169}\right]$

7) 113.2°

8) 1077.98 J

9) 2865.4 J

10) 779.4 J

W4 – Vectors in 3-Space

MCV4U

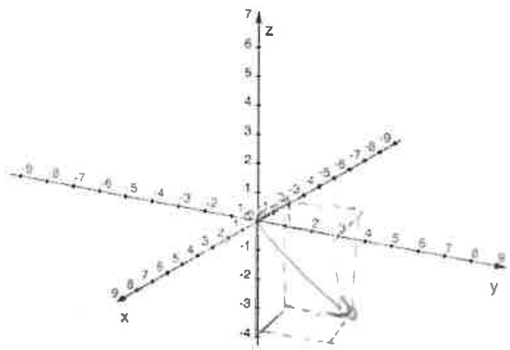
Jensen

Unit 5

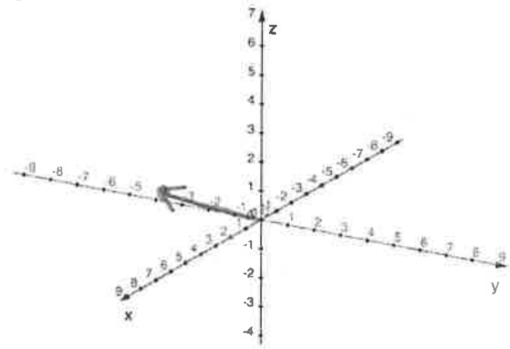
SOLUTIONS

1) Draw the position vectors.

a) [−2, 3, −4]



b) [2, −3, 1]



2) Express each vector as the sum of $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$.

a) [2, −1, 7]

$= 2\hat{\imath} - \hat{\jmath} + 7\hat{k}$

b) [−4, −6, 5]

$= -4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$

3) Express each vector in the form $[a, b, c]$.

a) $3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$

$= [3, -4, 5]$

b) $2\hat{\imath} + 3\hat{k}$

$= [2, 0, 3]$

c) $-8\hat{\imath} + 9\hat{\jmath} - 4\hat{k}$

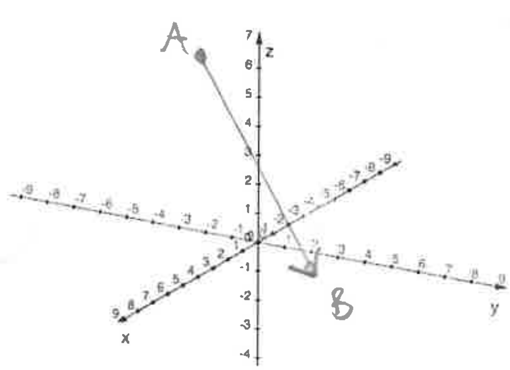
$= [-8, 9, -4]$

d) $-8\hat{\jmath} - 7\hat{k}$

$= [0, -8, -7]$

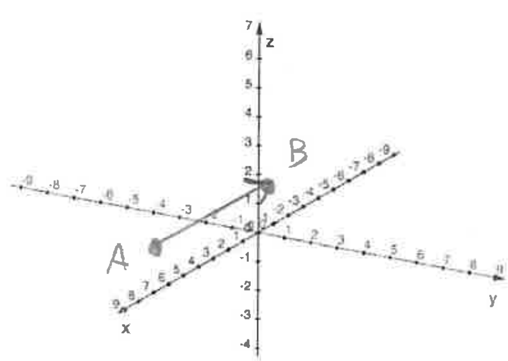
4) Draw vector $\overrightarrow{AB}$ joining each pair of points. Then write the vector in the form $[a, b, c]$.

a) A(2, −1, 7) and B(0, 2, −1)



$\overrightarrow{AB} = [-2, 3, -8]$

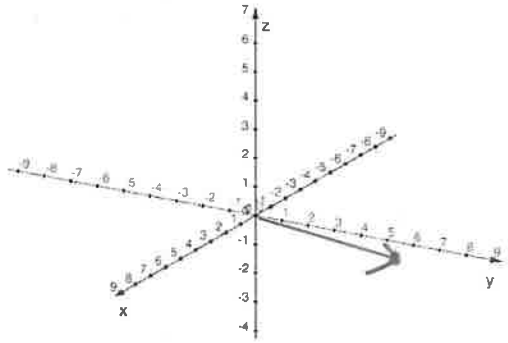
b) A(0, −4, −2) and B(−3, −1, 0)



$\overrightarrow{AB} = [-3, 3, 2]$

5) Draw each position vector. Then find its magnitude.

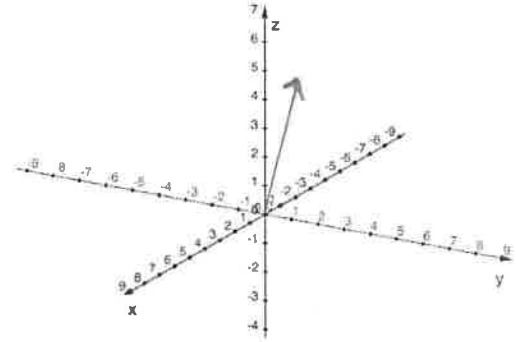
a) $[-1,5,-2]$



Magnitude $= \sqrt{(-1)^2+(5)^2+(-2)^2}$

$= \sqrt{30}$

b) $[-2,0,4]$



Magnitude $= \sqrt{(-2)^2+(0)^2+(4)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

6) Find a and b such that $\vec{u} = [a,3,6]$ and $\vec{v} = [-8,12,b]$ are collinear.

x:

$a = -8k$

$a = -8(\frac{1}{4})$

$\boxed{a = -2}$

y:

$3 = 12k$

$k = \frac{1}{4}$

z:

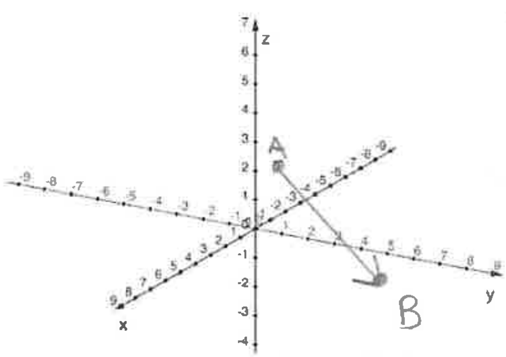
$6 = bk$

$6 = b(\frac{1}{4})$

$\boxed{b = 24}$

7) Draw the vector $\overrightarrow{AB}$ joining each pair of points. Write the vector in the form $[x,y,z]$. Then determine the exact magnitude of the vector.

a) $A(2,1,3)$ and $B(5,7,1)$

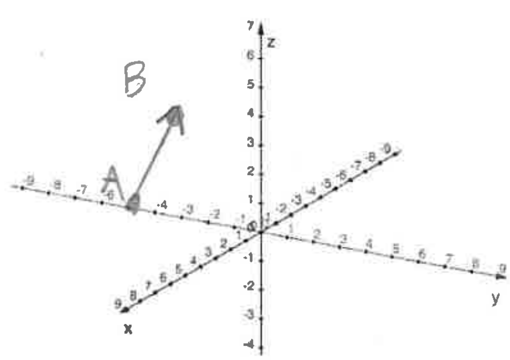


$\overrightarrow{AB} = [3,6,-2]$

$|\overrightarrow{AB}| = \sqrt{(3)^2+(6)^2+(-2)^2}$

$|\overrightarrow{AB}| = 7$

b) $A(3,-4,1)$ and $B(6,-1,5)$



$\overrightarrow{AB} = [3,3,4]$

$|\overrightarrow{AB}| = \sqrt{(3)^2+(3)^2+(4)^2}$

$|\overrightarrow{AB}| = \sqrt{34}$

8) Evaluate each given the vectors $\vec{a}$= [–2, 1, 8], $\vec{b}$= [3, 1, –2], and $\vec{c}$ = [2, –3, 4].

a) $3\vec{b}$

$= 3[3,1,-2]$

$= [9,3,-6]$

b) $\vec{b} - \vec{c}$

$= [3,1,-2] - [2,-3,4]$

$= [1,4,-6]$

c) $2\vec{a} - 3\vec{c} + 4\vec{b}$

$= 2[-2,1,8] - 3[2,-3,4] + 4[3,1,-2]$

$= [-4,2,16] - [6,-9,12] + [12,4,-8]$

$= [-10,11,4] + [12,4,-8]$

$= [2,15,-4]$

d) $(\vec{a} + \vec{b}) - (\vec{a} + \vec{c})$

$= ([-2,1,8] + [3,1,-2]) - ([-2,1,8] + [2,-3,4])$

$= [1,2,6] - [0,-2,12]$

$= [1,4,-6]$

e) $\vec{b} \cdot \vec{c}$

$= 3(2) + 1(-3) + (-2)(4)$

$= 6 - 3 - 8$

$= -5$

f) $\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}$

$= -2(3) + 1(1) + 8(-2) - (-5)$

$= -6 + 1 - 16 + 5$

$= -16$

Let $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, $\vec{b} = 7\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\vec{c} = -2\hat{\imath} + 5\hat{\jmath} + 9\hat{k}$.

a) $(\vec{a} + \vec{b}) \cdot \vec{c}$

$= ([3,-2,4] + [7,4,-1]) \cdot [-2,5,9]$

$= [10,2,3] \cdot [-2,5,9]$

$= 10(-2) + 2(5) + 3(9)$

$= 17$

b) $2\vec{a} \cdot (4\vec{b} - 3\vec{c})$

$= 2[3,-2,4] \cdot (4[7,4,-1] - 3[-2,5,9])$

$= [6,-4,8] \cdot [34,1,-31]$

$= 6(34) + (-4)(1) + 8(-31)$

$= -48$

10) Determine the values of k such that $\vec{u}$ and $\vec{v}$ are orthogonal.

a) $\vec{u}$ = [2, k, –1] and $\vec{v}$ = [3, –2, 7]

$\vec{u} \cdot \vec{v} = 0$

$2(3) + k(-2) + (-1)(7) = 0$

$6 - 2k - 7 = 0$

$-2k = 1$

$\boxed{k = \frac{-1}{2}}$

b) $\vec{u}$ = [–3, 1, k] and $\vec{v}$ = [4, –k, k]

$\vec{u} \cdot \vec{v} = 0$

$-3(4) + 1(-k) + k(k) = 0$

$-12 - k + k^2 = 0$

$k^2 - k - 12 = 0$

$(k-4)(k+3) = 0$

$\boxed{k_1 = 4 \quad k_2 = -3}$

11) Find a vector orthogonal to each vector.

a) [2, −1, 7]

$[0, 7, 1]$

b) [8, −3, 4]

$[0, 4, 3]$

12) Consider the vectors $\vec{u}$ = [3, −5, 8] and $\vec{v}$ = [3, 1, −2].

a) Find $\vec{u} \cdot \vec{v}$.

$= 3(3) + (-5)(1) + 8(-2)$

$= -12$

b) Calculate the angle between $\vec{u}$ and $\vec{v}$.

$$\cos\theta = \frac{-12}{(\sqrt{98})(\sqrt{14})}$$

$\theta \simeq 108.9°$

13) Determine the projection of $\vec{a}$ on $\vec{b}$.

a) $\vec{a}$ = [2, 1, −3] and $\vec{b}$ = [1, 7, 6]

$$\text{proj}_{\vec{b}}\vec{a} = \frac{2(1) + 1(7) + (-3)(6)}{(1)^2 + (7)^2 + (6)^2}[1, 7, 6]$$

$$= \frac{-9}{86}[1, 7, 6]$$

$$= \left[\frac{-9}{86}, \frac{-63}{86}, \frac{-27}{43}\right]$$

b) $\vec{a}$ = [3, 4, 7] and $\vec{b}$ = [2, −1, 1]

formula:

$$\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}(\vec{b})$$

$$\text{proj}_{\vec{b}}\vec{a} = \frac{3(2) + 4(-1) + 7(1)}{(2)^2 + (-1)^2 + (1)^2}[2, -1, 1]$$

$$= \frac{9}{6}[2, -1, 1]$$

$$= \left[3, -\frac{3}{2}, \frac{3}{2}\right]$$

14) The initial point of vector $\overrightarrow{CD} = [2, -9, 1]$ is $C(-3, 2, 2)$ determine the coordinates of D.

$x - (-3) = 2$

$x + 3 = 2$

$x = -1$

$y - 2 = -9$

$y = -7$

$z - 2 = 1$

$z = 3$

$\boxed{D(-1, -7, 3)}$

15) Find 2 unit vectors that are parallel to $\vec{a} = [9, -7, 2]$.

$\pm \frac{1}{|\vec{a}|} \vec{a}$

$|\vec{a}| = \sqrt{(9)^2 + (-7)^2 + (2)^2}$

$|\vec{a}| = \sqrt{134}$

The unit vectors are:

1) $\frac{1}{\sqrt{134}}[9, -7, 2] = \left[\frac{9}{\sqrt{134}}, \frac{-7}{\sqrt{134}}, \frac{2}{\sqrt{134}}\right]$

2) $\frac{-1}{\sqrt{134}}[9, -7, 2] = \left[\frac{-9}{\sqrt{134}}, \frac{7}{\sqrt{134}}, \frac{-2}{\sqrt{134}}\right]$

16) A triangle has vertices at the points $D = (3, -2, -3)$, $E(7, 0, 1)$ and $F(1, 2, 1)$. What type of triangle is $\triangle DEF$? Explain.

$\vec{DE} = [4, 2, 4]$ $\quad |\vec{DE}| = 6$

$\vec{DF} = [-2, 4, 4]$ $\quad |\vec{DF}| = 6$

$\vec{EF} = [-6, 2, 0]$ $\quad |\vec{EF}| = \sqrt{40} = 2\sqrt{10}$

Isosceles

$\vec{DE} \cdot \vec{DF} = 4(-2) + 2(4) + 4(4) = 16$

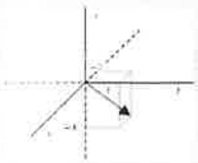
$\vec{DE} \cdot \vec{EF} = 4(-6) + 2(2) + 4(0) = -20$

$\vec{DF} \cdot \vec{EF} = -2(-6) + 4(2) + 4(0) = 20$

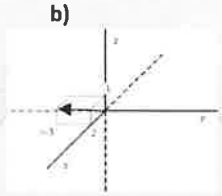
No right angles.

ANSWER KEY:

1. a)



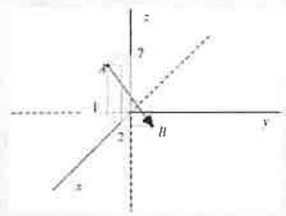
b)



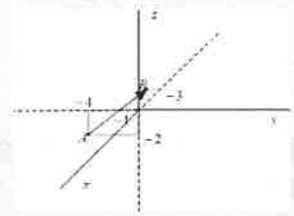
2. a) $2\hat{\imath} - \hat{\jmath} + 7\hat{k}$ **b)** $-4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$

3. a) [3, –4, 5] **b)** [2, 0, 3] **c)** [–8, 9, –4] **d)** [0, –8, –7]

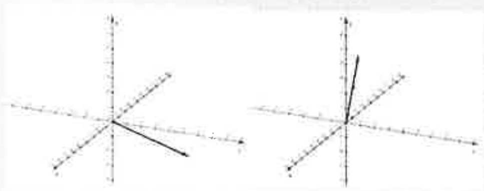
4. a) [–2, 3, –8]



b) [–3, 3, 2]



5)a) $\sqrt{30}$ **b)** $2\sqrt{5}$



6) $a = -2, b = 24$

7)a) $[3,6,-2], 7$ **b)** $[3,3,4], \sqrt{34}$

8) a) [9, 3, –6] **b)** [1, 4, –6] **c)** [2, 15, –4] **d)** [1, 4, –6] **e)** –5 **f)** –16

9) a) 17 **b)** ~~92~~ -46

10)a) $k = -0.5$ **b)** $k = 4, k = -3$

11)a) [4, 8, 0] **b)** [1,0,–2]

12)a) –12 **b)** 108.9°

13)a) [–0.10, –0.73, –0.63] **b)** [3, –1.5, 1.5]

14) $D(-1,-7,3)$

15) $[\frac{9}{\sqrt{134}}, -\frac{7}{\sqrt{134}}, \frac{2}{\sqrt{134}}]$ and $[-\frac{9}{\sqrt{134}}, \frac{7}{\sqrt{134}}, -\frac{2}{\sqrt{134}}]$

16) This is a non-right isosceles triangle because 2 sides of the triangle are the same length but no 2 vectors that make up the sides of the triangle dot to 0, this tells us there are no perpendicular vectors and therefore no right angles.

W5 – Cross Product of Vectors — Unit 5

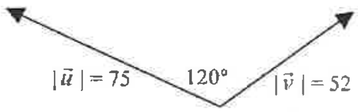
MCV4U

Jensen

SOLUTIONS

1) Determine $\vec{u} \times \vec{v}$.

a)

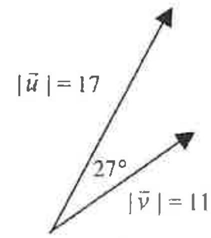


$\vec{u} \times \vec{v} = [75(52)\sin(120)](-\hat{n})$

$= -3377.5\hat{n}$

OR 3377.5 in to the page.

b)



$\vec{u} \times \vec{v} = [17(11)\sin(27)](-\hat{n})$

$= -84.9\hat{n}$

OR 84.9 in to the page

c) $\vec{u} = [2, -1, 7]$, $\vec{v} = [2, 1, 3]$

$u_1 \; u_2 \; u_3 \qquad v_1 \; v_2 \; v_3$

$$\begin{array}{cc} -1 & 1 \\ 7 & 3 \\ 2 & 2 \\ -1 & 1 \end{array}$$

$\vec{u} \times \vec{v} = [-1(3) - 7(1), 7(2) - 2(3), 2(1) - (-1)(2)]$

$= [-10, 8, 4]$

d) $\vec{u} = [-3, 4, 7]$, $\vec{v} = [4, 3, -5]$

$$\begin{array}{cc} 4 & 3 \\ 7 & -5 \\ -3 & 4 \\ 4 & 3 \end{array}$$

$\vec{u} \times \vec{v} = [4(-5) - 7(3), 7(4) - (-3)(-5), -3(3) - 4(4)]$

$= [-41, 13, -25]$

e) $\vec{u} = 3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ $\quad \vec{v} = 5\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$= [3, 4, -1] \qquad = [5, 1, -2]$

$$\begin{array}{cc} 4 & 1 \\ -1 & -2 \\ 3 & 5 \\ 4 & 1 \end{array}$$

$\vec{u} \times \vec{v} = [4(-2) - (-1)(1), -1(5) - 3(-2), 3(1) - 4(5)]$

$= [-7, 1, -17]$

f) $\vec{u} = 2\hat{\imath} - 3\hat{\jmath} + 7\hat{k}$ $\quad \vec{v} = -\hat{\imath} + \hat{\jmath}$

$= [2, -3, 7] \qquad = [-1, 1, 0]$

$$\begin{array}{cc} -3 & 1 \\ 7 & 0 \\ 2 & -1 \\ -3 & 1 \end{array}$$

$\vec{u} \times \vec{v} = [-3(0) - 7(1), 7(-1) - 2(0), 2(1) - (-3)(-1)]$

$= [-7, -7, -1]$

2) Find a vector perpendicular to each of the following pairs of vectors. Use the dot product to check your answer.

a) [5, 0, 1] and [–2, 5, 8]

$$\begin{array}{cc} 0 & 5 \\ 1 & 8 \\ 5 & -2 \\ 0 & 5 \end{array}$$

$[0(8) - 1(5), 1(-2) - 5(8), 5(5) - 0(-2)]$

$= [-5, -42, 25]$

$[-5,-42,25] \cdot [5,0,1] = -25 + 0 + 25 = 0$

$[-5,-42,25] \cdot [-2,5,8] = 10 - 210 + 200 = 0$

$\therefore [-5,-42,25]$ is orthogonal to both.

b) [1, 4, –2] and [–4, 9, 0]

$$\begin{array}{cc} 4 & 9 \\ -2 & 0 \\ 1 & -4 \\ 4 & 9 \end{array}$$

$[4(0) - (-2)(9), -2(-4) - 1(0), 1(9) - 4(-4)]$

$= [18, 8, 25]$

$[18,8,25] \cdot [1,4,-2] = 18 + 32 - 50 = 0$

$[18,8,25] \cdot [-4,9,0] = -72 + 72 + 0 = 0$

$\therefore [18,8,25]$ is orthogonal to both.

3) Find a unit vector perpendicular to $\vec{a}$= [6, –2, –3] and $\vec{b}$ = [5, 1, –4].

$\vec{a} \times \vec{b}$

$$\begin{array}{cc} -2 & 1 \\ -3 & -4 \\ 6 & 5 \\ -2 & 1 \end{array}$$

$\vec{a} \times \vec{b} = [-2(-4) - (-3)(1), -3(5) - 6(-4), 6(1) - (-2)(5)]$

$= [11, 9, 16]$

$|\vec{a} \times \vec{b}| = \sqrt{458}$

unit vector $= \dfrac{1}{|\vec{a} \times \vec{b}|} [\vec{a} \times \vec{b}]$

$= \dfrac{1}{\sqrt{458}} [11, 9, 16]$

$= \left[\dfrac{11}{\sqrt{458}}, \dfrac{9}{\sqrt{458}}, \dfrac{16}{\sqrt{458}}\right]$

4) Given $\vec{a}$ = [1, –2, –1], $\vec{b}$ = [2, 2, –1] and $\vec{c}$ = [2, –3, –4], evaluate each of the following:

a) $\vec{a} \times (\vec{b} \times \vec{c})$

$\vec{b} \times \vec{c}$

$$\begin{array}{cc} 2 & -3 \\ -1 & -4 \\ 2 & 2 \\ 2 & -3 \end{array}$$

$\vec{a} \times (\vec{b} \times \vec{c})$

$$\begin{array}{cc} -2 & 6 \\ -1 & -10 \\ 1 & -11 \\ -2 & 6 \end{array}$$

$\vec{b} \times \vec{c} = [2(-4) - (-1)(-3), -1(2) - 2(-4), 2(-3) - 2(2)]$

$= [-11, 6, -10]$

$\vec{a} \times (\vec{b} \times \vec{c}) = [-2(-10) - (-1)(6), -1(-11) - (1)(-10), 1(6) - (-2)(-11)]$

$= [26\ 21, -16]$

b) $(\vec{a} \times \vec{b}) \times \vec{c}$

$\vec{a} \times \vec{b}$

$$\begin{array}{cc} -2 & 2 \\ -1 & -1 \\ 1 & 2 \\ -2 & 2 \end{array}$$

$\vec{a} \times \vec{b} \quad \times \quad \vec{c}$

$$\begin{array}{cc} -1 & -3 \\ 6 & -4 \\ 4 & 2 \\ -1 & -3 \end{array}$$

$\vec{a} \times \vec{b} = [-2(-1) - (-1)(2), -1(2) - 1(-1), 1(2) - (-2)(2)]$

$= [4, -1, 6]$

$(\vec{a} \times \vec{b}) \times \vec{c}$

$= [-1(-4) - 6(-3), 6(2) - 4(-4), 4(-3) - (-1)(2)]$

$= [22, 28, -10]$

$\vec{a} = [1,-2,-1]$ $\vec{b} = [2,2,-1]$ $\vec{c} = [2,-3,-4]$

c) $\vec{a} \times \vec{c} - \vec{a} \times \vec{b}$

$\vec{a} \times \vec{c}$

$$\begin{array}{cc} -2 & -3 \\ -1 & -4 \\ 1 & 2 \\ -2 & -3 \end{array}$$

$\vec{a} \times \vec{c} = [-2(-4) - (-1)(-3), (-1)(2) - 1(-4), 1(-3) - 2(-2)]$

$= [5, 2, 1]$

$\vec{a} \times \vec{b} = [4, -1, 6]$

$(\vec{a} \times \vec{c}) - (\vec{a} \times \vec{b}) = [1, 3, -5]$

d) $\vec{b} \times 3\vec{c}$

$\vec{b} \times 3\vec{c}$

$$\begin{array}{cc} 2 & -9 \\ -1 & -12 \\ 2 & 6 \\ 2 & -9 \end{array}$$

$\vec{b} \times 3\vec{c} = [2(-12) - (-1)(-9), (-1)(6) - 2(-12), 2(-9) - 2(6)]$

$= [-33, 18, -30]$

e) $(\vec{a} \times \vec{c}) \cdot \vec{b}$

$\vec{a} \times \vec{c} = [5, 2, 1]$

$(\vec{a} \times \vec{c}) \cdot \vec{b} = [5,2,1] \cdot [2,2,-1]$

$= 5(2) + 2(2) + 1(-1)$

$= 13$

f) $(\vec{a} \times \vec{b}) \cdot \vec{c}$

$= [4, -1, 6] \cdot [2, -3, -4]$

$= 4(2) + (-1)(-3) + 6(-4)$

$= -13$

g) $|\vec{a} \times \vec{b}|$

$= \sqrt{(4)^2 + (-1)^2 + (6)^2}$

$= \sqrt{53}$

h) $|\vec{a} \times (\vec{b} - \vec{c})|$

$\vec{b} - \vec{c} = [2,2,-1] - [2,-3,-4]$

$= [0, 5, 3]$

$\vec{a} \times (\vec{b} - \vec{c})$

$$\begin{array}{cc} -2 & 5 \\ -1 & 3 \\ 1 & 0 \\ -2 & 5 \end{array}$$

$\vec{a} \times (\vec{b} - \vec{c}) = [-2(3) - (-1)(5), -1(0) - 1(3), 1(5) - (-2)(0)]$

$= [-1, -3, 5]$

$|\vec{a} \times (\vec{b} - \vec{c})| = \sqrt{35}$

5) Use the cross product to determine the angles between the vectors $\vec{a} = [2, 1, -3]$ and $\vec{b} = [5, -4, 3]$. Consider ambiguous case. Use dot product to confirm or use graphing software to inspect.

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} \qquad \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\begin{array}{cc} \vec{a} & \times \quad \vec{b} \\ 1 & -4 \\ -3 & 3 \\ 2 & 5 \\ 1 & -4 \end{array}$$

$$\sin\theta = \frac{\sqrt{691}}{(\sqrt{14})(\sqrt{50})} \qquad \cos\theta = \frac{2(5) + 1(-4) + (-3)(3)}{(\sqrt{14})(\sqrt{50})}$$

$$\vec{a} \times \vec{b} = [1(3) - (-3)(-4), -3(5) - 2(3), 2(-4) - 1(5)]$$

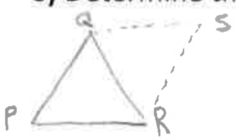
$$\vec{a} \times \vec{b} = [-9, -21, -13]$$

$$\theta_1 \simeq 83.5° \qquad \cos\theta = \frac{-3}{\sqrt{700}}$$

$$\boxed{\begin{aligned} \theta_2 &= 180 - \theta_1 \\ &= 96.5° \end{aligned}} \qquad \theta = 96.5°$$

verified using dot product.

6) Determine the area of ΔPQR with vertices of P(3, −2, 7), Q(2, 2, −3), and R(1, 1, 2).



$$\vec{PQ} = [-1, 4, -10] \qquad \vec{PR} = [-2, 3, -5]$$

$$\vec{PQ} \times \vec{PR} = [4(-5) - (-10)(3), -10(-2) - (-1)(-5), -1(3) - 4(-2)]$$

$$\begin{array}{cc} 4 & 3 \\ -10 & -5 \\ -1 & -2 \\ 4 & 3 \end{array} \qquad = [10, 15, 5]$$

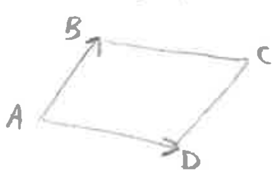
$$\text{Area of } \Delta PQR = \frac{1}{2}|\vec{PQ} \times \vec{PR}| = \frac{1}{2}\sqrt{350}$$

$$= \frac{1}{2}(5)\sqrt{14}$$

$$= 2.5\sqrt{14} \text{ units}^2$$

(2, −1, −1) (−4, −2, 3) (2, 3, 2)

7) Determine the area of the parallelogram ABCD defined by the vertices A(~~2, 1, 5~~), B(~~−2, 7, 8~~), C(~~1, 3, 8~~), and D(~~4, −3, 7~~). (8, 4, −2)



$$\vec{AB} = [-6, -1, 4] \qquad \vec{AD} = [6, 5, -1]$$

$$\vec{AB} \times \vec{AD} = [-1(-1) - 4(5), 4(6) - (-6)(-1), -6(5) - (-1)(6)]$$

$$\begin{array}{cc} -1 & 5 \\ 4 & -1 \\ -6 & 6 \\ -1 & 5 \end{array} \qquad = [-19, 18, -24]$$

$$\text{Area} = |\vec{AB} \times \vec{AD}| = \sqrt{1261} \text{ units}^2$$

ANSWER KEY:

1)a) $-3377.5\hat{n}$ or 3377.5 in to the page **b)** $-84.9\hat{n}$ or 84.9 in to the page **c)** [−10, 8, 4] **d)** [−41, 13, −25] **e)** [−7, 1, −17] **f)** [−7, −7, −1]

2)a) [−5, −42, 25] **b)** [18, 8, 25]

3) $\frac{1}{\sqrt{458}}$ [11, 9, 16]

4)a) [26, 21, −16] **b)** [22, 28, −10] **c)** [1, 3, −5] **d)** [−33, 18, −30] **e)** 13 **f)** −13 **g)** $\sqrt{53}$ **h)** $\sqrt{35}$

5) 96.5°

6) 2.5 $\sqrt{14}$ units²

7) ~~$\sqrt{1556}$ units²~~ $\sqrt{1261}$ units²

W6 – Applications of Dot and Cross Product Unit 5

MCV4U

Jensen

SOLUTIONS

1) Given $\vec{a}$ = [2, 4, –5], $\vec{b}$ = [–1, 3, 7], and $\vec{c}$ = [–2, 7, 3], evaluate each expression.

a) $\vec{a} \times \vec{b} \cdot \vec{c}$

$\vec{a} \times \vec{b} = [4(7) - (-5)(3), -5(-1) - 2(7), 2(3) - 4(-1)]$

$= [43, -9, 10]$

$\vec{a} \times \vec{b} \cdot \vec{c} = [43, -9, 10] \cdot [-2, 7, 3]$

$= 43(-2) + (-9)(7) + 10(3)$

$= -119$

b) $\vec{a} \times \vec{c} \cdot \vec{b}$

$\vec{a} \times \vec{c} = [4(3) - (-5)(7), -5(-2) - 2(3), 2(7) - 4(-2)]$

$= [47, 4, 22]$

$\vec{a} \times \vec{c} \cdot \vec{b} = [47, 4, 22] \cdot [-1, 3, 7]$

$= 47(-1) + 4(3) + 22(7)$

$= 119$

2) Determine the projection, and its magnitude of $\vec{u}$ on $\vec{v}$.

a) $\vec{u}$ = [2, 1, 7], $\vec{v}$ = [–7, 2, 6]

$proj_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}(\vec{v})$

$= \frac{2(-7) + 1(2) + 7(6)}{(-7)^2 + (2)^2 + (6)^2}[-7, 2, 6]$

$= \frac{30}{89}[-7, 2, 6]$

$|proj_{\vec{v}}\vec{u}| = \frac{30}{89}\sqrt{89} = \frac{30}{\sqrt{89}}$

b) $\vec{u} = 7\hat{\imath} - 6\hat{\jmath} + 5\hat{k},\ \vec{v} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$= [7, -6, 5]$ $= [3, -2, 1]$

$proj_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}(\vec{v})$

$= \frac{7(3) + (-6)(-2) + 5(1)}{(3)^2 + (-2)^2 + (1)^2}[3, -2, 1]$

$= \frac{38}{14}[3, -2, 1]$

$|proj_{\vec{v}}\vec{u}| = \frac{38}{14}\sqrt{14} = \frac{38}{\sqrt{14}}$

3) Determine the work done in the direction of travel.

a) $\vec{F}$ = [200, 150, 75], $\vec{s}$ = [2, –1, 8]

$W = \vec{F} \cdot \vec{s}$

$W = 200(2) + 150(-1) + 75(8)$

$W = 850$ J.

b) $\vec{F} = -3\hat{\imath} + 9\hat{\jmath} + 5\hat{k},\ \vec{s} = 2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$

$= [-3, 9, 5]$ $= [2, 5, 3]$

$W = \vec{F} \cdot \vec{s}$

$W = -3(2) + 9(5) + 5(3)$

$W = 54$ J.

4) Find the area of the parallelogram with sides consisting of the vectors.

a) $\vec{a} = [-4, 5, -8]$, $\vec{b} = [1, -2, 3]$

$\vec{a} \times \vec{b} = [5(3) - (-8)(-2), -8(1) - (-4)(3), -4(-2) - 5(1)]$

$= [-1, 4, 3]$

5 -2
-8 3
-4 1
5 -2

Area $= |\vec{a} \times \vec{b}| = \sqrt{26}$ units^2

b) $\vec{a} = [9, -5, 7]$, $\vec{b} = [3, -2, 5]$

$\vec{a} \times \vec{b} = [-5(5) - 7(-2), 7(3) - 9(5), 9(-2) - (-5)(3)]$

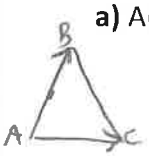
$= [-11, -24, -3]$

-5 -2
7 5
9 3
-5 -2

Area $= |\vec{a} \times \vec{b}| = \sqrt{706}$ units^2

5) Find the area of the triangle with the given vertices.

a) A(0, 2, 4), B(3, −2, 1), C(4, −2, 5)



$\vec{AB} = [3, -4, -3]$ $AC = [4, -4, 1]$

$\vec{AB} \times \vec{AC}$

-4 -4
-3 1
3 4
-4 -4

$\vec{AB} \times \vec{AC} = [-4(1) - (-3)(-4), -3(4) - 3(1), 3(-4) - (-4)(4)]$

$= [-16, -15, 4]$

Area $= \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{497}$ units^2

b) A(−2, 4, 5), B(1, 4, 2), C(7, 4, 9)

$\vec{AB} = [3, 0, -3]$ $\vec{AC} = [9, 0, 4]$

$\vec{AB} \times \vec{AC} = [0(4) - (-3)(0), -3(9) - 3(4), 3(0) - 0(9)]$

$= [0, -39, 0]$

0 0
-3 4
3 9
0 0

Area $= \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}(39)$

$= \frac{39}{2}$ units^2

6) Determine the volume of the parallelepiped determined by the vectors.

a) $\vec{a} = [2, 5, -8]$, $\vec{b} = [7, -2, 3]$, and $\vec{c} = [8, 2, -1]$

$\vec{a} \times \vec{b} = [5(3) - (-8)(-2), -8(7) - 2(3), 2(-2) - 5(7)]$

$= [-1, -62, -39]$

5 -2
-8 3
2 7
5 -2

Volume $= |\vec{a} \times \vec{b} \cdot \vec{c}|$

$= |[-1, -62, -39] \cdot [8, 2, -1]|$

$= |-1(8) + (-62)(2) + (-39)(-1)|$

$= |-93|$

$= 93$ units^3

b) $\vec{a} = [1, -5, 9]$, $\vec{b} = [3, 4, -7]$, and $\vec{c} = [1, 0, 2]$

$\vec{a} \times \vec{b} = [-5(-7) - 9(4), 9(3) - 1(-7), 1(4) - (-5)(3)]$

$= [-1, 34, 19]$

-5 4
9 -7
1 3
-5 4

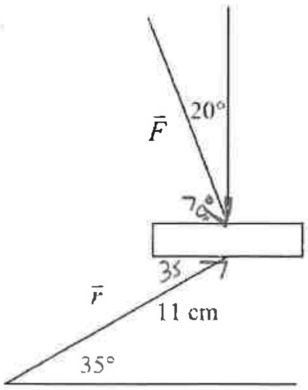
Volume $= |\vec{a} \times \vec{b} \cdot \vec{c}|$

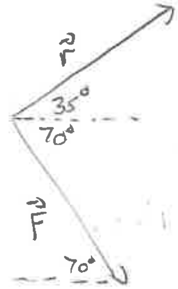
$= |[-1, 34, 19] \cdot [1, 0, 2]|$

$= |-1(1) + 34(0) + 19(2)|$

$= 37$ units^3

7) Find the torque produced by a cyclist exerting a force of 85 N on the pedal in the position shown in the diagram, if the shaft of the petal is 11 cm long.





$T = \vec{r} \times \vec{F}$

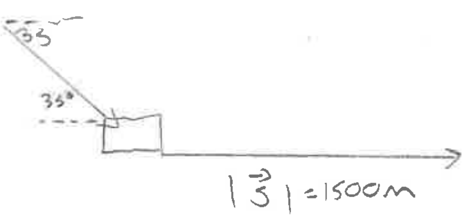
$T = [|\vec{r}||\vec{F}| \sin\theta](-\hat{n})$

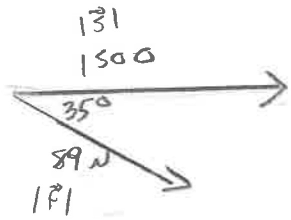
$T = [0.11(85) \sin 105](-\hat{n})$

$T = -9.03\hat{n}$ N·m

9.03 N·m in. to the bike.

8) A woman pushes her baby stroller a distance of 1500 m by a force of 89 N applied at an angle of 35° to the roadway. Calculate the work done.



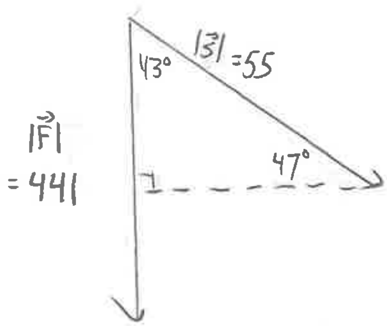


$W = \vec{F} \cdot \vec{s}$

$W = 89(1500)\cos(35)$

$W \simeq 109\,356.8$ N·m

9) Determine the work done by gravity in causing a 45 kg child to slide down a 55 m slope, which has an angle of 47° to the horizontal.



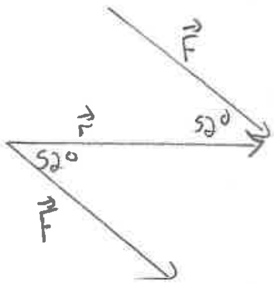
$W = \vec{F} \cdot \vec{s}$

$W = 441(55)\cos(43)$

$W = 17738.98$ N·m

10) A force of 75 N is applied to a wrench in a clockwise direction at 52° to the handle, 17 cm from the centre of the bolt.

a) Calculate the magnitude of the torque.



$$|\vec{T}| = \vec{r} \times \vec{F}$$

$$|\vec{T}| = |\vec{r}||\vec{F}|\sin\theta$$

$$|\vec{T}| = 0.17(75)\sin(52)$$

$$|\vec{T}| = 10.05 \text{ N}\cdot\text{m}$$

b) In what direction does the bolt move?

In to the material.

ANSWER KEY:

1. a) -119 **b)** 119

2. a) $\frac{30}{89}[-7, 2, 6]$; $\frac{30}{\sqrt{89}}$ **b)** $\frac{38}{14}[3, -2, 1]$; $\frac{38}{\sqrt{14}}$

3. a) 850 J **b)** 54 J

4. a) $\sqrt{26}$ units^2 **b)** $\sqrt{706}$ units^2

5. a) $\frac{\sqrt{447}}{2}$ units^2 **b)** $\frac{39}{2}$ units^2

6. a) 93 units^3 **b)** 37 units^3

7. 9.03 N•m

8. 109 356.8 J

9. 17 738.98 J

10. a) 10.05 N•m **b)** The bolt is being tightened into the material